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MEAN FIELD BOSON TREATMENT OF THE ELECTRON SCATTERING FORM FACTORS IN ^{18}O

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1. INTRODUCTION

Bosonic description of nuclear properties has become very popular in recent years[1]. However, in order for such a treatment to be useful in practice, it must satisfy the following requirements:

1) The bosonization procedure must be dictated exclusively by the mapping of fermion operators, i.e. once the boson images of fermion operators are found, it must be possible to obtain all the necessary physical information within the space of simple boson states;
2) The mapped boson operators must preserve the properties of the original fermion operators, i.e., for example, a hermitian one-plus-two-body fermion Hamiltonian should be mapped onto a hermitian one-plus-two-body boson Hamiltonian;
3) There must exist a relatively small boson subspace in which (a part of) the complicated fermion dynamics can be well reproduced;
4) The boson states spanning this subspace must not be contaminated by spurious components, i.e. every boson state must have its counterpart in the original fermion space.

To meet all these requirements simultaneously is a very difficult problem which has only been solved in certain model cases up to now[2]. Recently, we have demonstrated [3,4] that the conditions 1)-4) can be satisfied in some realistic situation as well. Our approach is based on a combined use of the exact Dyson mapping[5] of the shell-model Hamiltonian and an approximate treatment of the resulting boson Hamiltonian in the framework of the mean field techniques[6]. This procedure has been shown to describe successfully the energy spectra[3] as well as the electron scattering form factors[4] for the $0^+_1 \to 0^+$ and $0^+ \to 2^+$ transitions in ^{20}Ne and ^{24}Mg. Here we extend the relevant formalism to nuclei with $n_\pi \neq n_\nu$ and apply it to ^{18}O.

2. THE EXTENDED MEAN FIELD APPROACH

We start from the microscopically derived boson Hamiltonian

$$\hat{H}_B = \sum_{\substack{JTM\tau\\ abcd}} E^{JT}_{abcd}\, B^+_{JTM\tau}(ab)\, B_{JTM\tau}(cd) + \sum_{J_1J_2J_3J_4}\ \sum_{T_1T_2T_3T_4}\ \sum_{abcdef}\ \sum_{JT} \bar{W}^{JT}_{J_1T_1J_2T_2J_3T_3J_4T_4}(abcdef)\ \times$$
$$\times\left([B^+_{J_1T_1}(ae)\times B^+_{J_2T_2}(bf)]^{JT}\cdot[\tilde{B}_{J_3T_3}(de)\times\tilde{B}_{J_4T_4}(cf)]^{JT}\right), \qquad (1)$$

where the functions E^{JT}_{abcd}, $\bar{W}^{JT}_{\dots\dots}(abcdef)$ are fully specified in terms of the quantities characterizing the original fermion system, i.e. the single-particle energies and the matrix elements of an effective nucleon-nucleon interaction. For detailes and notation we refer the reader to ref. [3].

The mean field approach in the boson picture [6] *assumes* a definite form of the ground-state (GS) wave function. For the present system of n_π protons and n_ν neutrons ($n_\pi \neq n_\nu$, n_π, n_ν even) we propose to represent the ground state by the following axially symmetric boson condensate

$$|\,GS;\ K=0,\ M_T\rangle_B = \frac{1}{\sqrt{N_1!\,N_0!}}\,(B^+_{\pm 1})^{N_1}(B^+_0)^{N_0}\,|\,0\,\rangle_B, \qquad (2)$$

where

$$B^+_\tau = \sum_{JTab} \chi^{(\tau)}_{JT}(ab)\, B^+_{JT0\tau}(ab);\quad \tau = 0,\ \pm 1 \qquad (3a)$$

$$N_1 = \tfrac{1}{2}\,|\,n_\nu - n_\pi|,\quad N_0 + N_1 = \tfrac{1}{2}\,(n_\pi + n_\nu),\quad M_T = \pm N_1 \qquad (3b)$$

and the sign + (-) holds for $n_\nu > n_\pi$ and $n_\nu < n_\pi$, respectively. Our assumed form of the GS wave function thus corresponds to the situation when protons are first paired with neutrons to generate N_0 $\pi\nu$-bosons with isospin projection $\tau = 0$, while the remaining $2N_1 = |n_\nu - n_\pi|$ identical nucleons give rise to N_1 $\nu\nu$-bosons ($\tau = +1$) or $\pi\pi$-bosons ($\tau = -1$). The total isospin projection of the state (2) is then clearly $M_T = +N_1, -N_1$ for $n_\nu > n_\pi$ and $n_\nu < n_\pi$, respectively. This concept of building the GS wave function should be contrasted with the usual approach [7] where only identical nucleons are paired to give $\pi\pi$- and $\nu\nu$-bosons. The latter approach can be well justified in heavy nuclei where protons and neutrons occupy very different single-particle orbits. However, for applications to lighter nuclei with protons and neutrons filling the same shell, the explicit inclusion of $\pi\nu$-bosons is expected to be important.

The amplitudes $\chi^{(\tau)}_{JT}(ab)$, appearing in (3a), are determined variationally by minimizing the expectation value of the boson Hamiltonian (1) in the model ground state (2). This leads to the following system of coupled non-linear eigenvalue equations for $\chi^{(0)}_{JT}(ab)$, $\chi^{(1)}_{JT}(ab)$

$$\sum_{\substack{J'T'\\ cd}} h^{(0)}_{JTab,J'T'cd}\,\chi^{(0)}_{J'T'}(cd) = \epsilon_0\,\chi^{(0)}_{JT}(ab), \qquad (4a)$$

$$\sum_{\substack{J'T'\\ cd}} h^{(1)}_{JTab,J'T'cd}\,\chi^{(1)}_{J'T'}(cd) = \epsilon_1\,\chi^{(1)}_{JT}(ab), \qquad (4b)$$

$$h^{(0)}_{JTab,J'T'cd} = \delta_{JJ'}\delta_{TT'}E^{JT}_{abcd} + \delta_{ab}\sum_{\substack{J_1T_1J_2T_2\\ efg}}\Big\{(N_0-1)\,G^{(abcJTJ'T')}_{efgJ_1T_1J_2T_2}(0)\cdot\chi^{(0)}_{J_1T_1}(eg)\,\chi^{(0)}_{J_2T_2}(fg) + 2N_1\cdot G^{(abcJTJ'T')}_{efgJ_1T_1J_2T_2}(1)\,\chi^{(1)}_{J_1T_1}(eg)\,\chi^{(1)}_{J_2T_2}(fg), \qquad (4c)$$

$$h^{(1)}_{JTab,J'T'cd} = \delta_{JJ'}\delta_{TT'}E^{JT}_{abcd} + \delta_{ab}\sum_{\substack{J_1T_1J_2T_2\\ efg}}\Big\{2N_0\,G^{(abcJTJ'T')}_{efgJ_1T_1J_2T_2}(1)\cdot\chi^{(0)}_{J_1T_1}(eg)\,\chi^{(0)}_{J_2T_2}(fg) + (N_1-1)\,G^{(abcJTJ'T')}_{efgJ_1T_1J_2T_2}(2)\,\chi^{(1)}_{J_1T_1}(eg)\,\chi^{(1)}_{J_2T_2}(fg), \qquad (4d)$$

where

$$G^{(abcJTJ'T')}_{efgJ_1T_1J_2T_2}(0) = \sum_R g^{(R)}_{J_1T_1J'T'J_2T_2JT}(ecafgb)\,\langle T_1 0\, T'0\,|\,R0\rangle\langle T_2 0\, T0\,|\,R0\rangle$$

$$G^{(abcJTJ'T')}_{efgJ_1T_1J_2T_2}(1) = \sum_R g^{(R)}_{J_1T_1J'T'J_2T_2JT}(ecafgb)\,\langle T_1 1\, T'0\,|\,R1\rangle\langle T_2 1\, T0\,|\,R1\rangle$$

$$G^{(abcJTJ'T')}_{efgJ_1T_1J_2T_2}(2) = \sum_R g^{(R)}_{J_1T_1J'T'J_2T_2JT}(ecafgb) \langle T_1 1T'1 | R2\rangle \langle T_2 1T1 | R2 \rangle$$

and

$$g^{(R)}_{J_1T_1J'T'J_2T_2JT}(ecafgb) =$$

$$= \sum_L \bar{W}^{LR}_{J_1T_1J'T'J_2T_2JT}(ecafgb)(-)^{L+R} \langle J_1 0 J'0 | L0 \rangle \langle J_2 0 J 0 | L0 \rangle$$

with the functions $\bar{W}^{LR}_{J_1T_1J'T'J_2T_2JT}(ecafgb)$ appearing in the boson Hamiltonian (1). The system of equations (4) is solved using standard iterative techniques. Together with $\chi^{(\tau)}_{JT}(ab)$ one obtains simultaneously the solutions ${}^{\sigma}\chi^{(\tau)}_{JT}(ab)$ orthogonal to $\chi^{(\tau)}_{JT}(ab)$,

$$\sum_{JTab} {}^{\sigma}\chi^{(\tau)}_{JT}(ab)\chi^{(\tau)}_{JT}(ab) = 0, \tag{5}$$

which will be useful later.

Once the GS-structure-amplitudes $\chi^{(\tau)}_{JT}(ab)$ are determined, we use the Tamm-Dancoff (TD) approximation to construct the excited states [6,7]. This amounts to diagonalizing the boson Hamiltonian (1) in the "basis"

$$|(\pm 1), K \neq 0, M_T; JTab)_B = \frac{1}{\sqrt{(N_1-1)!\,N_0!}} B^+_{JTK\pm 1}(ab)(B^+_{\pm 1})^{N_1-1}(B^+_0)^{N_0}|0)_B$$

$$|(0), K \neq 0, M_T; JTab)_B = \frac{1}{\sqrt{N_1!\,(N_0-1)!}} B^+_{JTK0}(ab)(B^+_{\pm 1})^{N_1}(B^+_0)^{N_0-1}|0)_B \tag{6}$$

in which one of the bosons building the condensate (2) has been "broken" and promoted to an excited configuration with angular momentum projection $K \neq 0$. For $K = 0$, the corresponding "basis" is given by

$$|(\pm 1), K = 0, M_T; \sigma)_B = \frac{1}{\sqrt{(N_1-1)!\,N_0!}} B^+_{\pm 1}(\sigma)(B^+_{\pm 1})^{N_1-1}(B^+_0)^{N_0}|0)_B$$

$$|(0), K = 0, M_T; \sigma)_B = \frac{1}{\sqrt{N_1!\,(N_0-1)!}} B^+_0(\sigma)(B^+_{\pm 1})^{N_1}(B^+_0)^{N_0-1}|0)_B, \tag{7}$$

where

$$B^+_\tau(\sigma) = \sum_{JTab} {}^{\sigma}\chi^{(\tau)}_{JT}(ab) B^+_{JT0\tau}(ab), \quad \tau = 0, \pm 1$$

are the boson excitations orthogonal to B^+_τ /see eq.(5)/. The eigenvectors resulting from the diagonalization then define the i-th excited state in the form

$$|i, K \neq 0, M_T)_B = \sum_{JTab} \{ a^{(i)}_{JTab}(\pm 1) |(\pm 1) K \neq 0, M_T; JTab)_B + a^{(i)}_{JTab}(0) |(0), K \neq 0, M_T; abJT)_B \}, \tag{8}$$

$$|i; K = 0, M_T)_B = \sum_\sigma \{ a^{(i)}_\sigma(\pm 1) |(\pm 1), K = 0, M_T; \sigma)_B + a^{(i)}_\sigma(0) |(0), K = 0, M_T; \sigma)_B \}. \tag{9}$$

The ground state (2) as well as the excited states (8),(9) represent what is ussually called intrinsic states [6,7], from which the actual nuclear states with definite angular momentum and isospin are obtained by projecting out the desired component. In the following we will try to describe some of the nuclear properties in terms of only three intrinsic boson states, namely

$$\begin{aligned} |1)_B &\equiv |GS;\ K = 0, M_T)_B \\ |2)_B &\equiv |i = 1;\ K = 2, M_T)_B \\ |3)_B &\equiv |i = 1;\ K = 0, M_T)_B, \end{aligned} \tag{10}$$

corresponding to the ground-, γ- and β-bands [8], respectively. As will be seen, these states are sufficient to provide a good approximation both to the low-lying energy spectra and to the electron scattering form factors for the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions in ^{18}O.

In order to be sure that the boson states (10) are indeed physical, we construct their fermion analogues with a fermion pair operator on place of each boson operator and diagonalize the corresponding norm matrix $\langle \lambda | \lambda' \rangle_F$, $\lambda = 1,2,3$. For the parameters described in sect.3 we have found that this matrix has no zero eigenvalues, which means that the respective fermion states are linearly independent. Consequently, they can be put into one-to-one correspondence with the boson states (10), thereby quaranteeing that the results obtained from the boson calculation are physically meaningful.

3. APPLICATION TO ^{18}O

The extended boson formalism developed in the preceding section has been applied to the study of the electron scattering form factors for the $0^+ \to 0^+$ and $0^+ \to 2^+$ (i = 1,2,3) transitions in ^{18}O. Electron scattering form factors are generally considered [9] to provide a stringent test for our conceptions on the structure of nuclear states because the momentum transfer dependence of the nuclear matrix elements contains much more detailed information about this structure than the energy levels and electromagnetic rates alone. The nucleus ^{18}O has been chosen because of a considerable amount of experimental data and shell-model calculations available [10, 11]. Although the shell-model treatment proved to be rather successful in reproducing the gross features of experimental data, it remains unable to account for certain subtleties such as the structure of the low-lying collective states. This can most likely be attributed to the fact that the above shell-model calculations were performed within the ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$) configuration space (hereafter called the restricted single-particle space), thereby neglecting possible excitations of the closed-shell ^{16}O core. In fact, it has been known for a long time [12] that an "effective" nucleon charge must be used in shell-model calculations to provide an approximate way of allowing for polarization of the core. Recent investigations have shown [13] that the core polarization effects may indeed be important if one is interested in a correct description of electron scattering form factors. In principle, it is possible to take into account the excitations of the core by enlarging the shell-model space involved [14], but such calculations become prohibitively time consuming even for a small number af additional configurations included. On the other hand, the present approach allows one to consider all nucleons to be active in the space consisting of $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$ orbitals (hereafter referred to as the full single-particle space) without any serious computational difficulties. Of course, this calculation is no longer exact in the shell-model sense but, as will be seen, this drawback is overwhelmingly exceeded by the possibility of including a larger model space. Moreover, there is no need to use an effective nucleon charge instead of the bare charge of a proton.

The energies associated with the single-particle orbitals given above are taken from ref. [15]. The nucleon-nucleon interaction is that of ref. [16] with the parameters V_0 = = 68.3 (MeV fm^3) η = 0.294, μ = 0.495, ξ_1 = −1.21, ξ_2 = 0.16, determined by the way proposed in [3]. The oscillator length parameter is b = 1.71 fm.

In order to estimate the role of the ^{16}O core in the description of ^{18}O, the calculations have been carried out twice. In the first calculation (referred to as B1 in the following), all nucleons are allowed to be active in the full single-particle space defined above. Correspondingly, the boson numbers N_0, N_1 in (2) take the values N_0 = 8, N_1 = 1 /see (3b)/. In the second calculation (hereafter referred to as B2), only the particles outside the inert ^{16}O core are taken into account, so that the nucleus ^{18}O is treated as a one-boson system (N_0 = 0, N_1 = 1).

The results of our calculations are summarized in figs.1-5. In fig.1 we compare the experimental excitation energies of the low-lying even-parity states in ^{18}O with those resulting

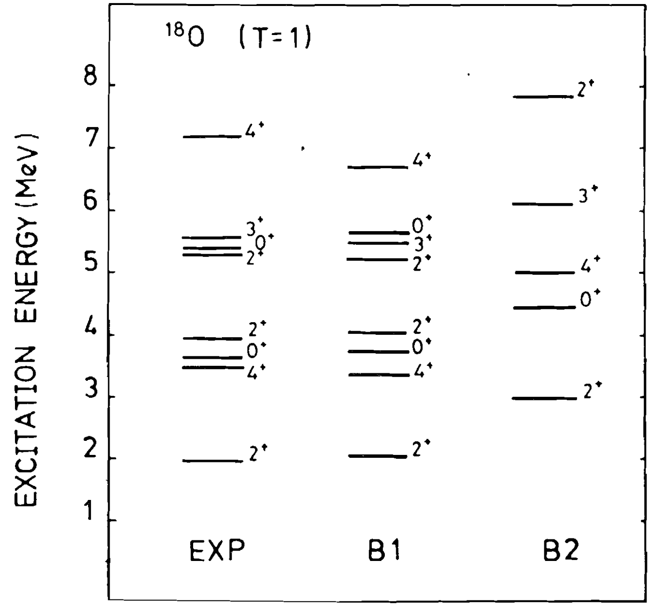

Fig.1. Even-parity T = 1 levels of ^{18}O. Experimental data are from ref. [18].

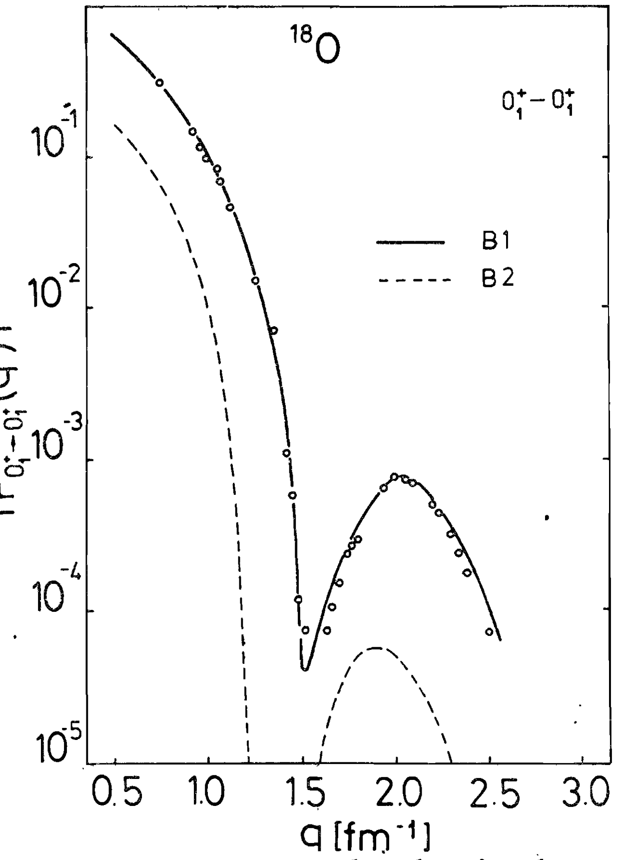

Fig.2. Form factor for the elastic electron scattering by ^{18}O. Experimental data are from ref. /11/. For further details see the text.

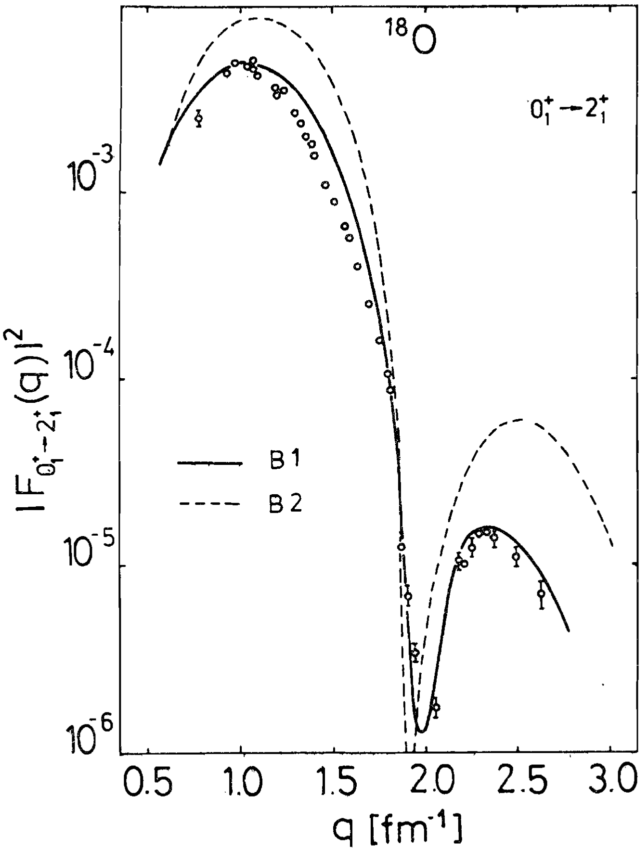

Fig.3. Electron scattering form factor for the excitation of the 2_1^+-state in ^{18}O.

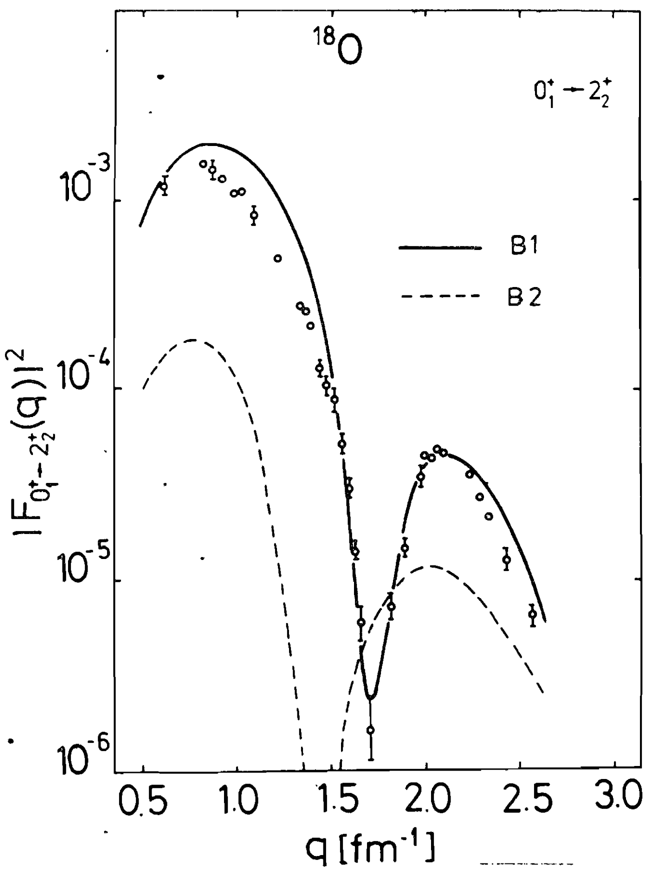

Fig.4. The same as in fig.3 for the 2_2^+ -state.

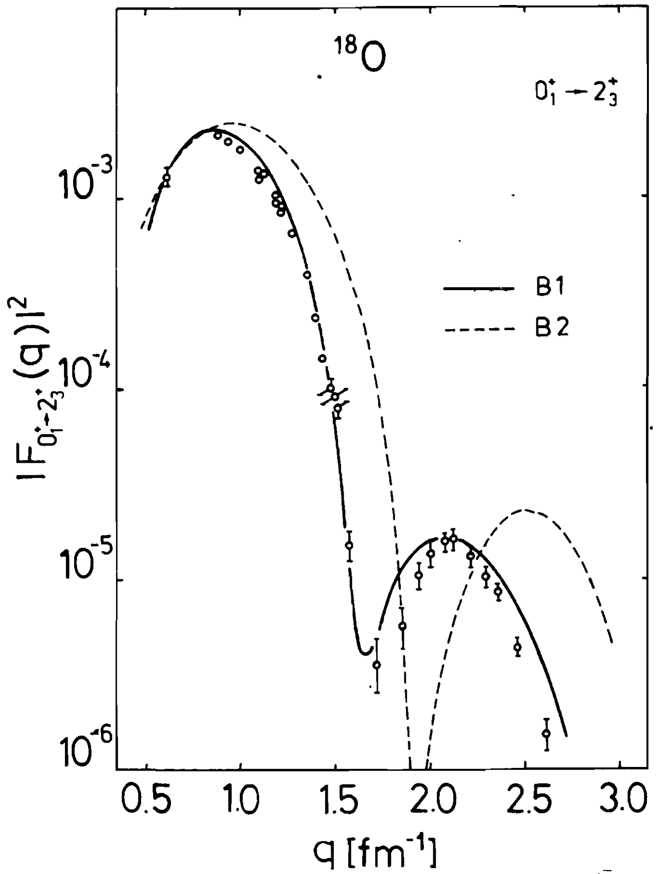

Fig.5. The same as in fig.3 for the 2_3^+-state.

from the B1 and B2 calculations, respectively. As is seen from the figure, the results of the B1 calculation agree quite well with the experimental spectra, whereas the B2 results are much worse. This indicates that the role of the core excitations in the description of the low-lying spectra of ^{18}O is non-negligible. In the following we pay attention to the 2^+_1, 2^+_2, 2^+_3 states and examine their structure by studying the respective electron scattering form factors. Our interest in the 2^+ states is motivated by the fact that the low-lying quadrupole states have always played an important role in nuclear spectroscopy [12] but a detailed understanding of their structure is still far from complete [11]. In figs.2-5 we show the calculated form factors both for the elastic scattering (fig.2) and for the excitation of 2^+_1, 2^+_2, 2^+_3 states (figs.3,4,5) in ^{18}O. The solid and dashed curves correspond to the B1 and B2 calculations, respectively. While the B1 results are in a good agreement with experimental data, the same is not true for the B2 results. This is consistent with the above suggestion that the core excitations are important for a correct description of the low-lying states in ^{18}O. It is also apparent from figs.2-5 that the core polarization effects are essential in all three of the excited 2^+-states and even in the ground state.

4. CONCLUSION

We have shown that the microscopic boson approach proposed in [3] and extended here to proton-neutron systems with $n_\pi \neq n_\nu$ provides a powerful tool for obtaining a reasonably accurate description of the observed data on the elastic as well as inelastic ($0^+ \to 2^+$) electron scattering form factors in ^{18}O. The main advantage of the present approach consists in that it can be applied without any drastic increase of computational effort to situations in which the large-scale shell-model calculations are hopeless.

In view of the achieved success, it is desirable to extend our method to odd-A nuclei as well as to apply it in other regions of the periodic table. Of special interest is an application to medium mass and heavy nuclei for which there exists a highly successful phenomelology due to the interacting boson model (IBM) [17]. Taking into account the results of our investigations, the present approach seems to be well suited to provide a microscopic understanding of this phenomenological success in terms of a large number of nucleon configurations. Work on this subject is now in progress.

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Формфакторы электронного рассеяния на ^{18}O в приближении бозонного среднего поля

Предлагается расширенный бозонный подход среднего поля к описанию протон-нейтронных систем с $n_\pi \neq n_\nu$. Рассчитаны энергетические спектры и формфакторы для электронного рассеяния в 0^+ и 2^+ состояния ^{18}O. Если взять одночастичные уровни из нижайших трех главных осцилляторных оболочек и считать все нуклоны активными, получается хорошее согласие с экспериментальными данными. Результаты также сравниваются с результатами, полученными в ограниченном подпространстве sd-оболочки. Это дает возможность оценить эффекты возможных возбуждений остова ^{16}O. Найдено, что этими эффектами пренебречь нельзя.

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Mean Field Boson Treatment of the Electron Scattering Form Factors in ^{18}O

An extended mean field boson approach to proton-neutron systems with $n_\pi \neq n_\nu$ is proposed and applied to calculating the energy spectra as well as the electron scattering form factors for the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions in ^{18}O. Provided the single-particle orbitals are taken from the lowest three major oscillator shells and all nucleons are allowed to be active, the agreement with experimental data is very good. The results are also compared with those obtained in the restricted sd-shell subspace, which enables one to estimate the effects of possible excitations of the ^{16}O core. It is found that such effects are non-negligible.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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