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**EFFECT
OF THE PARTICLE-PARTICLE
INTERACTION
ON THE $K^\pi = 2^+$ STATES
IN DEFORMED NUCLEI**

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Introduction

Effective interactions between quasiparticles in complex nuclei can be classified as particle-hole and particle-particle ones. Particle-hole interactions are responsible for the formation of collective vibrational low-lying states and giant resonances ^{/1/}. An important role in nuclei is played by interactions leading to superconducting pairing correlations. It is of interest to elucidate the effect of particle-particle interactions on low-lying vibrational states in deformed nuclei.

The effect of particle-particle interactions on the first 2_1^+ and 3_1^- states in spherical nuclei has been studied in a number of papers, for example ref. ^{/2/}. Particle-particle interactions influence greatly the double β decay ^{/3/}, β^+ decays of neutron-deficient spherical nuclei and the strength functions of (n,p) transitions ^{/4/}.

The present paper is aimed at studying in the random phase approximation (RPA) the effect of particle-particle interactions on low-lying states with $K^\pi = 2^+$ in doubly even deformed nuclei.

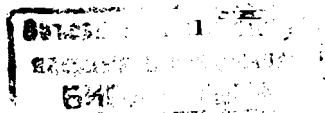
I. RPA with the particle-hole and particle-particle interaction

Apart from the particle-hole interaction and monopole pairing the quasiparticle-phonon nuclear model (QPNM) ^{/5-8/} takes into account the particle-particle interaction. In this case, the following terms

$$H^{PP} = -\frac{1}{2} \sum_{\lambda\mu\tau\sigma} G_{\tau}^{(\lambda\mu)} P_{\lambda\mu\sigma}^+(\tau) P_{\lambda\mu\sigma}(\tau), \quad (1)$$

$$P_{\lambda\mu\sigma}^+(\tau) = \sum_{q_1 q_2 \sigma_1 \sigma_2} \langle q_1 \sigma_1 \frac{dV(r)}{dr} Y_{\lambda\sigma\mu}(q_1, q_2) | q_2 \sigma_2 \rangle a_{q_1 \sigma_1}^+ a_{q_2 \sigma_2}^+ \sigma. \quad (2)$$

are added to the QPNM Hamiltonian. Here $G_{\tau}^{(\lambda\mu)}$ is the particle-particle interaction constant, $q\sigma$ are quantum particle of single-particle states, $\sigma = \pm 1$, $\tau = p$ and n , $a_{q\sigma}^+$ is the nucleon creation operator; the matrix element in (2) is denoted by $f^{\lambda\mu}(q_1, q_2)$. Then, we perform the Bogolubov canonical transformation



$$Q_{q\sigma} = u_q \alpha_{q\sigma} + \sigma v_q \alpha_{q-\sigma}^+$$

introduce the phonon creation operator

$$Q_{\lambda\mu i\sigma}^+ = \frac{1}{2} \sum_{qq'} \{ \psi_{qq'}^{\lambda\mu i} A^+(qq'; \mu\sigma) - \varphi_{qq'}^{\lambda\mu i} A(qq'; \mu-\sigma) \} \quad (3)$$

and rewrite (2) in the form

$$P_{\lambda\mu\sigma}^+(t) = \frac{1}{2} \sum_i \{ D_{g\tau}^{\lambda\mu i} (Q_{\lambda\mu i\sigma}^+ + Q_{\lambda\mu i-\sigma}^+) + D_{w\tau}^{\lambda\mu i} (Q_{\lambda\mu i\sigma}^+ - Q_{\lambda\mu i-\sigma}^+) \} - \sum_{qq'} f^{\lambda\mu}(qq') (u_{qq'}^{(+)} - u_{qq'}^{(-)}) B(q'q; \mu\sigma), \quad (2)$$

where

$$D_{g\tau}^{\lambda\mu i} = \sum_{qq'} f^{\lambda\mu}(qq') v_{qq'}^{(-)} g_{qq'}^{\lambda\mu i},$$

$$D_{w\tau}^{\lambda\mu i} = \sum_{qq'} f^{\lambda\mu}(qq') v_{qq'}^{(+)} w_{qq'}^{\lambda\mu i},$$

$$g_{qq'}^{\lambda\mu i} = \psi_{qq'}^{\lambda\mu i} + \varphi_{qq'}^{\lambda\mu i}, \quad w_{qq'}^{\lambda\mu i} = \psi_{qq'}^{\lambda\mu i} - \varphi_{qq'}^{\lambda\mu i},$$

$$u_{qq'}^{\pm} = u_q v_{q'} \pm v_q u_{q'}, \quad v_{qq'}^{\pm} = u_q u_{q'} \pm v_q v_{q'}.$$

the remaining notation is given in ref. /9/ .

After transformations the total QPM Hamiltonian for the multipole states with $K^\pi \neq 0^+$ becomes

$$H_H^P = \sum_q \varepsilon_q B(q) + H_{M\nu}^P + H_{M\nu q}^P, \quad (4)$$

$$H_{M\nu}^P = \sum_{\lambda\mu} \sum_{i i' \sigma} W_{ii'}^{\lambda\mu} Q_{\lambda\mu i\sigma}^+ Q_{\lambda\mu i'\sigma}, \quad (5)$$

$$W_{ii'}^{\lambda\mu} = \frac{1}{4} \sum_{\tau} \left\{ \sum_{p=\pm 1} (\alpha_0^{(\lambda\mu)} + p \alpha_1^{(\lambda\mu)}) D_{\tau}^{\lambda\mu i} D_{p\tau}^{\lambda\mu i'} + \right. \quad (5')$$

$$\left. + G_{\tau}^{(\lambda\mu)} [D_{g\tau}^{\lambda\mu i} D_{g\tau}^{\lambda\mu i'} + D_{w\tau}^{\lambda\mu i} D_{w\tau}^{\lambda\mu i'}] \right\}$$

$$H_{M\nu q}^P = - \sum_{\lambda\mu i\sigma} \sum_{\tau} \left\{ V_{\tau}^{\lambda\mu i}(qq') [(Q_{\lambda\mu i\sigma}^+ + Q_{\lambda\mu i-\sigma}^+) B(qq'; \mu-\sigma) + (Q_{\lambda\mu i\sigma}^+ + Q_{\lambda\mu i-\sigma}^+) B(q'q; \mu\sigma)] + V_{w\tau}^{\lambda\mu i}(qq') [(Q_{\lambda\mu i\sigma}^+ - Q_{\lambda\mu i-\sigma}^+) B(qq'; \mu-\sigma) + (Q_{\lambda\mu i\sigma}^+ - Q_{\lambda\mu i-\sigma}^+) B(q'q; \mu\sigma)] \right\}, \quad (6)$$

$$V_{\tau}^{\lambda\mu i}(qq') = \frac{1}{4} \sum_{p=\pm 1} (\alpha_0^{(\lambda\mu)} + p \alpha_1^{(\lambda\mu)}) D_{p\tau}^{\lambda\mu i} f^{\lambda\mu}(qq') U_{qq'}^{-} - \frac{1}{4} G_{\tau}^{(\lambda\mu)} f^{\lambda\mu}(qq') D_{g\tau}^{\lambda\mu i} (u_{qq'}^{(+)} - u_{qq'}^{(-)}),$$

$$V_{w\tau}^{\lambda\mu i} = \frac{1}{4} G_{\tau}^{(\lambda\mu)} f^{\lambda\mu}(qq') D_{w\tau}^{\lambda\mu i} (u_{qq'}^{(+)} - u_{qq'}^{(-)}),$$

where

$$D_{p\tau}^{\lambda\mu i} = \sum_{qq'} p^{\tau} f^{\lambda\mu}(qq') u_{qq'}^{(+)} g_{qq'}^{\lambda\mu i},$$

$\alpha_0^{(\lambda\mu)}$ and $\alpha_1^{(\lambda\mu)}$ are the isoscalar and isovector constants of the particle-hole multipole $\lambda\mu$ interaction and ε_q is the quasi-particle energy.

Now we derive equations in the RPA. For this purpose we calculate an average (4) over the one-phonon state

$$\langle Q_{\lambda\mu i\sigma} \left\{ \sum_q B(q) \varepsilon_q + H_{M\nu}^P \right\} Q_{\lambda\mu i\sigma}^+ \rangle = \frac{1}{4} \sum_{qq'} \varepsilon_{qq'} \{ (g_{qq'}^{\lambda\mu i})^2 + (w_{qq'}^{\lambda\mu i})^2 \} - \quad (7)$$

$$- \frac{1}{4} \sum_{\tau} \left\{ \sum_p (\alpha_0^{(\lambda\mu)} + p \alpha_1^{(\lambda\mu)}) D_{p\tau}^{\lambda\mu i} + G_{\tau}^{(\lambda\mu)} [(D_{g\tau}^{\lambda\mu i})^2 + (D_{w\tau}^{\lambda\mu i})^2] \right\}$$

and using the variational principle get the following equations

$$\varepsilon_{qq'} g_{qq'}^{\lambda\mu i} - \omega_{\lambda\mu i} w_{qq'}^{\lambda\mu i} - (\alpha_0^{(\lambda\mu)} + \alpha_1^{(\lambda\mu)}) f^{\lambda\mu}(qq') u_{qq'}^{(+)} D_{\tau}^{\lambda\mu i} - \quad (8)$$

$$- (\alpha_0^{(\lambda\mu)} - \alpha_1^{(\lambda\mu)}) f^{\lambda\mu}(qq') u_{qq'}^{(+)} D_{-T}^{\lambda\mu i} - G_{\tau}^{(\lambda\mu)} f^{\lambda\mu}(qq') U_{qq'}^{(-)} D_{g\tau}^{\lambda\mu i} = 0,$$

$$\varepsilon_{qq'} w_{qq'}^{\lambda\mu i} - \omega_{\lambda\mu i} g_{qq'}^{\lambda\mu i} - G_{\tau}^{(\lambda\mu)} f^{\lambda\mu}(qq') U_{qq'}^{(+)} D_{w\tau}^{\lambda\mu i} = 0. \quad (8')$$

where $\varepsilon_{qq'} = \varepsilon_q + \varepsilon_{q'}$ and $\omega_{\lambda\mu i}$ are the energies of one-phonon states. After transformations the secular equation for the energies $\omega_{\lambda\mu i}$ of one-phonon states is

$$\mathcal{F}(\omega_{\lambda\mu i}) = \begin{vmatrix} (\alpha_0^{(\lambda\mu)} + \alpha_1^{(\lambda\mu)})X(p)-1 & G_p^{(\lambda\mu)}X_{+-}(p) & G_p^{(\lambda\mu)}X_{++}(p) & (\alpha_0^{(\lambda\mu)} - \alpha_1^{(\lambda\mu)})X(p) & 0 & 0 \\ (\alpha_0^{(\lambda\mu)} + \alpha_1^{(\lambda\mu)})X_{+-}(p) & G_p^{(\lambda\mu)}X_{+-}(p)-1 & G_p^{(\lambda\mu)}X_{+-}(p) & (\alpha_0^{(\lambda\mu)} - \alpha_1^{(\lambda\mu)})X_{+-}(p) & 0 & 0 \\ (\alpha_0^{(\lambda\mu)} + \alpha_1^{(\lambda\mu)})X_{++}(p) & G_p^{(\lambda\mu)}X_{+-}(p) & G_p^{(\lambda\mu)}X_{+-}(p)-1 & (\alpha_0^{(\lambda\mu)} - \alpha_1^{(\lambda\mu)})X_{++}(p) & 0 & 0 \\ (\alpha_0^{(\lambda\mu)} - \alpha_1^{(\lambda\mu)})X(n) & 0 & 0 & (\alpha_0^{(\lambda\mu)} + \alpha_1^{(\lambda\mu)})X(n)-1 & G_n^{(\lambda\mu)}X_{+-}(n) & G_n^{(\lambda\mu)}X_{++}(n) \\ (\alpha_0^{(\lambda\mu)} - \alpha_1^{(\lambda\mu)})X_{+-}(n) & 0 & 0 & (\alpha_0^{(\lambda\mu)} + \alpha_1^{(\lambda\mu)})X_{+-}(n) & G_n^{(\lambda\mu)}X_{+-}(n)-1 & G_n^{(\lambda\mu)}X_{+-}(n) \\ (\alpha_0^{(\lambda\mu)} - \alpha_1^{(\lambda\mu)})X_{++}(n) & 0 & 0 & (\alpha_0^{(\lambda\mu)} + \alpha_1^{(\lambda\mu)})X_{++}(n) & G_n^{(\lambda\mu)}X_{+-}(n) & G_n^{(\lambda\mu)}X_{+-}(n)-1 \end{vmatrix} = 0, \quad (9)$$

where

$$X(\tau) = (1 + \delta_{\mu 0}) \sum_{qq'} \tau \frac{(f^{\lambda\mu}(qq'))^2 U_{qq'}^+ \varepsilon_{qq'}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{V\pm}(\tau) = (1 + \delta_{\mu 0}) \sum_{qq'} \tau \frac{(f^{\lambda\mu}(qq'))^2 U_{qq'}^{(\pm)} \varepsilon_{qq'}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{+-}(\tau) = (1 + \delta_{\mu 0}) \sum_{qq'} \tau \frac{(f^{\lambda\mu}(qq'))^2 U_{qq'}^{(+)} U_{qq'}^{(-)} \varepsilon_{qq'}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{++}(\tau) = (1 + \delta_{\mu 0}) \sum_{qq'} \tau \frac{(f^{\lambda\mu}(qq'))^2 U_{qq'}^{(+)} U_{qq'}^{(+)} \omega_{\lambda\mu i}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{V-}(\tau) = (1 + \delta_{\mu 0}) \sum_{qq'} \tau \frac{(f^{\lambda\mu}(qq'))^2 U_{qq'}^{(-)} U_{qq'}^{(+)} \omega_{\lambda\mu i}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2}.$$

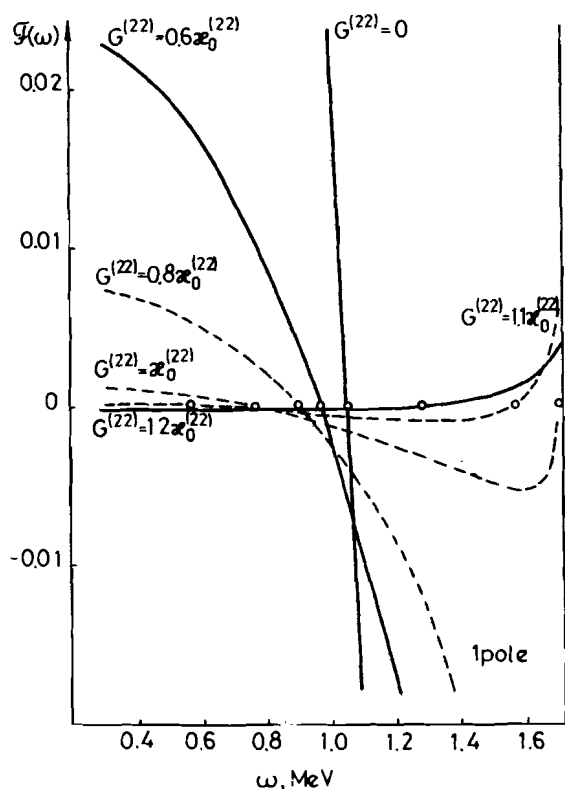
An explicit form of the functions $g_{qq'}^{\lambda\mu i}$, $w_{qq'}^{\lambda\mu i}$ or $\psi_{qq'}^{\lambda\mu i}$, $\varphi_{qq'}^{\lambda\mu i}$ can be found from eqs. (8) and (8') taking account of the normalisation condition of the one-phonon state $Q_{\lambda\mu i}^+ \Psi_0$.

2. Dependence of characteristics of $K^\pi = 2^+$ states on the particle-particle interaction constant $G^{(22)}$

Let us study the influence of the particle-particle interaction on the properties of low-lying vibrational states using as an example $K^\pi = 2^+$ states in ^{168}Er . The calculations will be made with the single-particle energies and wave functions of the Saxon-Woods potential whose parameters and pairing constants are taken the same as in ref. /10/ and other papers. In all the calculations the relation $\alpha_1^{(22)} = -1.2\alpha_0^{(22)}$ is taken into account. The reduced E2 transition probabilities from the ground state to 2^+ states are calculated with the effective charge equal to 0.2. To reduce the number of free parameters, we assume the constants of particle-particle interactions in neutron and proton systems to be the same, i.e., $G_p^{(22)} = G_n^{(22)} = G^{(22)}$.

Now let us analyse how changes the behaviour of the function $\mathcal{F}(\omega)$, defined by formula (9), depending on $G^{(22)}$ at fixed constant $\alpha_0^{(22)} = 0.0221 \text{ fm}^2/\text{MeV}$. The results of calculations are given in the Figure. The scale of the figure is chosen so as to clearly represent the behaviour of $\mathcal{F}(\omega)$ at $G^{(22)}$ close to $\alpha_0^{(22)}$; therefore, the behaviour of $\mathcal{F}(\omega)$ at $G^{(22)} = 0$ is almost a vertical line. At $G^{(22)} < 0.5\alpha_0^{(22)}$ the influence of the particle-particle interaction is negligible and it can be neglected. With increasing $G^{(22)}$ the energy of the first 2_1^+ state decreases and the $B(E2)$ -value is reduced. A similar effect has been obtained earlier in ref. /2/, in studying the influence of the particle-particle interaction on the 2_1^+ and 3_1^- states in spherical nuclei. With increasing $G^{(22)}$ the function $\mathcal{F}(\omega)$ decreases and becomes close to zero. At $G^{(22)} = \alpha_0^{(22)}$ the first root ω_{221} turned out to be smaller than the experimental value. The energies and $B(E1)$ -values for the first and second 2^+ states at different values of $\alpha_0^{(22)}$ and $G^{(22)}$ are given in Table 1. It is seen from it that with increasing $G^{(22)}$ the second state energy ω_{222} decreases whereas $B(E2)$ increases. At $G^{(22)} = \alpha_0^{(22)}$ the energy ω_{222} turned out to be lower than the first pole energy of the secular equation (9). Note that in this case the total number of the calculated 2^+ states has not changed and thus the 2_2^+ state is not spurious.

At $G^{(22)} = 1.1\alpha_0^{(22)}$ the energy ω_{221} and $B(E2)$ -values are small and contradict experimental data for ^{168}Er . Note that the



Dependence $\mathcal{F}(\omega)$ on the energy ω for the $K^\pi=2^+$ states in ^{168}Br for some values of the particle-particle interaction constant $G^{(22)}$. The circles denote the roots of eq. $\mathcal{F}(\omega_{22i})=0$, lying below the first pole.

condition for conservation of the number of particles in this case is fulfilled approximately with the same accuracy as in the case $G^{(22)}=0$. At $G^{(22)}=1.1\alpha_0^{(22)}$ the energy ω_{222} is less than the first pole energy and $B(E2)$ -value has noticeably increased. In this case the first two 2^+ states are lower than the first pole. Probably, an analogous situation may take place in some nuclei.

At $G^{(22)}=1.2\alpha_0^{(22)}$ the first root disappears, the second root energy decreases and the $B(E2)$ -value increases. The condition for conservation of the number of particles becomes worse. With increasing $G^{(22)}$ the energy decreases and the $B(E2)$ - value increases. A further increase in $G^{(22)}$ leads to the disappearance of the second root, then third, the function $\mathcal{F}(\omega)$ below the first pole fluctua-

tes near zero. In this case the total number of roots of the secular equation (9) does not change. There arises a question whether the RPA is applicable in this case.

A similar behaviour of the function $\mathcal{F}(\omega)$, demonstrated in the Figure, occurs in other nuclei, for example, in ^{158}Gd , ^{172}Yb , ^{230}Th and ^{234}U .

Table I
Energies and $B(E2)$ - values for the first two states depending on $\alpha_0^{(22)}$ and $G^{(22)}$

| $\alpha_0^{(22)}$ $\frac{\text{fm}^2}{\text{MeV}}$ | $G^{(22)}$ | ω_{221}, MeV | $B(E2)_1$ s.p.u. | ω_{222}, MeV | $B(E2)_2$ s.p.u. |
|---|-----------------------|----------------------------|---------------------|----------------------------|---------------------|
| 0.0221 | 0 | 1.05 | 5.78 | 1.723 | 0.008 |
| 0.0221 | $0.6\alpha_0^{(22)}$ | 0.96 | 5.45 | 1.722 | 0.006 |
| 0.0221 | $0.8\alpha_0^{(22)}$ | 0.90 | 5.15 | 1.720 | 0.008 |
| 0.0221 | $1.0\alpha_0^{(22)}$ | 0.76 | 4.37 | 1.708 | 0.069 |
| 0.0221 | $1.1\alpha_0^{(22)}$ | 0.56 | 3.02 | 1.577 | 0.636 |
| 0.0221 | $1.2\alpha_0^{(22)}$ | - | - | 1.287 | 2.62 |
| 0.0221 | $2.0\alpha_0^{(22)}$ | - | - | 1.720 | 0.136 |
| 0.0180 | $1.0\alpha_0^{(22)}$ | 1.32 | 2.08 | 1.721 | 0.013 |
| 0.0180 | $1.2\alpha_0^{(22)}$ | 1.17 | 1.86 | 1.713 | 0.052 |
| 0.0200 | $1.05\alpha_0^{(22)}$ | 1.06 | 2.39 | 1.716 | 0.026 |
| 0.0200 | $1.1\alpha_0^{(22)}$ | 1.00 | 2.61 | 1.709 | 0.07 |
| 0.0210 | $0.8\alpha_0^{(22)}$ | 1.07 | 3.86 | 1.721 | 0.008 |
| 0.0210 | $1.0\alpha_0^{(22)}$ | 0.96 | 3.47 | 1.716 | 0.023 |
| 0.0230 | $0.6\alpha_0^{(22)}$ | 0.77 | 7.60 | 1.722 | 0.006 |
| 0.0230 | $0.8\alpha_0^{(22)}$ | 0.72 | 7.07 | 1.720 | 0.008 |
| 0.0230 | $1.0\alpha_0^{(22)}$ | 0.57 | 5.59 | 1.679 | 0.229 |
| 0.0240 | $0.5\alpha_0^{(22)}$ | 0.45 | 15.8 | 1.722 | 0.005 |

The particle-particle interaction affects the 2_1^+ states with $1 \geq 3$. Thus, between the fourth and fifth poles at $G^{(22)}$ there is one root of eq. (9) and at $G^{(22)} = \alpha_0^{(22)}$ there are two roots; between the ninth and tenth poles there is one root at $G^{(22)}$; and at $G^{(22)} = 0.8 \alpha_0^{(22)}$, two poles; between the 35th and 36th poles at $G^{(22)} = 0$ there are two roots and at $G^{(22)} = \alpha_0^{(22)}$ there is one root and so on.

Now we choose the constants $\alpha_0^{(22)}$ and $G^{(22)}$ so as to obtain approximately the same ω_{221} . As is seen from Table 1, this takes place at $\alpha_0^{(22)} = 0.0221 \text{ fm}^2/\text{MeV}$, $G^{(22)} = 0$, at $\alpha_0^{(22)} = 0.021 \text{ fm}^2/\text{MeV}$, $G^{(22)} = 0.8 \alpha_0^{(22)}$ as well as at $\alpha_0^{(22)} = 0.020 \text{ fm}^2/\text{MeV}$ and $G^{(22)} = 1.05 \alpha_0^{(22)}$. Hence it is seen that the particle-particle interaction decreases $B(E2)$ - value at the same energies of one-phonon states.

Let us now study how does the wave function of the 2_1^+ state change with increasing $G^{(22)}$. The largest in the absolute value functions $\Psi_{q_1 q_2}^{221}$ and $\Psi_{q_1 q_2}^{224}$ for ^{168}Er are shown in Table 2. It demonstrates that with increasing $G^{(22)}$ the $\Psi_{q_1 q_2}^{221}$ somewhat change. An increase in $G^{(22)}$ from zero to $\alpha_0^{(22)}$ decreases the $\Psi_{q_1 q_2}^{221}$. At $G^{(22)} > \alpha_0^{(22)}$ with increasing $G^{(22)}$ for some components $\Psi_{q_1 q_2}^{221}$ increase and take the sign opposite to the sign of $\Psi_{q_1 q_2}^{221}$. This takes place in the cases when ω_{221} and $B(E2)$ are small. Note that an analogous situation took place in the case of charge-exchange Gamov-Teller states in spherical nuclei when the $\log ft$ -values for β^+ decays and the strength functions of (n, p) transitions were described [4].

The interacting boson model takes effectively into account interactions between nucleons in open shells. Thus, their special role is pointed out. Let us consider whether one can restrict oneself to particle-particle interactions between quasiparticles in the limited space of single-particle states or should work in the whole space. For this purpose, we shall calculate the properties of 2_1^+ and 2_2^+ states when the whole space of single-particle states is used for particle-hole interactions or its various truncations for particle-particle interactions. The results of some of these calculations are shown in Table 3. First we take 18 neutron and 18 proton states, 9 from each group being below the Fermi level. Then we take 40 single-particle neutron and 40 proton states with 20 states from each group below the Fermi level and then we use the whole space of single-particle space as in the above described results. The investigations have shown that the truncation of the number of single-particle states for particle-particle interactions can mainly be compensated by the renormalisation of the $G^{(22)}$ constant. Note that if the number of proton and neutron states is limited, the change of the 2_1^+ state

Table 2. state structure with increasing $G^{(22)}$
Change in the $K_{2_1^+}^{\pi} = 2_1^+$

| $\alpha_0^{(22)}$ fm ² /MeV | 0.0221 | 0.0221 | 0.0221 | 0.020 | 0.0221 |
|--|--------|------------------------|------------------------|------------------------|------------------------|
| $G^{(22)}$ | 0 | 1.0 $\alpha_0^{(22)}$ | 1.05 $\alpha_0^{(22)}$ | 1.1 $\alpha_0^{(22)}$ | 1.1 $\alpha_0^{(22)}$ |
| q_1 | q_2 | $\Psi_{q_1 q_2}^{221}$ | $\Psi_{q_1 q_2}^{224}$ | $\Psi_{q_1 q_2}^{221}$ | $\Psi_{q_1 q_2}^{224}$ |
| nn 512↑ | 510↑ | 0.15 | 0.08 | 0.22 | 0 |
| nn 523↓ | 521↓ | -0.44 | -0.13 | -0.47 | -0.05 |
| nn 521↑ | 521↓ | -0.31 | -0.12 | -0.35 | -0.05 |
| pp 413↓ | 411↓ | 0.24 | 0.12 | 0.21 | 0.10 |
| pp 411↑ | 411↓ | 0.77 | 0.21 | 0.58 | 0.18 |
| ω_{221} , MeV | 1.05 | 0.764 | 1.06 | 0.557 | 0.17 |

Table 3

Dependence of the energies and B(E2)-values for the first two $K^\pi = 2^+$ states on the number of single-particle states taken into account in particle-particle interactions at $\alpha_0^{(2)} = 0.0221 \text{ fm}^2/\text{MeV}$ and $G^{(2)} = \alpha_0^{(2)}$.

| Single-particle basis | ω_{221} , MeV | $B(E2)_1$, s.p.u. | ω_{222} , MeV | $B(E2)_2$, s.p.u. |
|-----------------------|----------------------|--------------------|----------------------|--------------------|
| 18 states | 0.966 | 5.51 | 1.722 | 0.006 |
| 40 states | 0.882 | 5.05 | 1.719 | 0.009 |
| the whole space | 0.764 | 4.37 | 1.709 | 0.070 |

properties depending on $G^{(2)}$ appears to be not so sharp, and therefore, it can be preferable in calculations.

3. Conclusion

One can observe inconsistency in calculating the energies ω_{221} and B(E2) of the first $K^\pi = 2^+$ states in the RPA: If the calculated energies ω_{221} coincide with the experimental ones, the calculated B(E2) values exceed them. Thus, in the calculations in ref. /11/ an excess of B(E2) - values amounts to factor of 3. In the calculations /7, 10/ in the RPA with inclusion of single-particle states from the bottom of the Saxon-Woods potential up to 50 MeV in the quasi-continuous spectrum and effective charge equal to zero or 0.1, this excess is 1.5-1.8. The difference lies in the use of the single-particle energies and wave functions of the Saxon-Woods potential instead of the Nilsson potential in ref. /11/ as well as in taking account of the blocking effect and radial dependence of a quasi-quadrupole interaction in the form $\frac{dV(r)}{dr}$, where $V(r)$ is the central part of the Saxon-Woods potential instead of r^2 in ref. /11/. In the calculations within the QPNM with the wave function containing one- and two-phonon terms the calculated B(E2) -values exceed the experimental ones 10-20%. In usual calculations, as in ref. /7, 10/, all single-particle states from the bottom of the well up to 5 MeV in the quasi-continuous spectrum are taken into account and the effective charge equal to 0.2 is used. The (10-40)% excess of the

calculated B(E1) values over the experimental ones can be avoided by introducing a particle-particle interaction with $G^{(2)} = (0.6 - 0.8)\alpha_0^{(2)}$,

The formulae presented here and investigation of the solutions may serve as the basis for including particle-particle interactions in the QPNM. It is hoped that the inclusion of particle-particle interactions in the QPNM will improve the description of low-lying states with $K^\pi \neq 0^+$ of doubly even deformed nuclei.

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