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ELECTRON SCATTERING FROM ^{20}Ne AND ^{24}Mg IN A MICROSCOPIC BOSON MODEL

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1. INTRODUCTION

Form factors of electron scattering are known to provide a deeper insight into the structure of nuclear states than the energy spectra and electromagnetic transition probabilities alone[1]. This is so because the momentum-transfer dependence of the associated nuclear matrix elements contains information about the spatial structure of nuclear states, thereby yielding a more stringent test on the reliability of the model wave functions. Various methods have been proposed to study the electron scattering form factors, both in heavy[2,3] and in light[4-6] nuclei. We have recently developed a new approach[7] which consists in applying the mean field (MF) approximation[8] to a microscopically derived boson Hamiltonian. This approach has proved to be rather successful in describing the energy spectra of some sd-shell nuclei, and therefore, one is naturally tempted to try a more ambitious task, namely to use the proposed method for the description of the electron scattering form factors.

Due to the obvious simplicity, the search for a bosonic treatment of fermion systems has been a long one[9] and it has been even more intensified[10] after the success of the phenomenological interacting boson model (IBM)[11]. Recently, there have also appeared some attempts toward a boson description of the electron scattering form factors [3,5,6]. These attempts are based either on the IBM[3,5] or on a specific realization of the sp(3, R) algebra in terms of the harmonic oscillator boson operators[6]. Our approach is conceptually closer to that using the IBM, in the sense that it treats the bosons as counterparts of nucleon pairs[12]. However, we go beyond the conventional IBM[11,12] because the MF approximation enables us to take into account the bosons with all possible angular momenta as well as those corresponding to photon-neutron pairs.

2. THE ELECTRON SCATTERING FORM FACTORS

Our aim is to calculate the charge (Coulomb) electron scattering form factor for a transition from an initial nuclear state characterized by angular momentum J_i and isospin T_i to a final state with J_f and T_f. In the plane-wave Born approximation (PWBA) this form factor squared at momentum transfer q is given by[1]

$$|F_{i\to f}(q)|^2 = \frac{4\pi}{z^2}\frac{1}{(2J_i+1)(2T_i+1)} |<J_f\, T_f\, |||\, \hat{\mathcal{T}}_L(q)\, |||\, J_i\, T_i>|^2 , \quad (1)$$

where z is the atomic number of the target nucleus, the transition operator $\mathcal{T}_L(q)$ has the second quantized form

$$\hat{\mathcal{T}}_L(q) = e \int_0^\infty dr \sum_{\alpha\beta} <\alpha\,|\, j_L(qr)\, Y_{LM}(\frac{1}{2} - t_3)\,|\,\beta>\, c^+_\alpha c_\beta \quad (2)$$

and the symbol $<\ldots\, |||\, ..\, |||\, \ldots>$ means that the matrix element is reduced both in the ordinary and isospin spaces. In (2), represents the charge of a nucleon, $j_L(qr)$ and Y_{LM} stand for the spherical Bessel functions and the spherical harmonics, respectively, and t_3 is the operator of the third isospin component,

$$t_3\,|\,\text{proton}> = -\frac{1}{2}\,|\text{proton}> , \quad t_3\,|\,\text{neutron}> = +\frac{1}{2}\,|\,\text{neutron}> . \quad (3)$$

In our boson formalism, the states $|J_i\, T_i>$, $|J_f\, T_f>$ are taken to be those given by eqs. (27a), (27b) of ref. /7/ and the boson image $(\hat{\mathcal{T}}_L(q))_B$ of the transition operator (2) is obtained by the replacement

$$c^+_\alpha c_\beta \to (c^+_\alpha c_\beta)_B = \sum_\gamma b^+_{\alpha\gamma} b_{\beta\gamma} . \quad (4)$$

More explicitly, we get

$$(\hat{\mathcal{T}}_L(q))_B = \sum_{\substack{J_1 T_1 J_2 T_2 \\ T\, abc}} Q^{(abc)}_{J_1 T_1 J_2 T_2\, ;\, T}(q)\, [\, B^+_{J_1 T_1}(ac) \times \tilde{B}_{J_2 T_2}(cb)]^{LT}_{M0} , \quad (5)$$

where

$$Q^{(abc)}_{J_1 T_1 J_2 T_2\, ;\, T}(q) = e(-1)^{j_a + j_b + L} \sqrt{\frac{2(2J_2+1)(2J_1+1)(2T_2+1)(2T_1+1)}{(2L+1)}} \times$$

$$\times \begin{Bmatrix} j_b & j_c & J_2 \\ J_1 & L & j_a \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T_2 \\ T_1 & T & \frac{1}{2} \end{Bmatrix} \int_0^\infty dr < a\, ||\, j_L(qr)\, Y_L\, ||\, b> . \quad (6)$$

For details of the boson representation see ref. /7/.

3. SPURIOUS EXCITATIONS

Any microscopic boson theory is accompanied by two kinds of spurious states. The first kind arises from the overcompleteness of the boson basis with respect to the fermion space. It has been shown in ref. /7/ that our approximate treatment of the boson Hamiltonian is completely free from such spurious solutions. The second kind of spuriosity is associated with the underlying shell-model Hamiltonian and it arises due to the oscillations of the nucleus as a whole in the shell-model potential (the centre-of-mass (c.m.) excitations). In the case when some approximations are introduced, these c.m. excitations can mix with the actual intrinsic excitations and their mutual separation is often very difficult or even impossible. A unitary transformation of the shell-model Hamiltonian has been invented /13/ which distributes the strength of c.m. excitations among various states in such a way that the amount of c.m. components in certain states becomes enhanced while in others it turns out to be suppressed. The only task one is then left with is to identify which states belong to the first category (c.m. excitations) and which to the second (intrinsic excitations). This is done by evaluating the quantities

$$n_{c.m.}(JT) = <JT\,|\,\hat{N}_{c.m.}\,|\,JT> , \quad (7)$$

where

$$\hat{N}_{c.m.} = \sum_{k=1}^{+1} a^+_k a_k \quad (8)$$

is the operator that counts the number of c.m.-motion quanta /13/ and $|JT>$ represents the respective nuclear state. The creation operator a^+_k in (8) is defined as

$$a^+_k = \frac{1}{\sqrt{2hmA\omega}} \{\hat{P}_k + imA\omega \hat{X}_k\} , \quad (9)$$

where $\hat{P}_k$ and $\hat{X}_k$ are the c.m. momentum and coordinate, respectively.

4. RESULTS AND DISCUSSION

We have applied the boson wave functions of nuclear states obtained in ref. /7/ to the study of the electron scattering form factors of ^{20}Mg and

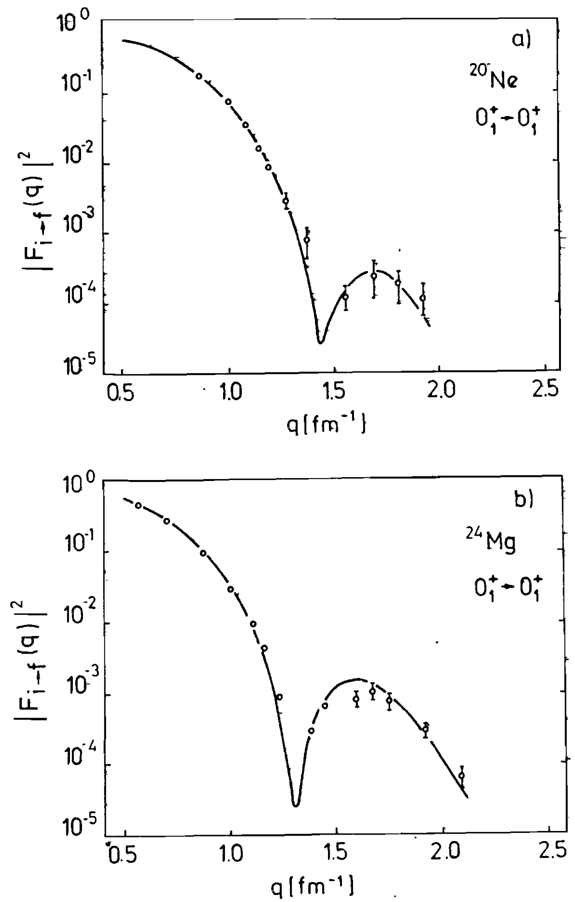


Fig. 1. Calculated form factors for the elastic electron scattering by ^{20}Ne *and* ^{24}Mg. *Experimental data are from* /14/ *and* /20/, *respectively.*

^{24}Mg. In figs. 1 and 2 we show respectively the calculated form factors for the elastic scattering and for the excitation of the two 2^+ states in the above nuclei. In all cases the agreement with experimental data is very good. Special attention was devoted to the form factor for the excitation of the third 2^+ state in ^{20}Ne because it exhibits anomalous momentum transfer dependence /14/ and the shell-model calculations in the 0d1s space were unable to reproduce it /15/. In fig. 3 we compare our results (solid curve) with those of Singhal et al. (dashed curve) /15/. As is apparent from the figure, our results agree with experimental data much better. However, we have to be aware that this comparison is only qualitative since we used a different effective interaction as well as different single-particle wave functions. Nevertheless, it suggests that configurations involving the 0p shell (which are included in our approach but were neglected in /15/) play an important role in the structure of the 2_3^+ state in ^{20}Ne. Indeed, there is some evidence /16/ for such an observation. A more detailed discussion of excitations from the 0p shell (or/and even from the 0s shell) will be given elsewhere /17/.

While the experimental form factors for the $0_1^+ \to 0_1^+$ and $0_1^+ \to 2_i^+$ (i = 1,2,3) transitions are reproduced nicely in our approach, the same is not true for the form factors corresponding to the $0_1^+ \to 4_1^+$ transitions. This is shown in fig. 4 from which one can see that the calculated form factors disagree with the experimental ones both in shape and magnitude. Possible

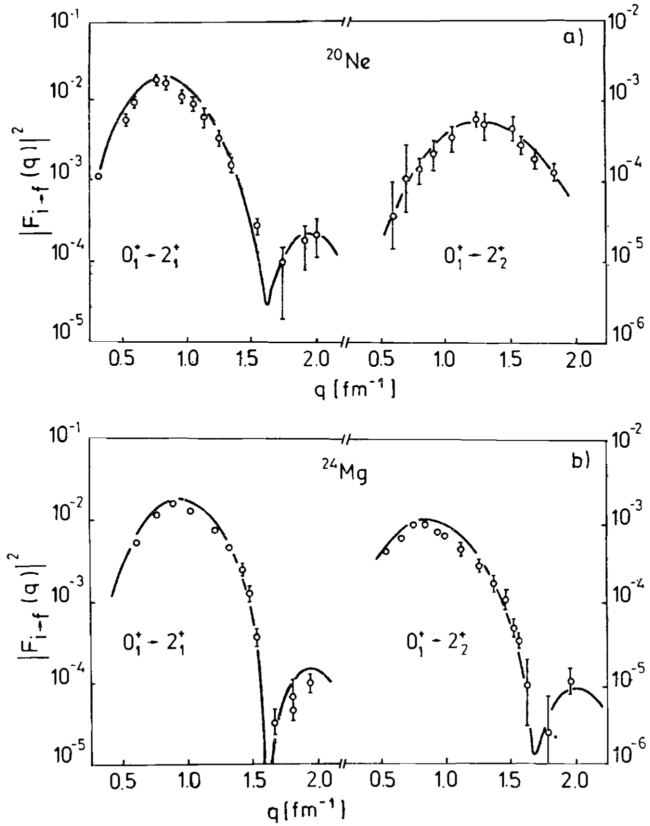


Fig. 2. Calculated form factors for inelastic electron scattering to the first two 2^+*-states in* ^{20}Ne *and* ^{24}Mg. *For experimental data see caption to fig. 1.*

sources of the observed discrepancies are the following. First, we have performed the variation before projection in this work. It is well known /18/ that such an approach does not allow for changes of the self-consistent internal field within a rotational band. Since there exists no a priori decoupling between rotational and intrinsic motion, the method of variation before projection can be expected to work well only in cases when the coupling terms are relatively small, which occurs most likely for not-too-large angular momenta. How large they may be depends in an essential way on the choice of the intrinsic system. It is indeed possible that in the present case the values J = 0, J = 2 (for which we have obtained excellent results) still permit a sufficiently good decoupling between intrinsic and rotational motion, whe-

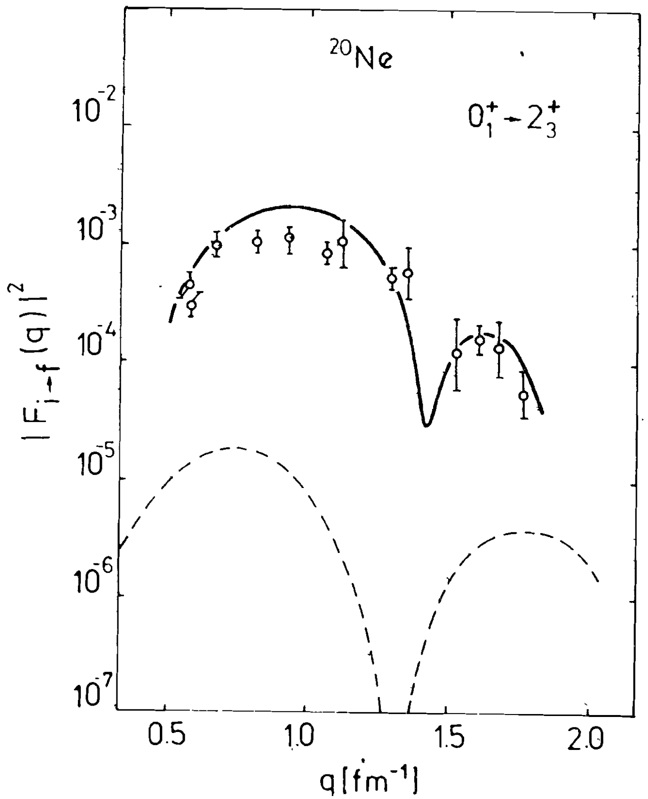
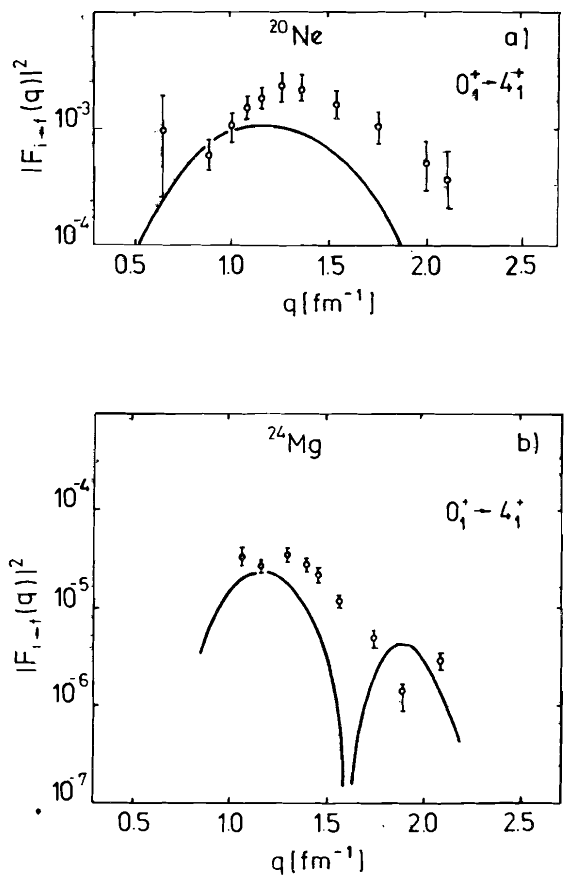


Fig. 3. Electron scattering form factor for the excitation of the 2^+_3 state in ^{20}Ne . The solid curve is our result obtained in the present boson approach, the dashed curve represents the result of the shell-model calculation in the restricted sd-shell space, taken from /15/. Experimental data are from /14/.

reas the value J = 4 is already too large to allow the variation before projection to be correct. The second reason for the failure of our approach in reproducing the $0^+_1 \to 4^+_1$ form factors can be traced back to the occurrence of c.m. components in the 4^+_1 state. In the table we give the quantities (7), which measure the amount of c.m. excitations in a given state, for the relevant cases (i.e. the 0^+_1, 2^+_1, 2^+_2, 4^+_1 states). It is seen that the values of these quantities for the 4^+_1 states are by two orders of magnitude larger than those for the 0^+_1 and 2^+_i (i = 1, 2) states, irrespective of the fact that the former are still very small (~ 10^{-4}). It remains to be investigated to what extent such an admixture of c.m. excitations destroys the structure of a given sta-

Fig. 4. Calculated form factors for the excitation of the first 4^+ state in ^{20}Ne and ^{24}Mg. For experimental data see caption to fig. 1.



te. Since the electron scattering form factors are known /1-6/ to be very sensitive to the actual form of the wave function, even a small admixture of the c.m. excitation may have large influence on the final result.

We have thus suggested two possible sources of the discrepancies observed between the experimental and calculated form factors for the excitation of the 4^+ states in ^{20}Ne and ^{24}Mg . While the implementation of the first point (i.e. projection before variation) is rather straightforward, an answer to the question of how to remove the c.m. excitations from the physical states is still far from being clear. Of course, we could resort to an exact treatment in a restricted single-particle space and use the special techniques for isolating the c.m. excitations

Table

Expectation values of the c.m.-quanta counting operator (8) in some $|J^\pi_i, T=0>$ states of ^{20}Ne and ^{24}Mg.

J^π_i	0^+_1	2^+_1	2^+_2	4^+_1
^{20}Ne	0.000 001	0.000 001	0.000 002	0.000 135
^{24}Mg	0.000 001	0.000 000	0.000 001	0.000 172

which are availabe in this case [19]. However, the main advantage of our approximate method, namely the possibility of including a large single-particle space without serious difficulties, would then be lost. We are therefore currently engaged in an effort to develop a sound and reliable method for dealing with this problem.

5. CONCLUSION

In this report we have studied the electron scattering form factors of some sd-shell nuclei (^{20}Ne , ^{24}Mg) using a second quantized boson representation of the relevant operators. Good agreement with experimental data was obtained both for the elastic scattering and for excitation of $2^+_i (i = 1,2,3)$ states. Since the (e , e′) form factors are rather sensitive to the structure of nuclear wave functions, the above observation shows that the present boson approach is able to incorporate many nucleon correlations in a very efficient way. At the same time, however, we have observed some discrepancies between the calculated results and experimental data for the $0^+_1 \to 4^+_1$ form factors. Possible explanation was suggested, indicating the urgent need for a careful and detailed treatment of the effects associated with the symmetry restoration and the interplay of intrinsic and c.m. excitations.

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Рассеяние электронов на ^{20}Ne и ^{24}Mg в микроскопической бозонной модели

Показано, что приближение среднего поля, примененное к микроскопически построенному бозонному гамильтониану, предоставляет разумное описание формфакторов для упругого и неупругого рассеяния на ядрах sd-оболочки (^{20}Ne, ^{24}Mg). Результаты хорошо согласуются с экспериментальными данными для $0^+ \to 0^+$ и $0^+ \to 2^+$ переходов, но гораздо хуже для $0^+ \to 4^+$ переходов. Предлагается возможное объяснение наблюдаемых различий.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Electron Scattering from ^{20}Ne and ^{24}Mg in a Microscopic Boson Model

It is shown that a mean-field approximation applied to the microscopically derived boson Hamiltonian yields a reasonable description of the form factors for both elastic and inelastic electron scattering from some sd-shell nuclei (^{20}Ne, ^{24}Mg). The results agree well with experimental data for the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions but much less so for the $0^+ \to 4^+$ transitions. Possible sources of the observed discrepancies are suggested.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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