объединенный
институт
ядерных
исследований
дубна

E4-88-229

M.Trajdos

A NEW ISOSPIN INVARIANT FORM OF THE INTERACTING BOSON MODEL

Submitted to "Journal of Physics G"

1988

I. Introduction

In sufficiently light nuclei when the valence neutrons and protons occupy the same shell-model levels the competition between two-nucleon interactions in $J=0,2,\ldots 2j-1$, $T=1$ and $J=1,3,\ldots 2j$, $T=0$ states leads to dynamical many-nucleon clusters. Among them two-nucleon and α -clusters are often discussed in different experimental and theoretical papers, e.g. [1-3, 4 and references therein]. For even isotopes between berilium and sulfur the correlation energy [1] defined by

$$E^{cor}_{\delta A_1+\delta A_2}(A,Z) = S_{\delta A_1}(A,Z) + S_{\delta A_2}(A,Z) - S_{\delta A_1+\delta A_2}(A,Z) ,$$

where $S_{\delta A}$ is the separation energy of a cluster δA from (A,Z) nucleus, is on average 2-3 times greater for neutron-proton pairs than for pairs of the same type of nucleons [3]. This effect arises because the interactions in the $T=0$ channel are stronger as compared with the channel $T=1$ $(G_{T=0}/G_{T=1} \simeq 5/3)$. It can be traced even for heavier nuclei. Table I of the paper [5] shows that for $^{42}_{21}Sc$ all effective two-nucleon interactions in the $T=0$ states are stronger than in the $J=2$, $T=1$ state and those with $J=1$ and $J=7$ are almost as strong as in $J=0$. So, if only s and d bosons represent the nucleon pairs in light nuclei the main part of the two-nucleon interactions ($T=0$ channel) will be omitted.

The additional bosons were taken into account in many papers [6–10 and others] . If we build an interacting boson model based on bosons $b_{\lambda\mu}$ with $\lambda = 0,1,2,\ldots r$, $-\lambda \le \mu \le \lambda$, the general Hamiltonian of such a system has the $U(n)$, $n=(r+1)^2$ group structure. The main advantage of the IBM 1 [11]–a relative simplicity of calculations obtained at the price of the Pauli principle – is lost. Instead, substantial difficulties appear in a construction of a physical basis and in a calculation of one-boson matrix elements for large λ . Those inconveniences can be overcome by a choice of some special two-boson interactions which lead to the so-called "dynamical symmetries" [11,12]. In these cases the Hamiltonian can be written in terms of only Casimir operators of a complete group chain. Now the eigenvalues of H can be calculated without the recourse to numerical studies but to obtain other properties of the system we must know the matrix elements of the appropriate operators .

In order to take into consideration two-nucleon interactions in the T=0 channel and recover simplicity of calculations, the model was proposed [3] with the $U(6)$ group structure following from the choice of bosons with the quantum members (LST)= (001) and (010). It can repwoduce 0^+, 1^+ and some $2^+, 3^+, \ldots$ states but all odd parity states and the electromagnetic E1 and M2 transitions are absent in the model. So, in this paper the model based on the bosons with quantum numbers $(J^\pi,T)=(0^+,1)$, $(1^+,0)$ and $(1^-,0)$ is proposed to search light nuclei (sect.2). Sect.3 contains the complete group theory analysis of the model and sect.4 provides the eigeneneriges of different limiting cases.

2. Hamiltonian of the interacting r-, s- and p-bosons

Let us to consider the bosons s, r, p with quantum numbers of total angular momentum J, parity π and isospin T as in the table

	J^π	T
s	0^+	1
r	1^+	0
p	1^-	0

The boson s corresponds to a pair of nucleons coupled by pairing forces, r describes deuteron - like correlations, so besides the orbital angular momentum L = 0 it includes a few percent admixture L = 2; boson p substitutes for a neutron--proton pair found on single particle shell model levels with $|l_1 - l_2| = 1$ or on one level l with L = 1, S = 0. These are "the pairs" only in the meaning of quantum numbers. Such initial bosons constitute a minimum set from which multi-boson states of positive and negative parity and all electromagnetic transition operators with multipolarity from 0 to 2 can be formed. Choice of two bosons of T = 0 makes it easier to consider stronger two nucleon interactions in the channel T = 0 as compared with the channel T = 1.
The most general Hamiltonian for a system of interacting r , s and p bosons is:

$$\begin{aligned} H = {} & \varepsilon_s \hat{n}_s + \varepsilon_p \hat{n}_p + \varepsilon_r \hat{n}_r + \\ & + \varepsilon_1 [s^+ s^+]^{J=0,T=0} \cdot [\tilde{s}\tilde{s}]^{00} + \varepsilon_2 [s^+ s^+]^{02} \cdot [\tilde{s}\tilde{s}]^{02} \\ & + \varepsilon_3 [r^+ r^+]^{00} \cdot [\tilde{r}\tilde{r}]^{00} + \varepsilon_4 [r^+ r^+]^{20} \cdot [\tilde{r}\tilde{r}]^{20} + \\ & + \varepsilon_5 [p^+ p^+]^{00} \cdot [\tilde{p}\tilde{p}]^{00} + \varepsilon_6 [p^+ p^+]^{20} \cdot [\tilde{p}\tilde{p}]^{20} + \\ & + \varepsilon_7 \left([s^+ s^+]^{00} \cdot [\tilde{r}\tilde{r}]^{00} + h.c. \right) + \varepsilon_8 \left([s^+ s^+]^{00} \cdot [\tilde{p}\tilde{p}]^{00} + h.c. \right) + \\ & + \varepsilon_9 \left([r^+ r^+]^{00} \cdot [\tilde{p}\tilde{p}]^{00} + h.c. \right) + \varepsilon_{10} \left([r^+ r^+]^{20} \cdot [\tilde{p}\tilde{p}]^{20} + h.c. \right) + \\ & + \varepsilon_{11} [p^+ s^+]^{11} \cdot [\tilde{s}\tilde{p}]^{11} + \varepsilon_{12} [s^+ r^+]^{11} \cdot [\tilde{r}\tilde{s}]^{11} + \\ & + \varepsilon_{13} [r^+ p^+]^{10} \cdot [\tilde{p}\tilde{r}]^{10} + \varepsilon_{14} [r^+ p^+]^{00} \cdot [\tilde{p}\tilde{r}]^{00} + \varepsilon_{15} [r^+ p^+]^{20} \cdot [\tilde{p}\tilde{r}]^{20}, \end{aligned} \tag{1}$$

where b^+_μ (b_μ), $b \equiv$ s, r or p, is the b-boson creation (annihilation) operator with $\mu = 0, \pm 1$ for the z-component of $K \equiv J_1$, J_2 or $\tilde{b}_\mu = (-)^{K+\mu} b_{-\mu}$; square brackets denote coupling and $T^k \cdot T^k = (-)^k \sqrt{2k+1}\, [T^k T^k]^0$.

Hamiltonian (I) is invariant under the total angular momentum and isospin rotations independently and conserves the total number of bosons $N = n_s + n_p + n_r$. It means that a core nucleus which will be taken into consideration is enough hard to neglect its excitations or that a nucleus is enough light to treated as a system of N=A/2 bosons. This Hamiltonian has U(9) group structure because it can be rewritten in its generators

$$\begin{array}{lll} [s^+\tilde{s}]^{J=0;T=0,1,2}_{\lambda} & [s^+\tilde{r}]^{J=1;T=1}_{\mu\ \lambda} & [r^+\tilde{s}]^{J=1;T=1}_{\mu\ \lambda} \\ {[r^+\tilde{r}]}^{J=0,1,2;T=0}_{\mu} & [s^+\tilde{p}]^{J=1;T=1}_{\mu\ \lambda} & [p^+\tilde{s}]^{J=1,T=1}_{\mu\ \lambda} \\ {[p^+\tilde{p}]}^{J=0,1,2;T=0}_{\mu} & [r^+\tilde{p}]^{J=0,1,2;T=0}_{\mu} & [p^+\tilde{r}]^{J=0,1,2;T=0}_{\mu} . \end{array} \qquad (2)$$

However, it is convenient for the sake of calculations to re-write Hamiltonian in Casimir operators of appropriate subgroups of U(9) group.

3. Subgroups and bases

There are four complete chains of subgroups (and sub-algebras) of U(9) group which end up with the direct product of rotational isospin and total angular momentum algebras $SO_T(3) \times SO_J(3)$. One of them is

$$U(9) \supset U_{n_p}(3) \times U_{n_r}(3) \times U_{n_s}(3) \supset SO_{J_1}(3) \times SO_{J_2}(3) \times SO_T(3) \supset$$
$$\supset SO_J(3) \times SO_T(3) \qquad (3)$$

and another has the form:

$$U(9) \supset SO(9) \supset SO(6) \times SO(3) \supset SO_{J_1}(3) \times SO_{J_2}(3) \times SO_T(3) \supset$$
$$\supset SO_J(3) \times SO_T(3). \qquad (4)$$

The group chain (4) can be realized in 3 ways because in the $SO(6) \times SO(3)$ group the generators of SO(3) may be chosen as $\sqrt{2}\,[p^+\tilde{p}]^1_\mu$ or $\sqrt{2}[r^+\tilde{r}]^1_\mu$ or $\sqrt{2}[s^+\tilde{s}]^1_\lambda$. The first case is taken into further considerations. The generators of the appropriate SO(6) group and other subgroups mentioned in (3) and (4) are shown in Table 1.

Table 1. The subgroups of U(9) group.

Group	Generators
SO(9)	$\sqrt{2}[s^+\tilde{s}]^{01}_{\lambda}$, $\sqrt{2}\lvert r^+\tilde{r}\rvert^{10}_{\mu}$, $\sqrt{2}\lvert p^+\tilde{p}\rvert^{10}_{\mu}$ $[s^+\tilde{r} - r^+\tilde{s}]^{11}_{\mu\lambda}$, $[r^+\tilde{p} - p^+\tilde{r}]^{J=0,1,2,T=0}_{\mu}$, $[p^+\tilde{s} - s^+\tilde{p}]^{M}_{\mu\lambda}$
SO(6)	$\sqrt{2}[s^+\tilde{s}]^{01}_{\lambda}$, $\sqrt{2}[r^+\tilde{r}]^{10}_{\mu}$, $[r^+\tilde{s} - s^+\tilde{r}]^{11}_{\mu\lambda}$
$U_{n_p}(3)$	$[p^+\tilde{p}]^{J=0,1,2,T=0}_{\mu}$
$U_{n_r}(3)$	$[r^+\tilde{r}]^{J=0,1,2,T=0}_{\mu}$
$U_{n_s}(3)$	$[s^+\tilde{s}]^{J=0,T=0,1,2}_{\lambda}$
$SO_{J_1}(3)$	$\sqrt{2}[p^+\tilde{p}]^{10}_{\mu}$
$SO_{J_2}(3)$	$\sqrt{2}\,[r^+\tilde{r}]^{10}_{\mu}$
$SO_J(3)$	$\sqrt{2}\left([p^+\tilde{p}]^{10}_{\mu} + [r^+\tilde{r}]^{10}_{\mu}\right)$
$SO_T(3)$	$\sqrt{2}\,[s^+\tilde{s}]^{01}_{\lambda}$

Having determined the group chains we are now in a position to construct the corresponding bases.

We start from the totally symmetric U(9) irreducible representation (ir) [N] so we obtain N - boson state uniquely specified by quantum numbers labelled, in this case, only by totally symmetric irs of the groups in chain (3) or (4). The basis resulting from (3) is

$$|[N]\ n_r n_p J_1 J_2 J M T M_T\rangle , \tag{5}$$

where: $n = N, N-1, \ldots 0, \quad n \equiv n_r, n_s$ or n_p

$N = n_r + n_p + n_s$

$J_1 = n_p, n_p-2, \ldots 1$ or 0

$J_2 = n_r, n_r-2, \ldots 1$ or 0

$T = n_s, n_s-2, \ldots 1$ or 0

$|J_1-J_2| \leqslant J \leqslant J_1 + J_2$

$M = \pm J, \pm(J-1), \ldots 0, \quad M_T = \pm T, \pm(T-1), \ldots 0.$

It can be constructed in a relatively simple way. As matrix elements of the Hamiltonian and other physical operators can be written in terms of the single boson reduced matrix elements it is enough to obtain a part of basis (6), for example the following states:

$$|[N] n_r n_p J_1 J_2 J J_1 - J_2 T T\rangle = \left\{ [n_p J_1 J_1\rangle\ [n_r J_2 - J_2\rangle \right\}_{J, J_1-J_2} |N-n_p-n_r T T\rangle , \tag{6}$$

where

$$|n_p J_1 J_1\rangle = \left[3^{\frac{n_p-J_1}{2}} \frac{(2J_1+1)!!}{J_1!(n_p+J_1+1)!!(n_p-J_1)!!} \right]^{1/2} \left([p^+p^+]^0\right)^{\frac{n_p-J_1}{2}} (p_1^+)^{J_1} |0\rangle$$

In the same way $|n_r J_2 - J_2\rangle$ and $|n_s T T\rangle$ are read.

$\{\ \}_{JJ_1-J_2}$ means coupling to the total angular momentum J and its projection $M = J_1 - J_2$. Table 2 shows the complete set of one boson reduced matrix elements in (6) and

$$\langle n+1, K' || b^+ || nK\rangle \equiv \sqrt{2K'+1}\ \frac{\langle n+1, K' M+\mu | b_\mu^+ | nKM\rangle}{(KM;\ 1\mu | K'\ M+\mu)} , \tag{7}$$

where:
$b_\mu^+ \equiv r_\mu^+, p_\mu^+, s_\mu^+;$

$K \equiv J_1, J_2, T$

$(KM;\ 1\mu|K' M+\mu)$ - Clebsch-Gordan coefficient of SO(3).

Table 2. The complete set of one boson reduced matrix elements.

$\langle n+1, K+1 \| b^+ \| nK\rangle$	=	$[(K+1)(n+K+3)]^{1/2}$
$\langle n+1, K-1 \| b^+ \| nK\rangle$	=	$[K(n-K+2)]^{1/2}$
$\langle n-1, K+1 \| \tilde{b} \| nK\rangle$	=	$[(K+1)(n-K)]^{1/2}$
$\langle n-1, K-1 \| \tilde{b} \| nK\rangle$	=	$[K(n+K+1)]^{1/2}$

The groups chain (4) provides another basis:

$$|[N] v \omega J_1 J_2 J M T M_T\rangle . \tag{8}$$

Here v labels SO(9) ir and it means the maximum number of bosons (r, s and p) not coupled in pairs with $J_1 = J_2 = T = 0$; ω means the same but only for r- and s- bosons and it signs SO(6) ir. Simultaneously ω determines parity π of any state and $\pi = (-)^{\omega}$ for N even and $\pi = (-)^{\omega+1}$ for N odd. The quantum numbers in (8) fulfil the conditions:

$$
\begin{aligned}
v &= N,\ N-2, \ldots 1 \text{ or } 0 \\
\omega &= v,\ v-1, \ldots 0 \\
J_1 &= v-\omega,\ v-\omega-2, \ldots 1 \text{ or } 0 \\
J_2 &= \omega,\ \omega-1, \ldots 0 \\
T &= \omega - S,\ \omega - S - 2, \ldots 0.
\end{aligned}
\tag{9}
$$

Table 3. Classification scheme for the group chain (4) ($N = 4$).

v	ω	T	J_1	J_2	(J^π) multiplicity
4	4	0	0	0,2,4	$0^+,\ 2^+,\ 4^+$
		1	0	1, 3	$1^+,\ 3^+$
		2	0	0, 2	$0^+,\ 2^+$
		3	0	1	1^+
		4	0	0	0^+
4	3	0	1	1, 3	$0^-,\ 1^-,\ (2^-)^2,\ 3^-,\ 4^-$
		1	1	0, 2	$(1^-)^2,\ 2^-,\ 3^-$
		2	1	1	$0^-,\ 1^-,\ 2^-$
		3	1	0	1^-
4	2	0	0, 2	0, 2	$(0^+)^2,\ 1^+,\ (2^+)^3,\ 3^+,\ 4^+$
		1	0, 2	1	$(1^+)^2,\ 2^+,\ 3^+$
		2	0, 2	0	$0^+,\ 2^+$
4	1	0	1, 3	1	$0^+,\ 1^-,\ (2^-)^2,\ 3^-,\ 4^-$
		1	1, 3	0	$1^-,\ 3^-$
4	0	0	0,2,4	0	$0^+,\ 2^+,\ 4^+$
2	2	0	0	0, 2	$0^+,\ 2^+$
		1	0	1	1^+
		2	0	0	0^+
2	1	0	1	1	$0^-,\ 1^-,\ 2^-$
		1	1	0	1^-
2	0	0	0, 2	0	$0^+,\ 2^+$
0	0	0	0	0	0^+

Rules (9) allow for constructing Table 3 in which, as an example, the total number of bosons $N = 4$ was taken into account. States (8) can be obtained by diagonalisation in basis (5) of the second order Casimir operators of SO(9) and SO(6) groups. The relevant Casimir operators and their eigenvalues are given in Table 4.

Table 4. Casimir operators and their eigenvalues of the groups in (3) and (4).

$b^+_\mu \equiv s^+_\mu,\ r^+_\mu,\ p^+_\mu;\quad \hat{n} \equiv \hat{n}_s,\ \hat{n}_r,\ \hat{n}_p;\ K \equiv J_1,\ J_2,\ T.$

Group	Casimir operator	Eigen- value
SO(3)	$C_2 = 2\hat{n} - 2[b^+b^+]^0[\tilde{b}\tilde{b}]^0 + [b^+b^+]^2 \cdot [\tilde{b}\tilde{b}]^2$ $= 2[b^+\tilde{b}]^1 \cdot [b^+\tilde{b}]^1$	$K(K+1)$
SU(3)	$9C_2 = 4\hat{n} + [b^+b^+]^0[\tilde{b}\tilde{b}]^0 + [b^+b^+]^2 \cdot [\tilde{b}\tilde{b}]^2$ $= \hat{n}(\hat{n}+3)$	$n(n+3)$
SO(6)	$8C_2 = 3(\hat{n}_s + \hat{n}_r) + \hat{T}^2 + \hat{J}_2^2 + 2\hat{n}_s\hat{n}_r +$ $- 3\left([r^+r^+]^{00}[\tilde{s}\tilde{s}]^{00} + \text{h.c.}\right)$	$\omega(\omega+4)$
SO(9)	$14C_2 = 6\hat{N} + \hat{J}_1^2 + \hat{J}_2^2 + \hat{T}^2 + 2(\hat{n}_r\hat{n}_s + \hat{n}_s\hat{n}_p + \hat{n}_p\hat{n}_r) +$ $- 3\left\{\left([r^+r^+]^{00}[\tilde{s}\tilde{s}]^{00} + [s^+s^+]^{00}[\tilde{p}\tilde{p}]^{00} + [p^+p^+]^{00}[\tilde{r}\tilde{r}]^{00}\right) + \text{h.c.}\right\}$	$v(v+7)$
$SO_J(3)$	$C_2 = \hat{J}_1^2 + \hat{J}_2^2 + 2\hat{J}_1 \cdot \hat{J}_2 =$ $= \hat{J}_1^2 + \hat{J}_2^2 + 2[r^+p^+]^{20} \cdot [\tilde{r}\tilde{p}]^{20} + 2[r^+p^+]^{10} \cdot [\tilde{r}\tilde{p}]^{10} +$ $- 4[r^+p^+]^{00} \cdot [\tilde{r}\tilde{p}]^{00}$	$J(J+1)$

4. Dynamical symmetries

Dynamical symmetries arise when the Hamiltonian of the interacting boson system (or femions) can be written in terms of only Casimir invariants of a group complete chain. Thus, there are some special cases of the general Hamiltonian but of the main advantage that eigenvalues of

$$H' = \sum_i k_i \hat{C}_i \qquad (10)$$

and all other properties of the system can be obtained without any numerical calculations. The parameters k_i in expression (10) are functions of the boson strengths ε_i.

Now a few closed formulae for energies in various limiting cases are presented.

1. The most simplified version is obtained by choosing monopol interaction and degeneracy of one boson energies.

   For parameters

$$\varepsilon_s = \varepsilon_p = \varepsilon_r \equiv \varepsilon_o$$

$$\varepsilon_1 = \varepsilon_3 = \varepsilon_5 = \varepsilon_7 = \varepsilon_8 = \varepsilon_9 \equiv \varepsilon_M, \text{ (the remaining } \varepsilon_i = 0)$$

   and

$$M^+ \equiv [s^+s^+]^{00} + [r^+r^+]^0 + [p^+p^+]^0$$

   Hamiltonian

$$H_1 = \varepsilon_o\hat{N} + \varepsilon_M M^+\tilde{M}$$

$$= \varepsilon_o\hat{N} + {}^{\varepsilon_M}/_3\,[\hat{N}^2 + 7\hat{N} - 14C_2(O(9))]$$

   is diagonal in the basis $|Nv\omega J_1J_2JMTM_T\rangle$ with the eigenvalues

$$E_1 = \varepsilon_o N + {}^{\varepsilon_M}/_3\,[N(N+7) - v(v+7)].$$

2. Another choice of parameters

$$\varepsilon_s = \varepsilon_p = \varepsilon_r \equiv \varepsilon_o$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 \equiv \varepsilon$$

$$\varepsilon_7 = \varepsilon_8 = \varepsilon_9 = 3\varepsilon$$

$$\varepsilon_{11} = \varepsilon_{15} = 2\varepsilon, \quad \varepsilon_{14} = -4\varepsilon$$

   provides

$$H_2 = \varepsilon_o\hat{N} + \varepsilon\,[\hat{N}^2 + 5\hat{N} - 14C_2(O(9)) + \hat{J}^2 + \hat{T}^2]$$

   and

$$E_2 = \varepsilon_o N + \varepsilon\,[N(N+5) - v(v+7) + J(J+1) + T(T+1)].$$

3. A similar Hamiltonian is obtained with the parameters

$$\varepsilon_s = \varepsilon_p = \varepsilon_r \equiv \varepsilon_o$$

$$\varepsilon_1 = \varepsilon_3 = \varepsilon_5 \equiv -\varepsilon$$

$$\varepsilon_2 = \varepsilon_4 = \varepsilon_6 = \varepsilon_7 = \varepsilon_8 = \varepsilon_9 \equiv \varepsilon$$

$$\varepsilon_{13} = \varepsilon_{15} = \tfrac{2}{3}\varepsilon, \quad \varepsilon_{14} = -{}^4/_3\varepsilon$$

$$H_3 = \varepsilon_o\hat{N} + {}^{\varepsilon}/_3\,[\hat{N}^2 + \hat{N} - 14C_2(O(9)) + \hat{J}^2 + \hat{T}^2].$$

4. In the basis $|Nn_rn_pJ_1J_2JMTM_T\rangle$

$$H_4 = \varepsilon_s\hat{n}_s + \varepsilon_p\hat{n}_p + \varepsilon_r\hat{n}_r + {}^{\varepsilon_Q}/_3\,\{Q_1^+\cdot\tilde{Q}_1 + Q_2^+\cdot\tilde{Q}_2 + Q_T^+\cdot\tilde{Q}_T +$$

$$+ 2[r^+p^+]^{10}\cdot[\tilde{p}\tilde{r}]^{10} + 2[r^+p^+]^{20}\cdot[\tilde{p}\tilde{r}]^{20} - 4[r^+p^+]^{00}\cdot[\tilde{p}\tilde{r}]^{00}\}$$

   where: $Q_1^+ \equiv [r^+r^+]^{20}$; $Q_2^+ \equiv [p^+p^+]^{20}$; $Q_T^+ \equiv [s^+s^+]^{02}$

$$\text{and} \quad \varepsilon_2 = \varepsilon_4 = \varepsilon_6 = \tfrac{2}{3}\varepsilon_{13} = \tfrac{2}{3}\varepsilon_{15} = -\tfrac{4}{3}\varepsilon_{14} \equiv \varepsilon_Q$$

   is diagonal. Thus,

$$E_4 = \varepsilon_s n_s + \varepsilon_p n_p + \varepsilon_r n_r +$$

$$+ {}^{\varepsilon_Q}/_3\,\{2(n_s^2 + n_r^2 + n_p^2) - 4N + J(J+1) + T(T+1)\}.$$

5. The next version of the Hamiltonian, for the parameters

$$\varepsilon_1 = \varepsilon_3 = \varepsilon_5 \equiv -2\varepsilon$$

$$\varepsilon_2 = \varepsilon_4 = \varepsilon_6 = \varepsilon$$

$$\varepsilon_{13} = \varepsilon_{15} = 2\varepsilon , \quad \varepsilon_{14} = -4\varepsilon$$

contains a mixture of monopole and quadrupole interactions between b-bsons:

$$H_5 = \varepsilon_s \hat{n}_s + \varepsilon_p \hat{n}_p + \varepsilon_r \hat{n}_r + \varepsilon (-2\hat{N} + \hat{J}^2 + \hat{T}^2).$$

In this case

$$E_5 = \varepsilon_s n_s + \varepsilon_p n_p + \varepsilon_r n_r + \varepsilon \left[-2N + J(J+1) + T(T+1)\right].$$

Some more examples could be presented. The Hamiltonians $H_2 \div H_5$ contain the rotational energies in the total angular momentum and isospin spaces but an equal coefficient in both the terms can be untrue. This equality appears because the parameters ε_i are chosen so as to eliminate partial angular momenta $\mathcal{J}_1$ and $\mathcal{J}_2$.

More realistic situations arise after diagonalisation of the Hamiltonian (I) or its part in the basis (5). One of these possibilities is assumption $\varepsilon_p \to \infty$. Now low-lying states of a nucleus are built from s and r bosons and the Hamiltonian (I) reduces to

$$H_6 = H_0(\hat{N}, \hat{N}^2) + k_1 \hat{n}_r + k_2 \, 8C_2(SO(6)) + k_3 \, 9C_2(SU_{n_r}(3)) + k_4 \, 9C_2(SU_{n_s}(3)) + k_5 \hat{\mathcal{J}}_2^2 + k_6 \hat{T}^2.$$

In the last formulae only the operator $8C_2(SO(6))$ is not diagonal in (5). Its quasidiagonal matrix for N bosons has the largest submatrix of size $\frac{1}{2}(N+1)$ for N = odd and $\frac{1}{2}N+1$ for N = even, so any calculations are very simple. Figures 1 and 2 present the preliminary adaptation of the model to $^{26-28}_{13}$Al nuclei. The Hamiltonian H_6 was diagonalised in two cases: the levels signified by "a" refer to the boson number $N = \frac{1}{2}(A-16)$ and "b" stands for $N = \frac{1}{2}A$. The version "b" allows one to reproduce a greater number of excited levels but it occurs only for some nuclei far from the closed shells, e.g., $^{22-24}_{11}$Na, $^{26-28}_{13}$Al, $^{30-32}_{15}$P . Quite reasonable energetical values are obtained except for a few events with larger departures. They can arise from omission of p bosons (or with $L \gg 2$) or from using the same parameters for both $^{26-28}_{13}$Al nuclei.

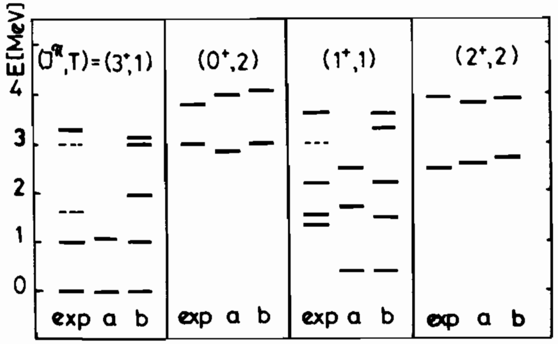

Fig.1. The experimental [13] and calculated spectrum of $^{28}_{13}$Al with H_0=0 and for case "a" ("b"): k_1=-1.934 (4.063) MeV, k_2= 0.038 (0.013) MeV, k_3=0.245 (-0.0063) MeV, k_4= 0.050 (0.202) MeV, k_5= -0.1 (-0.049) MeV, k_6= 0.521 (0.626) MeV.

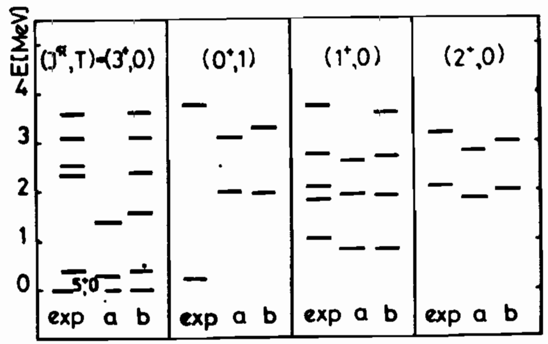

Fig.2. The experimental [13] and calculated spectrum of $^{26}_{13}$Al with the same parameters as in Fig.1.

A programme of numerical diagonalization of the general Hamiltonian (1) is being prepared. It will be used in order to find the strengths (ε_i in (I)) of the two-boson interactions and to check whether the nucleon-pairs, represented by r, s and p bosons, could be treated as building blocks [1] of $7 \leq Z \leq 16$ nuclei.

Acknowledgements

I am very grateful to Professor I.N.Mikhailov and Professor S.Szpikowski for helpful discussions.

References

1. Dussel G.G., Liotta R.I., Perazzo R.P.I. Nucl.Phys., 1982, A 388, 606.
2. Dunnweber W., Morinaga H., Alburger D. Phys.Lett., B, 1981, 106, p.47.
3. Trajdos M., Zajac L. J.Phys.G, 1988, in press.
4. Szpikowski S., Trajdos M. Nucl.Phys.A, 1976, 272, p.155.
5. Thompson M.J., Elliott J.P., Evans J.A. Phys.Lett.,1987, B195, p.511.
6. Engel J., Iachello F. Phys.Rev.Lett., 1985, 54, p.1126.
7. Gozdz A., Szpikowski S., Zajac K. Nuckeonika,1980, 25, p.1055.
8. Han C.S., Chun D.S., Hsieh St., Chiang H.C. Phys.Lett.B, 1985, 163, p.295.
9. Nadjakov E.G., Mikhailov I.N. J.Phys.G.,1987, 13, p.1221.
10. Bonatsos D., Klein A. Nucl.Phys.,1987, A 469, p.253.
11. Arima A., Iachello F. Ann.Phys., 1976, 99, p.253; 1978, Ann.Phys. 111, p.201; Ann.Phys., 1979, 123, p.468.
12. Qi-Zhi- Han, Hong-Zhou Sun, Quang-Hua Li. Phys.Rev.C, 1987, C35, p.786.
13. Endt P.M., Van der Leun C. Nucl.Phys.,1978, A310, p.1.

Received by Publishing Department
on April 8, 1988.

Трайдос М. E4-88-229
Новая изоспиново-инвариантная модель взаимодействующих бозонов

Предлагается простая версия модели взаимодействующих бозонов с групповой структурой U(9) для исследования четырехнуклонных корреляций, возникающих из-за конкуренции изовекторных и изоскалярных остаточных взаимодействий между нуклонами. Найдены групповые цепочки, базисы, однобозонные матричные элементы, операторы Казимира и собственные значения энергии в некоторых предельных случаях.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Trajdos M. E4-88-229
A New Isospin Invariant Form of the Interacting Boson Model

A simple version of the IBM with U(9) group structure is proposed to study the four-nucleon correlations induced by the interplay isovector and isoscalar components of the residual nuclear forces. Subgroup chains, bases, one-boson matrix elements, Casimir operators and eigenenergies of different limiting cases have been found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988