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DEVELOPMENT OF A MODEL FOR THE DESCRIPTION OF HIGHLY EXCITED STATES IN ODD-A DEFORMED NUCLEI

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1. To describe the structure complication with increasing excitation energy and to clear up general regularities of intermediate and highly excited states a model is used, based on the account of the quasiparticle-phonon interaction. Within the framework of such a model in order to solve the system of equations it is necessary to diagonalize matrices of the rank 10^4 and higher. This fact forces us to apply the approximate methods of solving. In ref.[1] rough approximation is used which takes into account only coherent terms. In this approximation there appear superfluous solutions, which are hardly separable from the true ones. In ref.[2] an approximate method of solving the system of equations is suggested which takes into account all the coherent terms and pole non-coherent terms. In this case we have no superfluous solutions. The approximate method developed in[2] is used in[3] for the case of doubly even deformed nuclei, and in[4] it is clarified and used for more complicated cases of odd-A deformed nuclei.

The approximate method[2] was used in[5,6] for studying the structure of odd-A deformed nuclei. These investigations have shown, firstly, that the approximate method mentioned above is useful for the study of the structure of states, secondly, that this method requires to be improved as it overestimates contribution to the secular equation from the separated pole.

The aim of the present paper is to develop a new approximate method of solving the equations of the model, free from the shortcomings of the former methods. The new approximation contains no superfluous solutions and special separation of some poles among all the other poles. The new method is made for the case of odd-A deformed nucleus, simultaneously considering several one-quasiparticle components.

2. The model Hamiltonian is taken to comprise an average field described by the potential, interactions leading to superconducting pairing correlations and multipole-multipole interactions. All the parameters were fixed earlier in studying the low-lying states of nuclei. Taking into account the secular equations for determining the phonon energies ω_t (where t denotes $\lambda\mu j$, j is the number of the secular equation root), the appropriate part of the Hamiltonian can be written in the form:

$$H = \sum_{\nu} \varepsilon(\nu) B(\nu\nu) - \frac{1}{2}\sum_{t} \frac{1}{Y_t} \sum_{\nu\nu'} \frac{\left(f^t(\nu\nu') u_{\nu\nu'}\right)^2 (\varepsilon(\nu)+\varepsilon(\nu'))}{(\varepsilon(\nu)+\varepsilon(\nu'))^2 - \omega_t^2} Q_t^+ Q_t -$$

$$- \frac{1}{2}\sum_{t} \sum_{\nu\nu'} \left\{ \Gamma^t(\nu\nu') B(\nu\nu')(Q_t^+ + Q_t) + h.c. \right\}, \qquad (1)$$

where

$$\Gamma^t(\nu\nu') = \frac{v_{\nu\nu'}}{2\sqrt{Y_t}} f^t(\nu\nu') \equiv \Gamma_{\nu\nu'},$$

$$B(\nu\nu') = \sum_{\sigma} \alpha^+_{\nu\sigma}\alpha_{\nu'\sigma} \text{ or } \sum_{\sigma} \sigma\alpha^+_{\nu\sigma}\alpha_{\nu'\sigma}.$$

Here the following notations are used; $f^t(\nu\nu')$ is the matrix element of the multipole moment operator $\lambda\mu$; Q_t^+ , Y_t is the phonon creation operator and its characteristic (see (8.67) in [7]); $\alpha^+_{\nu\sigma}$ is the quasiparticle creation operator, $\varepsilon(\nu) = \sqrt{C^2 + (E(\nu) - \lambda)^2}$, $E(\nu)$ is the single-particle energy, C is the correlation function, λ is the chemical potential, $u_{\nu\nu'} = u_\nu v_{\nu'} + u_{\nu'} v_\nu$, $v_{\nu\nu'} = u_\nu u_{\nu'} - v_\nu v_{\nu'}$, $(\nu\sigma)$ denotes the set of quantum numbers of the single-particle state, $\sigma = \pm 1$.

The wave function of the non-rotational state of odd-A deformed nucleus is written in the form:

$$\Psi_i(K^\pi) = \frac{1}{\sqrt{2}} \sum_\sigma \Big\{ \sum_\nu C_\nu^i \alpha^+_{\nu\sigma} + \sum_g D_g^i (\alpha^+ Q^+)_g + \frac{1}{\sqrt{2}} \sum_G F_G^i (\alpha^+ Q^+ Q^+)_G \Big\} \Psi_0 , \tag{2}$$

where Ψ_0 is the wave function of the ground state of the doubly even nucleus; i is the number of the state, $g = \nu i$, $G = \nu i_1 i_2$. The wave function (2) differs from [illegible] /1,2,5/ as it takes into account several one-particle components ν.

Now we calculate the average value H_M over the state (2) and by means of the variational principle obtain the following system of equations:

$$(P_g - \eta_i) D_g^i - \sum_\nu \Gamma_{\nu g} C_\nu^i - \sum_G \Gamma_{gG} F_G^i = 0 , \tag{3}$$

$$C_\nu^i = \frac{1}{\varepsilon(\nu) - \eta_i} \sum_g \Gamma_{\nu g} D_g^i , \tag{4}$$

$$F_G^i = \frac{1}{P_G - \eta_i} \sum_g \Gamma_{gG} D_g^i , \tag{5}$$

$$1 = \sum_\nu (C_\nu^i)^2 + \sum_g (D_g^i)^2 + \sum_G (F_G^i)^2 \tag{6}$$

We rewrite the equation (3) as follows;

$$D_g^i - \frac{1}{P_g - \eta_i} \sum_{g'} K(g,g') D_{g'}^i = 0 . \tag{7}$$

Here the following notations are used; η_i is the energy of the non-rotational state, $P_g = \varepsilon(\nu) + \omega_i$, $P_G = \varepsilon(\nu) + \omega_{i_1} + \omega_{i_2}$ are the fundamental poles,

$$K(g,g') = \sum_\nu \frac{\Gamma_{\nu g} \Gamma_{\nu g'}}{\varepsilon(\nu) - \eta_i} + \sum_G \frac{\Gamma_{gG} \Gamma_{g'G}}{P_G - \eta_i} , \tag{8}$$

$$\Gamma_{\nu\nu'} = \frac{1}{\sqrt{2}}\left\{\delta_1 \Gamma^{t_1}(\nu\nu_2)\delta_{t,t_2} + \delta_1' \Gamma^{-t_1}(\nu\nu_2)\delta_{t,t_2}\right\}, \qquad (8')$$

where multipliers $\delta_1 = \pm 1$, $\delta_1' = \pm 1$ are defined in[2].

Consider the system of N equations

$$x_n + \frac{1}{a_n}\sum_{n'} K(n,n')\, x_{n'} = \frac{y_n}{a_n} \; ; \qquad (9)$$

where $K(n,n')$ has the form of (8). The determinant of this system may be represented in the form:

$$\Delta = 1 + \sum_n \frac{K(n,n)}{a_n} + \frac{1}{2!}\sum_{n_1 n_2} \begin{vmatrix} K(n_1,n_1) & K(n_1,n_2) \\ K(n_2,n_1) & K(n_2,n_2) \end{vmatrix} \frac{1}{a_{n_1} a_{n_2}} + \qquad (10)$$

$$+ \ldots \begin{vmatrix} K(1,1) & \ldots & K(1,N) \\ \ldots & & \ldots \\ K(N,1) & \ldots & K(N,N) \end{vmatrix} \frac{1}{a_1 a_2 \ldots a_N} \, .$$

The determinants of the first order and higher due to their special form consist of non-coherent terms, containing only first-order poles but no second-order poles and higher. In the theory of nucleus one, first of all, deals with the coherent terms, thus in eq.(10) it is possible to limit oneself only to the first two terms, i.e., the determinant and the solution of the system (9) may be approximately given in the form:

$$\Delta = 1 + \sum_n \frac{K(n,n)}{a_n} \, , \qquad (11)$$

$$x_n = \frac{y_n / a_n}{1 + \sum_{n'} \frac{K(n',n')}{a_{n'}}} \qquad (12)$$

or in the more exact form:

$$x_n = \frac{1}{a_n}\left\{ y_n - \frac{\sum_{n'} K(n,n')\frac{y_{n'}}{a_{n'}}}{1 + \sum_{n'} \frac{K(n',n')}{a_{n'}}} \right\} . \qquad (12')$$

We return to the solution of the system of equations (7).

Using the exact equation (10). the determinant of the system of equations (7) *after some transformations may be represented* in the form:

$$\Delta = 1 - \sum_{\nu} \frac{A_\nu}{\varepsilon(\nu) - \eta} - \sum_{g} \frac{A_g}{P_g - \eta} - \sum_{G} \frac{A_G}{P_G - \eta}, \tag{13}$$

where the coefficients A_ν , A_g , A_G , being the sum of the determinants of various ranks from 1 to N_j , are independent of η . It follows from (13) that the secular equation

$$\Delta(\eta_i) = 0 \tag{14}$$

has only first-order poles.

We take an advantage of eq.(11) for determining the energies η_i . We get an approximate secular eq. in the form:

$$\Delta = 1 - \sum_{g} \frac{1}{P_g - \eta_i} \left\{ \sum_{\nu} \frac{\Gamma_{\nu g}^2}{\varepsilon(\nu) - \eta_i} + \sum_{G} \frac{\Gamma_{gG}^2}{P_G - \eta_i} \right\} = 0. \tag{15}$$

To find the explicit form of the functions C_ν , D_g , F_G we separate one single-particle state, denoted as ρ, and rewrite eq. (3) as:

$$(P_g - \eta_i) D_g^i - \sum_{g'} \left\{ \sum_{\nu \neq \rho} \frac{\Gamma_{\nu g} \Gamma_{\nu g'}}{\varepsilon(\nu) - \eta_i} + \sum_{G} \frac{\Gamma_{gG} \Gamma_{g'G}}{P_G - \eta_i} \right\} D_{g'}^i = \Gamma_{\rho g} C_\rho^i \tag{16}$$

Then, the exact solution of the system for the coefficients C_ν^i , D_g^i , F_G^i of wave function (2) may be written in a general form as in (13) for the determinant Δ of the system of equations. For example, for D_g^i we get:

$$D_g^i = \frac{C_\rho^i}{P_g - \eta_i} \left\{ \Gamma_{\rho g} - \frac{\sum\limits_{\nu \neq \rho} \frac{A_\nu^{(1)}}{\varepsilon(\nu) - \eta_i} + \sum\limits_{g'} \frac{A_{g'}^{(1)}}{P_{g'} - \eta_i} + \sum\limits_{G} \frac{A_G}{P_G - \eta_i}}{1 - \sum\limits_{\nu \neq \rho} \frac{A_\nu^{(2)}}{\varepsilon(\nu) - \eta_i} - \sum\limits_{g'} \frac{A_{g'}^{(2)}}{P_g - \eta_i} - \sum\limits_{G} \frac{A_G^{(2)}}{P_G - \eta_i}} \right\}. \tag{17}$$

Analogous expressions we obtain for C_ν^i and F_G^i . Yet, we use the approximation, having allowed to obtain an approximate solution (12):

$$D'_\nu = \frac{C_\nu'}{\mathcal{Y}_\nu} - \frac{\Gamma_{\nu g}}{\eta_g - \eta}, \tag{18}$$

$$\mathcal{Y}_\nu = \frac{1}{\varepsilon_\nu - \eta} \sum_g \frac{\Gamma_{\nu g}^2}{\eta_g - \eta}. \tag{18'}$$

Then, from equations (4), (5) and (6) we have:

$$C'_\nu = \frac{C_\nu}{\mathcal{Y}_\nu} \frac{1}{\varepsilon_\nu - \eta} \sum_g \frac{\Gamma_{g}\Gamma_{\nu g}}{\eta_g - \eta}, \tag{19}$$

$$\Gamma'_{i} = \frac{C_\nu}{\mathcal{Y}_\nu} \frac{1}{\varepsilon_i - \eta} \sum_g \frac{\Gamma_{\nu g}\Gamma_{g i}}{\eta_g - \eta}, \tag{20}$$

$$C_\nu'^{-2} = 1 + \frac{1}{\mathcal{Y}_\nu^2}\Big\{\sum_g \frac{\Gamma_{\nu g}^2}{(\eta_g - \eta)^2} + \sum_{\nu'} \frac{1}{(\varepsilon_{\nu'} - \eta)^2}\Big[\sum_g \frac{\Gamma_{\nu g}\Gamma_{g\nu'}}{\eta_g - \eta}\Big]^2 + \sum_i \frac{1}{(\varepsilon_i - \eta)^2}\Big[\sum_g \frac{\Gamma_{\nu g}\Gamma_{g i}}{\eta_g - \eta}\Big]^2\Big\} = -\frac{\partial}{\partial \eta}\Big\{\frac{\varepsilon_\nu - \eta}{\mathcal{Y}_\nu}\Delta\Big\}_{\eta=\eta_i} \tag{21}$$

The detailed study of fragmentation has been made within the framework of the former model with the wave function of the type (2), in which $\Gamma_i' \equiv 0$, and only one single-particle state ρ has been considered. In this case the single-particle state fragmentation is described by the function:

$$(C_\nu)^{-2} = 1 + \sum_g \frac{\Gamma_{\Omega}^2}{(\eta_g - \eta)^2} \tag{22}$$

By comparing C_ν' in the form (21) and (22) it is seen, that the expression in the form (21) is a natural generalization of a simple case, provided by eq.(22).

One has every reason to consider the approximate method of treating quasiparticle-phonon interaction to be useful for clearing up general regularities of fragmentation of the single-particle states, hence for the description of the structure of intermediate and highly exited states of complex nuclei in the language of various strength functions.

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