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ДУБНА

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R.V.Jolos, A.K.Nasirov\*, V.P.Permjakov

**TOTAL KINETIC ENERGIES  
OF THE DEEP INELASTIC HEAVY-ION  
COLLISION FRAGMENTS  
AND QUASI-FISSION**

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\* Institut of Nuclear Physics, Tashkent, UsSSR

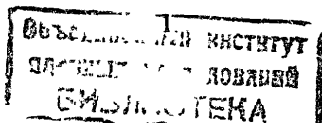
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## 1. Introduction

It has been shown in the first experiments on deep inelastic collisions that the most probable total fragment kinetic energy is approximately equal or even smaller than the Coulomb repulsion energy if it is calculated under the assumption that the reaction fragments are spherical. It was interpreted as an indication that an essential contribution to the kinetic energy dissipation comes from the effects of the fragment deformation in the exit channel. At the same time, it was found that the deformation is stronger when the fragments with approximately equal masses are created<sup>/1/</sup>.

The detailed investigation of the energy distributions of the heavy-ion reaction products when the total charge  $Z$  of the fragments is larger than 100 has recently been performed in<sup>/2/</sup>. The analysis of the obtained experimental data performed under the assumption that the main contribution to the kinetic energy comes from the Coulomb repulsion of the reaction products allows one to determine the dependence of the distance between the centers of the fragments at scission ( $D$ ) on the mass asymmetry in the exit channel. It was found that for the fragments with masses close to the half total mass of the system, the value of  $D$  is described by the expression  $D = \zeta_0 (A_3^{1/3} + A_4^{1/3})$ , where  $\zeta_0$  is the parameter equal to (1,8-2,0) fm and  $A_3, A_4$  are the masses of the fragments. With increasing mass-asymmetry in the exit channel, i.e. with increasing mass of the heavy fragment, the value of  $D$  deviates from such a dependence becoming smaller. The extrapolation of the experimental dependence of  $D$  on the heavy-fragment mass to the target mass gives  $D = 1.4 (A_3^{1/3} + A_4^{1/3})$ . For the products which are spherical at all values of mass-asymmetry in the exit channel the parameter  $\zeta_0$  must be a constant independent of mass-asymmetry.

These experimental data are in agreement with the current data on the fission-fragment kinetic energies. It has been found in<sup>/3/</sup> that the fission-fragment kinetic energies are determined not only by the Coulomb repulsion but depend also on the fissility-parameter.



The deviations from the simple Coulomb dependence ( $\sim Z_3 \cdot Z_4$ ) of the fission-fragment total kinetic energies have been observed in /4/.

Thus, the explanation of the observed dependence of D on mass-asymmetry in the exit channel in both deep inelastic collisions and fission requires investigation of a dynamical evolution of the form of a composite system during the reaction process. The form evolution of the composite system in fission has been investigated in several papers (see for example /5/). It was noted that with increasing mass of the composite system the scission configuration becomes more elongated.

In this paper we consider the most probable total kinetic energies of the heavy-ion reaction products investigating the problem under different assumptions on the dynamics of the process.

## 2. The Influence of the Quadrupole Deformation of the Fragments and the Nucleon Transfer on the Interaction Barrier

Most of the papers devoted to the description of the deep inelastic heavy-ion collisions are based on the model Hamiltonian containing as dynamical variables only the relative distance R and the conjugated momentum  $P_R$ . Other degrees of freedom have sometimes been taken into account but not in a direct way. For instance, the deformation effects have been simulated by the difference between the exit channel and the entrance channel potential. The nucleon exchange has also been considered in some papers but for the fixed configuration of the double nuclear system.

In some papers, together with the relative distance vector R, the dynamical variable characterizing the neck between the nuclei has also been taken into account /6/. In a more complete form such a model has been formulated in /7/.

In the following we shall analyse the problem of the most probable fragment kinetic energies using both the approaches.

In the framework of the first approach the kinetic energies of the products of deep inelastic collisions and quasifission are determined by the effective Coulomb barrier. The barrier depends on possible dynamical deformation of nuclei during an interaction, nucleon transfer between nuclei and so on. Since nuclear forces acting between the nuclei are proportional to the nuclear contact surface area, i.e. depend on mass asymmetry, it is possible that

the renormalized interaction barrier will have additional pure Coulomb dependence on mass asymmetry in the exit channel.

Let us take into account a possible dynamical deformation of the fragments when they go through the barrier to scission. When zero  $\beta$ -vibrations take place, the distance between fragment surfaces and therefore the nuclear and Coulomb interactions change. Therefore  $\beta$ -vibrations are coupled to relative motion

$$H = -\frac{\hbar^2}{2M} \frac{d^2}{dR^2} + U_L(R) - F(R) \cdot \beta + \frac{1}{2} C_4 \beta^2 - \frac{\hbar^2}{2B} \frac{d^2}{d\beta^2},$$

where  $U_L$  is the nucleus-nucleus interaction potential containing nuclear  $U_{Nucel}$ , Coulomb and centrifugal parts;

$$F(R) = (S/4\pi)^{1/2} (R_{04} dU_{Nucel}/dR - \gamma/(\sqrt{2} R^3)).$$

Here, for shortness, we include in the Hamiltonian only  $\beta$ -vibrations of one of the fragments ( $R_{04}$  is its equilibrium radius). The contribution from  $\beta$ -vibration of other fragment is easy to take into account in the final expressions. Since near the interaction barrier the contribution of the nuclear forces to F(R) is noticeably larger than that of the Coulomb force, only the nuclear force contribution will be taken into account.

Further it is important to find a relation between the characteristic times for radial motion and  $\beta$ -vibrations. The characteristic energy of  $\beta$ -vibration is  $\hbar\omega_\beta \approx 1$  MeV. If the radial friction is absent, the radial motion can be characterized by the frequency of the reversed parabola  $\omega_L$

$$\hbar\omega_L = \sqrt{\frac{\hbar^2}{M} \left( -d^2 U_L / dR^2 \Big|_{R=R_B} \right)},$$

determining the potential near the interaction barrier ( $R_B$  - barrier radius). The quantity  $\hbar\omega_L$  is approximately equal to 3-5 MeV. So, in the absence of the radial friction we can regard that the  $\beta$ -motion is slower than the radial motion (Below, we get the result for the opposite case too).

Let us introduce a new variable  $\zeta$

$$R = \zeta + \xi \cdot \beta$$

neglecting the fact that this transformation does not commute with the operator of the  $\beta$ -vibration kinetic energy which is small as assumed. Then

$$H = -\frac{\hbar^2}{2M} \frac{d^2}{dz^2} + U_L(z) + \left( \xi \frac{dU_L}{dz} - F(z) \right) \beta$$

$$+ \left( \frac{C_4}{2} - \xi \frac{dF}{dz} + \frac{1}{2} \xi^2 \frac{d^2 U_L}{dz^2} \right) \beta^2 - \frac{\hbar^2}{2B} \frac{d^2}{d\beta^2}$$

Consider the function  $(\xi \frac{dU_L}{dz} - F)$  near the barrier. The derivative  $dU_L/dz$  is linear in  $z$  and equals zero at  $z=R_B$ . The function  $F(z)$  is linear in  $z$  near the barrier and  $F(R_B) \neq 0$ . Let us write

$$\beta \left( \xi \frac{dU_L}{dz} - F(z) \right) = -F(R_B) \beta + \left[ \xi \frac{dU_L}{dz} - (F(z) - F(R_B)) \right] \beta.$$

We determine  $\xi$  so that the linear in  $z$  part of the function

$$\xi \frac{dU_L}{dz} - (F(z) - F(R_B))$$

is equal to zero. Then

$$\xi \left. \frac{d^2 U_L}{dz^2} \right|_{z=R_B} - \left. \frac{dF}{dz} \right|_{z=R_B} = 0,$$

where

$$\left. \frac{dF}{dz} \right|_{z=R_B} = \sqrt{\frac{5}{4\pi}} R_{04} \left. \frac{d^2 U_{Nuc}}{dz^2} \right|_{z=R_B} = \sqrt{\frac{5}{4\pi}} R_{04} \left[ -M\omega_L^2 - \left( \frac{d^2 V_{Coul}}{dz^2} + \frac{d^2 V_L}{dz^2} \right) \right]_{z=R_B},$$

$V_{Coul}$  and  $V_L$  are the Coulomb and centrifugal potentials. As a result, we get

$$H = -\frac{\hbar^2}{2M} \frac{d^2}{dz^2} + U_L(z) + \frac{1}{2} C_4 \left( 1 + \frac{M\omega_L^2 \xi^2}{C_4} \right) \beta^2 - F(R_B) \beta - \frac{\hbar^2}{2B} \frac{d^2}{d\beta^2}.$$

With the help of transformation

$$\beta = \left( 1 + \frac{M\omega_L^2 \xi^2}{C_4} \right)^{-\frac{1}{4}} \left[ \tilde{\beta} + \frac{F(R_B)}{C_4} \left( 1 + \frac{M\omega_L^2 \xi^2}{C_4} \right)^{-\frac{3}{4}} \right]$$

intrinsic Hamiltonian is reduced to the diagonal form and we get

$$H = -\frac{\hbar^2}{2M} \frac{d^2}{dz^2} + \left( U_L(z) - \frac{1}{2} \frac{F^2(R_B)}{C_4} \left( 1 + \frac{M\omega_L^2 \xi^2}{C_4} \right)^{-\frac{1}{4}} \right)$$

$$+ \left( \frac{1}{2} C_4 \tilde{\beta}^2 - \frac{\hbar^2}{2B} \frac{d^2}{d\tilde{\beta}^2} \right) \left( 1 + \frac{M\omega_L^2 \xi^2}{C_4} \right)^{1/2}.$$

Thus, in the new variables the radial and intrinsic motions are recoupled. The coupling of the initial variables creates, firstly, the decrease of the potential barrier

$$V_B^{eff} = V_B - \frac{F^2(R_B)}{2C_4} \left( 1 + \frac{M\omega_L^2 \xi^2}{C_4} \right)^{-1},$$

and secondly, the increase of the intrinsic vibration stiffness:

$$\hbar \omega_\beta^{eff} = \hbar \omega_\beta \left( 1 + \frac{M\omega_L^2 \xi^2}{C_4} \right)^{1/2}.$$

If the radial motion is slower than the  $\beta$ -vibrations due to radial friction, then for the renormalized potential barrier we get

$$V_B^{ad} = V_B - \frac{1}{2} \frac{F^2(R_B)}{C_4}.$$

Parametrizing  $V_B$  and  $V_B^{eff}$  by the expressions

$$V_B = e^2 Z_3 Z_4 / [z_{0B} (A_3^{1/3} + A_4^{1/3})],$$

$$V_B^{eff} = e^2 Z_3 Z_4 / [z_{0B}^{eff} (A_3^{1/3} + A_4^{1/3})]$$

we get for  $z_{0B}^{eff}$

$$\frac{z_{0B}^{eff}}{z_{0B}} = 1 + [5/8\pi \left( \frac{R_{03}}{C_3} + \frac{R_{04}}{C_4} \right) \frac{1}{V_B} \left( \frac{d(V_{Coul} + V_L)}{dz} \right)_{z=R_B}^2]^{1/2} /$$

$$\left[ 1 + \frac{5}{8\pi} \left( \frac{R_{03}^2}{C_3} + \frac{R_{04}^2}{C_4} \right) (2M\omega_L^2 \left( 1 + \frac{d^2(V_{Coul} + V_L)}{dz^2} \right)_{z=R_B} / M\omega_L^2)^2 \right]^{1/2}$$

$$- \frac{1}{V_B} \left( \frac{d(V_{Coul} + V_L)}{dz} \right)_{z=R_B}^2 \right]^{1/2}.$$

Here, we take into account the excitation of  $\beta$ -vibrations in both the fragments. The calculations for the reactions  $^{40}\text{Ar} + ^{232}\text{Th}$ ,  $^{64}\text{Zn} + ^{122}\text{Sn}$  show that  $z_{0B}^{\text{eff}} - z_{0B}$  is approximately equal to 0.05 fm. The quantity  $z_{0B}^{\text{eff}}$  is practically independent of mass-asymmetry in the exit channel.

In the adiabatic approximation the renormalization of  $z_{0B}$  is considerably larger. The difference  $z_{0B}^{\text{ad}} - z_{0B}$  is (0.3-0.5) fm. The variations of  $z_{0B}^{\text{eff}}$  with the exit channel mass asymmetry parameter do not exceed 0.05 fm. It is insufficient for explaining the experimental data.

The renormalization of the interaction barrier due to nucleon exchange is also weakly sensitive to variations of the exit channel mass-asymmetry parameter. So, the coupling of the relative motion to fragment surface vibrations and the nucleon exchange can renormalize appreciably the interaction barrier. But this renormalization weakly depends on mass-asymmetry and is insufficient to interpret the experimentally observed dependence of the most probable total fragment kinetic energies. The failure of the above consideration may be due to implicit assumption that the evolution from a combined double-nuclear system to scission is independent of the mass-asymmetry in the exit channel.

### 3. The Calculation of the Interfragment Distance at Scission on the Basis of Swiatecky's Model

The limitation mentioned at the end of section 2 is absent in Swiatecky's model, in which, together with the relative distance  $R$ , the parameter characterizing a neck size is introduced into consideration. Based on this model, let us imagine the following picture of the collision. Owing to the large mass-asymmetry in the entrance channel and the high enough initial kinetic energy, the system approaches a conditional saddle point where a neck size is enough to start an intensive nucleon exchange. So, the degree of freedom characterising mass-asymmetry gets unfrozen. The fragment mass difference decreases. As a result, the Coulomb repulsion increases (with increase in the product  $Z_3 \cdot Z_4$ ) and a double nuclear system changes the direction of evolution and starts a motion to scission. A turning point lies near the conditional saddle point corresponding to a new value of the mass-asymmetry parameter reached during the approach phase of the evolution. Since the characteristic time for variation of the mass-asymmetry parameter is large in comparison with the time-interval which is necessary for travelling from a saddle to

a scission point, we can regard that the mass parameter is fixed on the trajectory from the saddle point to scission. The assumption that the turning point of the trajectory lies near the conditional saddle point is very important for the following.

The consideration in the framework of Swiatecky's model is carried out in terms of the variables  $\nu$  and  $\sigma$ . The first of them is equal approximately to the neck radius; and the second one, to the length of the neck measured in units  $2\bar{R}$ , where  $\bar{R} = R_{03} R_{04} / (R_{03} + R_{04})$  and  $R_{0k}$  ( $k=3,4$ ) is the fragment radius. The evolution of  $\nu$  and  $\sigma$  is described by the equations

$$M \frac{d^2 \sigma}{d\tau^2} + \nu^2 \frac{d\sigma}{d\tau} + \nu - \chi = 0 \quad (3.1)$$

$$\frac{d\nu}{d\tau} = \frac{2\nu - 3\nu^2 - \sigma}{4\nu(\sigma + \nu^2)} \quad (3.2)$$

Here  $\chi = 0.041808 \frac{Z^2}{A} (1-\eta^2)^{2/3} [(1-\eta)^{1/3} + (1+\eta)^{1/3}]^{-1} \times [1 - 1.7826 (A-2Z)^2 / A^2]^{-1}$  is the effective fissility parameter;  $Z$  is the total charge and  $A$  is the total number of nucleons in a composite system;  $\eta = (A_4 - A_3) / A$  is the mass-asymmetry parameter;  $M$  is the reduced mass and  $\tau$  is the time measured in units  $\rho \bar{V} \bar{R}^2 / \gamma$  ( $\rho$  is the nuclear density,  $\bar{V}$  is the average velocity of nucleons in nuclei and  $\gamma = 0.9517 [1 - 1.7826 \times (A-2Z)^2 / A^2]$  (MeV fm<sup>2</sup>) is the surface energy parameter).

In deriving eqs. (3.1) and (3.2) it was assumed that the potential energy of the system is a sum of the surface and Coulomb energies and the dissipative function contains wall and window friction terms.

Thus, the model proposed to calculate the interfragment distance at scission can be formulated in the following way. The starting point is a conditional saddle point the position of which is determined by the exit channel mass-asymmetry parameter  $\eta$  in the following way

$$\nu_0 = \chi(\eta),$$

$$\sigma_0 = 2\chi(\eta) - 3\chi^2(\eta).$$

A further evolution of the quantities  $\nu$  and  $\sigma$  is described by eqs. (3.1) and (3.2). In the calculations we have assumed the initial value of  $d\delta/d\tau$  to be equal to zero.

For this reason we change slightly the initial position of the system in the  $\nu - \sigma$  plane from the point  $(\nu_0, \sigma_0)$  in the direction of scission.

The calculations have been performed under different assumptions about the critical neck radius  $r_{crit}$  at which the scission takes place

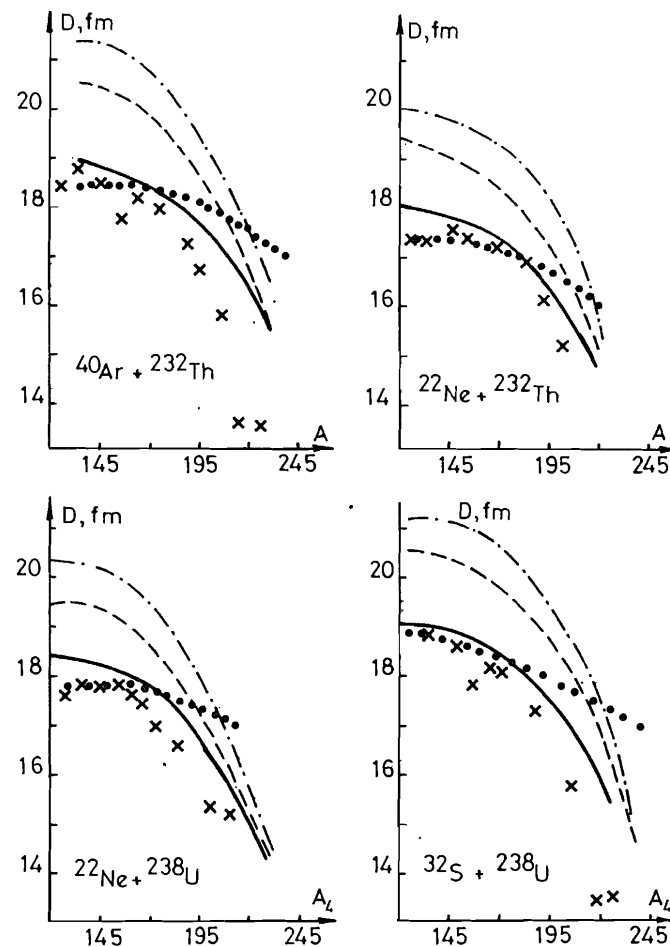
- $r_{crit} = 0$ ,
- $r_{crit} = r_0 = 1.15 \text{ fm}$
- $r_{crit} = \frac{2}{9} \times \text{neck length}.$

The last condition has been formulated in <sup>18/</sup> and reflects the stability condition found by Rayleigh. According to Rayleigh, a filament of liquid remains stable as long as the wave length  $\lambda$  of a perturbing vibration along filament's axis remains smaller than filament's circumference.

The calculations have been done for the following reactions:  $^{40}\text{Ar} + ^{232}\text{Th}$ ,  $^{22}\text{Ne} + ^{232}\text{Th}$ ,  $^{22}\text{Ne} + ^{238}\text{U}$ ,  $^{32}\text{S} + ^{238}\text{U}$ ,  $^{22}\text{Ne} + ^{197}\text{Au}$  and  $^{64}\text{Zn} + ^{122}\text{Sn}$ . The theoretical results for the interfragment distance at scission  $D$  together with the experimental results <sup>12/</sup> are shown in figs. 1 and 2. The total fragment kinetic energy at scission has been calculated as well.

What conclusions can be made? Firstly, in the case of very heavy composite systems the calculations based on Swiatecky's model are in good correspondence with the experimental data. The best results are obtained under the assumption that  $r_{crit} = \frac{2}{9}$  neck length. Secondly, if we go to more light composite systems like  $\text{Ne} + \text{Au}$  or  $\text{Zn} + \text{Sn}$  the agreement between the theoretical and the experimental results is destroyed. In the reaction  $^{22}\text{Ne} + ^{197}\text{Au}$  due to the decrease of the production cross-section with increasing mass-asymmetry parameter a sharp fall of  $D$  with mass-asymmetry has not been found experimentally. For the reaction  $^{64}\text{Zn} + ^{122}\text{Sn}$  we cannot explain the sharp fall of  $D$  with increasing mass-asymmetry at all.

The total fragment kinetic energy at scission amounts to 5-10 MeV for the considered systems, i.e. it is small in comparison with the energy of the Coulomb repulsion.



**Fig. 1.** Dependence of the distance  $D$  between the fragments at scission  $D$  on the heavy fragment mass  $A$  in the reactions  $^{40}\text{Ar} (220 \text{ MeV}) + ^{232}\text{Th}$ ,  $^{22}\text{Ne} (179 \text{ MeV}) + ^{232}\text{Th}$ ,  $^{22}\text{Ne} (179 \text{ MeV}) + ^{238}\text{U}$ ,  $^{32}\text{S} (192 \text{ MeV}) + ^{238}\text{U}$ .

- xxx experiment;
- - - theory,  $r_{crit} = 0$  ;
- — theory,  $r_{crit} = r_0 = 1.15 \text{ fm}$
- ..... theory,  $r_{crit} = \frac{2}{9} \times \text{neck length}$ ;
- .....  $D = \text{const} (A_3^{1/3} + A_4^{1/3})$

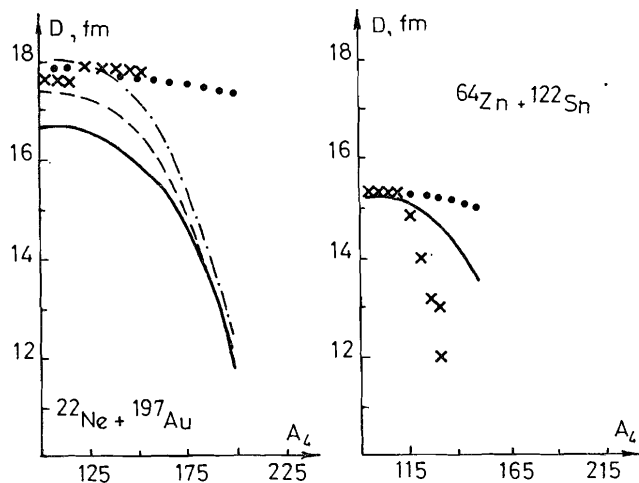


Fig. 2. The same as on fig. 1 but for the reactions  $^{22}\text{Ne}$  (179 MeV) +  $^{197}\text{Au}$  and  $^{64}\text{Zn}$  (315 MeV) +  $^{122}\text{Sn}$ .

Thus, in the case of very heavy composite systems with the total charge  $Z > 100$  the parametrization of the form of a double nuclear system suggested in [7] is successful. The problem of the choice of dynamical variables to describe the collisions of lighter nuclei must be investigated further. The above consideration has been performed under the assumption that the collisions are central or near central. For lighter systems due to the decrease of the moment of inertia the angular momentum plays a more important role and must be taken into account.

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Джолос Р.В., Насиров А.К., Пермяков В.П. E4-87-822

О полных кинетических энергиях продуктов  
глубокоупругих столкновений тяжелых ионов  
и квазиделения

Анализируется влияние динамической деформации фрагментов, обмена нуклонами и формирования "шейки", связывающей два ядра, на конфигурацию двойной ядерной системы в момент разрыва. Показано, что ни динамическая деформация фрагментов при прохождении барьера, ни обмен нуклонами не могут объяснить наблюдаемой зависимости наиболее вероятной полной кинетической энергии продуктов реакции от массовой асимметрии в выходном канале. Учет образования "шейки" позволяет объяснить экспериментальные данные о реакциях, в которых суммарный заряд сталкивающихся ядер превышает 100.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Jolos R.V., Nasirov A.K., Permjakov V.P. E4-87-822

Total Kinetic Energies of the Deep Inelastic  
Heavy-Ion Collision Fragments and Quasi-Fission

The most probable total kinetic energies of the heavy-ion collision fragments are considered. The influence of the fragment dynamical deformation, nucleon transfer and the neck formation on the scission configuration of the double nuclear system is analysed. It is shown that the dynamical deformation of the fragments on the way to scission and the nucleon transfer cannot explain the observed dependence of the most probable kinetic energies of the fragments on the exit channel mass asymmetry. The inclusion of the neck formation into consideration gives us a possibility of reproducing the experimental data on the kinetic energies for heavy composite systems with the total charge  $Z \sim 100$ .

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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