сообщения
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исследований
дубна

E4-87-747

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MICROSCOPIC BOSON APPROACH TO THE DESCRIPTION OF sd-SHELL NUCLEI

1987

1. INTRODUCTION

The widespread occurrence of boson-like features in nuclear collective spectra has stimulated a great deal of interest in various kinds of boson descriptions [1-10], both at the phenomenological [1,2] and microscopic [3-10] levels. All attempts to derive microscopically the boson description of the underlying fermion system are centered around the so-called mapping procedure [4] whose crucial point is the transition from the fermion space to the boson space.

There exist several mapping procedures [3-10], among which the Dyson transformation [3] seems to be the most attractive one. The main advantage of the Dyson mapping is its finiteness. The images of the fermion operators are expressed as finite series in the boson operators, so that there is no need to neglect the higher-order interaction terms whose significance is often hard to assess [4]. This attractive property is achieved at the expense of allowing the boson Hamiltonian to be nonhermitian. In spite of this fact, the Dyson mapping has become very popular in recent years [11-18], the main assertion being that the nonhermiticity of the boson Hamiltonian is in fact not serious drawback [16-18]. However, when calculating the electromagnetic transition probabilities in this approach, the correct normalization factors of the *boson* wave functions require an explicit knowledge of the *fermion* norm matrices [12,13]. This is rather inconvenient because one is tempted to believe [14] that once the boson images of fermion operators are found, any sensible microscopic boson theory must be able to provide all the desired physical information within the boson space only; i.e. with no additional recourse to the original fermion space. For this reason we consider it preferable to map the fermion Hamiltonian in such a way that the resulting boson Hamiltonian is *both* finite *and* hermitian. This is accomplished by transcribing the fermion Hamiltonian into the particle-hole (ph) form and exploiting the fact that the ph-operators $c^+_\alpha c_\beta$ are mapped exactly onto one-body boson operators, maintaining at the same time their original properties with respect to hermitian conjugation.

In order for the derived boson Hamiltonian to make practical sense, it must be treated within a suitable approximation. We propose to use the mean field (MF) approach [2] which has proved to be very successful in dealing with many-boson systems [19-21]. As will be seen, the results of our work indicate that this choice is indeed reasonable. Since any calculation in the boson space may in general be accompanied by the occurrence of nonphysical (spurious) solutions [14,15], we present a brief discussion of this problem as well.

2. BOZONIZATION OF THE FERMION SYSTEM

We consider a system of n_π protons and n_ν neutrons ($n_\pi = n_\nu$ = even). The nuclear Hamiltonian is taken to have the general form

$$\hat{H}_F = \sum_{\alpha\beta} T_{\alpha\beta} c^+_\alpha c_\beta + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\, c^+_\alpha c^+_\beta c_\delta c_\gamma , \tag{1}$$

where the indices $\alpha, \beta, \gamma, \delta$ run through a suitable complete set of single-particle states and $c^+_\alpha (c_\alpha)$ are the corresponding creation (annihilation) operators of nucleons. The quantities $T_{\alpha\beta}$ stand for the matrix elements of a one-body operator, such as the kinetic energy, while the $V_{\alpha\beta\gamma\delta}$ represent the matrix elements of an effective nucleon-nucleon interaction. These matrix elements are assumed to be real and to satisfy

$$V_{\alpha\beta\gamma\delta} = -V_{\beta\alpha\gamma\delta} = -V_{\alpha\beta\delta\gamma} \quad \text{(antisymmetry)} \tag{2}$$

$$V_{\alpha\beta\gamma\delta} = V_{\gamma\delta\alpha\beta}, \quad T_{\alpha\beta} = T_{\beta\alpha} \quad \text{(hermiticity)}. \tag{3}$$

Using the relation

$$c^+_\alpha c^+_\beta c_\delta c_\gamma = \delta_{\beta\delta} c^+_\alpha c_\gamma - c^+_\alpha c_\delta c^+_\beta c_\gamma , \tag{4}$$

we can rewrite (1) as

$$\hat{H}_F = \hat{H}_0 + \hat{H}_{int} , \tag{5a}$$

where

$$\hat{H}_0 = \sum_{\alpha\beta} (T_{\alpha\beta} + \frac{1}{4} \sum_\gamma V_{\alpha\gamma\beta\gamma})\, c^+_\alpha c_\beta , \tag{5b}$$

$$\hat{H}_{int} = -\frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\, c^+_\alpha c_\delta c^+_\beta c_\gamma . \tag{5c}$$

If we choose the single-particle states to form the Hartree-Fock (HF) basis, the one-body term (5b) is clearly diagonal because

$$T_{\alpha\beta} + \frac{1}{4} \sum_\gamma V_{\alpha\gamma\beta\gamma} = \epsilon_\alpha \delta_{\alpha\beta} \tag{6}$$

by definition of the HF basis. However, in cases when the single-particle states are restricted to those taken from neighbouring shells only, $\hat{H}_0$ may be diagonal in the harmonic oscillator basis as well [5]. We make this choice here, so that $\hat{H}_F$ acquires the form

$$\hat{H}_F = \sum_\alpha \epsilon_\alpha c^+_\alpha c_\alpha - \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\, c^+_\alpha c_\delta c^+_\beta c_\gamma \tag{7}$$

and the single-particle states $\alpha, \beta, \ldots$ are characterized by the oscillator quantum numbers in the isospin formalism [22]: $\alpha = (n_a, \ell_a, j_a, \frac{1}{2}, m_\alpha, \tau_\alpha)$, where $\tau_\alpha = +\frac{1}{2} (\tau_\alpha = -\frac{1}{2})$ for neutrons (protons). We will also use the letter a to denote the same set except m_α and τ_α.

Now, corresponding to the fermion pair operators $c^+_\alpha c^+_\beta$, $c_\beta c_\alpha$, we introduce a set of boson operators $b^+_{\alpha\beta}$, $b_{\alpha\beta}$ which obey the following antisymmetry and commutation relations:

$$b^+_{\alpha\beta} = -b^+_{\beta\alpha} , \tag{8}$$

$$\begin{aligned} &[b_{\alpha\beta}, b_{\alpha'\beta'}] = [b^+_{\alpha\beta}, b^+_{\alpha'\beta'}] = 0 , \\ &[b_{\alpha\beta}, b^+_{\alpha'\beta'}] = \delta_{\alpha\alpha'} \delta_{\beta\beta'} - \delta_{\alpha\beta'} \delta_{\beta\alpha'} . \end{aligned} \tag{9}$$

The boson vacuum $|0)_B$ is defined by the condition

$$b_{\alpha\beta} |0)_B = 0 . \tag{10}$$

In the Dyson representation, the boson images of the bifermion operators $c^+_\alpha c_\beta$ can be expressed as [3]

$$(c^+_\alpha c_\beta)_B = \sum_\gamma b^+_{\alpha\gamma} b_{\beta\gamma} . \tag{11}$$

The boson Hamiltonian $\hat{H}_B$, corresponding to the fermion Hamiltonian (7), is then simply

$$\hat{H}_B = \sum_{\alpha} \varepsilon_\alpha (c^+_\alpha c_\alpha)_B - \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} (c^+_\alpha c_\delta)_B (c^+_\beta c_\gamma)_B . \qquad (12)$$

By substituting (11) into (12) and arranging the boson operators in the interaction term into the normal order with respect to the boson vacuum (10), we obtain

$$\hat{H}_B = \sum_{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta}\, b^+_{\alpha\beta} b_{\gamma\delta} - \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \sum_{\rho\sigma} V_{\alpha\beta\gamma\delta}\, b^+_{\alpha\rho} b^+_{\beta\sigma} b_{\delta\rho} b_{\gamma\sigma} , \qquad (13)$$

where

$$\epsilon_{\alpha\beta\gamma\delta} = \delta_{\beta\delta} [\varepsilon_\alpha \delta_{\alpha\gamma} - \frac{1}{4} \sum_{\rho} V_{\alpha\rho\gamma\rho}] + \frac{1}{4} V_{\alpha\beta\gamma\delta} . \qquad (14)$$

With the help of (2), (3) and (8) it can easily be verified that the boson Hamiltonian (13) is hermitian.

As is well known /22, 14, 15/ , any boson image of a fermion Hamiltonian has not only the solutions corresponding to actual states of the underlying fermion system but also the spurious ones which are associated with the overcompleteness of the boson basis. However, since the operators (11) commute with the projector onto the physical boson subspace /3, 22/ , it is seen from (12) that $\hat{H}_B$ does not mix the physical and spurious boson states. Consequently, these two types of boson states are strictly separated from each other,

As a next step, we introduce a unitary transformation to bosons with good angular momentum and isospin

$$B^+_{JTM\tau}(ab) = \frac{1}{\sqrt{2}} \sum_{\substack{m_\alpha m_\beta \\ \tau_\alpha \tau_\beta}} \langle j_a m_\alpha j_b m_\beta | JM \rangle \langle \tfrac{1}{2}\tau_\alpha \tfrac{1}{2}\tau_\beta | T\tau \rangle\, b^+_{am_\alpha\tau_\alpha\, bm_\beta\tau_\beta} , \qquad (15)$$

where the quantity $\langle \ldots . | .. \rangle$ is the usual Clebsch-Gordan coefficient. The operators (15) with $T = 1$, $\tau = 1, -1$ correspond to the neutron-neutron $(\nu\nu)$ and proton-proton $(\pi\pi)$ bosons, respectively, those with $\tau = 0$ represent two types ($T = 0$, $T = 1$) of proton-neutron $(\pi\nu)$ bosons. Inverting the expression (15) and substituting into (13), we obtain after some angular-momentum algebra

$$\hat{H}_B = \sum_{JTM\tau} \sum_{abcd} E^{JT}_{abcd} B^+_{JTM\tau}(ab)\, B_{JTM\tau}(cd) +$$

$$+ \frac{1}{2} \sum_{\substack{J_1 J_2 J_3 J_4 \\ T_1 T_2 T_3 T_4}} \sum_{\substack{abcdef \\ JT}} \bar{W}^{JT}_{J_1 T_1 J_2 T_2 J_3 T_3 J_4 T_4}(abcdef) \times \qquad (16)$$

$$\times ([B^+_{J_1T_1}(ae) \times B^+_{J_2T_2}(bf)]^{JT} \cdot [\tilde{B}_{J_3T_3}(de) \times \tilde{B}_{J_4T_4}(cf)]^{JT}) .$$

Here,

$$\tilde{B}_{JTM\tau}(ab) = (-)^{J+T-M-\tau} B_{JT-M-\tau}(ab) ,$$

the symbol $[\ldots \times \ldots]^{JT}_{M\tau}$ denotes the standard angular momentum and isospin coupling, $([\ ]^{JT} \cdot [\ ]^{JT})$ stands for the scalar product of tensor operators and

$$E^{JT}_{abcd} = \delta_{ac}\, \delta_{bd} \left[(\varepsilon_a + \varepsilon_b) - \sum_{J'T'f} \frac{(2J'+1)(2T'+1)}{4(2ja+1)} V^{J'T'}_{afaf}\right] + \frac{1}{2} V^{JT}_{abcd} , \qquad (17a)$$

$$\bar{W}^{JT}_{1234} = W^{JT}_{1234} + (-)^{J_3+J_4+T_3+T_4+T+J}\, W^{JT}_{1234} , \qquad (17b)$$

$$1 \equiv (J_1 T_1), \quad 2 \equiv (J_2, T_2), \; \ldots .$$

$$W^{JT}_{1234}(abcdef) = \sum_{J'T'KS} (-)^{j_e+j_f+K+J_3+J_4+S+T_3+T_4} \times$$

$$\times \hat{J}_1 \hat{J}_2 \hat{J}_3 \hat{J}_4 \hat{T}_1 \hat{T}_2 \hat{T}_3 \hat{T}_4\, \hat{V}^{J'T'}_{abcd} (2J'+1)(2K+1)(2T'+1)(2S+1) \times \qquad (17c)$$

$$\begin{Bmatrix} j_b & j_a & J' \\ J_2 & J_1 & J \\ j_f & j_e & K \end{Bmatrix} \begin{Bmatrix} j_d & j_c & J' \\ J_3 & J_4 & J \\ j_e & j_f & K \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T' \\ T_2 & T_1 & T \\ \frac{1}{2} & \frac{1}{2} & S \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T' \\ T_3 & T_4 & T \\ \frac{1}{2} & \frac{1}{2} & S \end{Bmatrix}$$

In (17), $\hat{J} = \sqrt{2J+1}$, $\begin{Bmatrix} \ldots \\ \ldots \\ \ldots \end{Bmatrix}$ is the 9j-symbol and V^{JT}_{abcd} is the angular momentum- and isospin-coupled version of the matrix elements $V_{\alpha\beta\gamma\delta}$.

Summarizing this section, we have constructed a boson Hamiltonian [eq.(16)] which

i) is an exact image of the original shell-model Hamiltonian (1);

ii) preserves its hermiticity;
iii) contains at most two-body boson interactions;
iv) does not mix the physical and unphysical (spurious) boson states;
v) has the isospin invariant form, which enables one to include two types of $\pi\nu$-bosons in addition to the usual $\pi\pi$- and $\nu\nu$-bosons.

To our knowledge, no use of the boson Hamiltonian with all the five properties has been reported in the literature thus far.

3. APPROXIMATE TREATMENT OF THE BOSON HAMILTONIAN

Although the Hamiltonian (16) has the desirable properties given above, its exact diagonalization is quite a formidable problem which can only be handled in very special situations and for a substantially limited number of bosons. Moreover, even if an exact treatment is still possible on a computer, the underlying physical picture is very obscure due to the tremendous amount of terms arising from the diagonalization procedure. A way of circumventing these difficulties consists in the use of approximate mean field (MF) techniques /2, 19-22/ that are based upon few relevant physical ingredients and at the same time include many different degrees of freedom on an equal footing. The MF approach in the boson picture starts from the Hartree-Bose (HB) approximation to the ground state (GS) /2/. For the present system of n_π protons and n_ν neutrons ($n_\pi = n_\nu = N =$ even), the GS wave function is assumed to be represented by an axially symmetric condensate of the form

$$|GS)_B = \frac{1}{\sqrt{N!}}(B_g^+)^N\,|0)_B\,, \tag{18}$$

where

$$B_g^+ = \sum_{JTab} \chi_{JT}^{(g)}(ab)\, B_{JT00}^+(ab)\,, \tag{19}$$

$$\sum_{JTab} |\chi_{JT}^{(g)}(ab)|^2 = 1. \tag{20}$$

The coefficients $\chi_{JT}^{(g)}(ab)$ are determined variationally by minimizing the expectation value of the boson Hamiltonian (16) in the model ground state (18) under the constraint (20). This leads to the following system on nonlinear eigenvalue equations for $\chi_{JTab}^{(g)}$:

$$\sum_{\substack{J'T'\\cd}} \mathcal{H}^{(g)}_{JTab,J'T'cd}\, \chi^{(g)}_{J'T'}(cd) = \lambda_g\, \chi^{(g)}_{JT}(ab)\,, \tag{21a}$$

$$\mathcal{H}^{(g)}_{JTab,J'T'cd} = \delta_{JJ'}\delta_{TT'}E^{JT}_{abcd} + (N-1)\,\delta_{db}\,\times \tag{21b}$$

$$\times \sum_{J_1T_1J_2T_2}\sum_{efh} w^{(0)}_{abcefh}(J_1T_1J_2T_2JTJ'T')\,\chi^{(g)}_{J_1T_1}(eh)\,\chi^{(g)}_{J_2T_2}(fh)\,,$$

where

$$w^{(0)}_{abcefh}(J_1T_1J_2T_2JTJ'T') = \sum_{RL} \bar{W}^{LR}_{J_1T_1J'T'J_2T_2JT}(ecafhb)\,\times \tag{22}$$

$$\times\,(-)^{L+R}\langle J_1 0 J'0|L0\rangle\langle J_2 0 J 0|L0\rangle\langle T_1 0 T' 0|R0\rangle\langle T_2 0 T 0|R0\rangle$$

with $\bar{W}$ given by (17b,c).

In the second step, the MF approach considers the one-boson excitations of the condensate (18),

$$|iK)_B = \frac{1}{\sqrt{(N-1)!}}\, B^+_{iK}(B_g^+)^{N-1}|0)_B\,, \tag{23}$$

where

$$B^+_{iK} = \sum_{JTab}\psi^{(iK)}_{JT}(ab)\, B^+_{JTK0}(ab)\,. \tag{24}$$

The coefficients $\psi^{(iK)}_{JT}(ab)$ are determined by the requirement that the states (23) be eigenstates of H_B, i.e.

$$\hat{H}_B|iK)_B = \mathcal{E}_{iK}|iK)_B\,. \tag{25}$$

This leads to

$$\sum_{J'T'cd} \mathcal{K}^{(K)}_{JTab,J'T'cd}\,\psi^{(iK)}_{J'T'}(cd) = \mathcal{E}_{ik}\,\psi^{(iK)}_{JT}(ab)\,, \tag{26a}$$

$$\mathcal{K}^{(K)}_{JTab,J'T'cd} = \delta_{JJ'}\delta_{TT'}E^{JT}_{abcd} + (N-1)\sum_{\substack{ef\\J_1T_1J_2T_2}} \chi^{(g)}_{J_1T_1}(ed)\,\times \tag{26b}$$

$$\times\,[\chi^{(g)}_{J_2T_2}(fb) + \chi^{(g)}_{J_2T_2}(fa)]\, w^{(K)}_{aecfbd}(JTJ_1T_1J_2T_2J'T')\,.$$

Note that (26) represents a system of *linear* equations for $\psi^{(iK)}_{JT}(ab)$ because the matrix $\mathcal{K}^{(K)}_{JTab,J'T'cd}$ is fully specified by the quantities arising from the GS calculation ($\chi^{(g)}_{JT}(ab)$). In the following we will consider only the lowest (i = 1) solutions of (26) with K = 0 and K = 2, corresponding to the so-colled β- and γ-bands, respectively .

To proceed further, we must restore the spherical and isospin symmetries which are obviously broken in the boson wave functions of the form (18) and (23). This is accomplished by angular momentum- and isospin projection according to [22]

$$|GS; JT; MM_T)_B = n^{GS}_B \hat{P}^J_{MO} \hat{P}^T_{M_T O} |GS)_B , \tag{27a}$$

$$|i; JT; MM_T)_B = n^i_B \hat{P}^J_{MK} \hat{P}^T_{M_T O} |iK)_B , \tag{27b}$$

where n^{GS}_B, n^i_B are the normalization constants and $\hat{P}^J_{MM'}$, $\hat{P}^T_{\tau\tau'}$ stand for the projection operators onto states with definite angular momentum and isospin, respectively. The energies of different nuclear states $|JT>$ are then obtained by simply taking the expectation value of H_B in the states (27a,b)

4. PHYSICAL BOSON STATES

In order for the procedure described in the preceding section to be reliable, we must be sure that the basic boson states (18) and (23) are indeed physical,i.e.that they are in one-to-one correspondence with actual states of the underlying fermion system. We therefore construct the following fermion analogues (not images!) of the above-mentioned boson states:

$$|GS>_F = \mathcal{N}_{GS} (\Gamma^+_g)^N |0>_F , \tag{28}$$

$$|iK>_F = \mathcal{N}_{iK} \Gamma^+_{iK} (\Gamma^+_g)^{N-1} |0>_F , \tag{29}$$

where $\mathcal{N}_{GS}$, $\mathcal{N}_{iK}$ are the normalization constants, $|0>_F$ denotes the fermion vacuum ($c_\alpha|0>_F = 0$) and the operators Γ^+_g, Γ^+_{iK} are given by

$$\Gamma^+_g = \sum_{abJT} \chi^{(g)}_{JT}(ab) [c^+_a \times c^+_b]^{JT}_{00} , \tag{30a}$$

$$\Gamma^+_{iK} = \sum_{abJT} \psi^{(iK)}_{JT}(ab) [c^+_a \times c^+_b]^{JT}_{KO} \tag{30b}$$

with the same coefficients $\chi^{(g)}_{JT}(ab)$, $\psi^{(iK)}_{JT}(ab)$ as in (19), (24). At the present stage, it should be emphasized that the fermion states (28) and (29) *cannot* in general be considered as counterparts of the boson states (18) and (23), respectively. This is so because the former may be linearly dependent (as a consequence of the Paul principle) while the latter are always linearly independent. It is thus necessary to check explicitly whether or not the linear dependence among fermion states occurs. This is done by diagonalizing the norm matrix

$$\mathcal{M} = \begin{pmatrix} <GS|GS>_F ; & <i'K|GS>_F \;\cdots \\ <GS|iK>_F & \cdots\cdots \\ \vdots & \mathcal{M}^{(K)}_{i',i} \end{pmatrix} \tag{31}$$

with $\mathcal{M}^{(K)}_{i',i} = <i'K|iK>_F$ and looking at the eigenvalues. In the case investigated in this paper (i = 1, K = 0; i = 1, K = 2) we have found (for parameters of the calculation see sect.5) that none of the eigenvalues is zero, which implies linear *independence* of the fermion states $|GS>_F$, $|iK>_F$. Consequently, the boson states (18) and (23) can be put into one-to-one correspondence with the fermion states (28) and (29), respectively, thereby showing that they indeed represent certain physical states of the fermion system considered.

5. APPLICATION TO ^{20}Ne AND ^{24}Mg

As a first realistic example, the general results of the previous sections have been applied to calculating the energy spectra of ^{20}Ne and ^{24}Mg. The calculations were performed using the oscillator model single-particle space consisting of the $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ - shells with energies [23] $\varepsilon(0p_{3/2}) = -21.8$ MeV, $\varepsilon(0p_{1/2}) = -15.65$ MeV, $\varepsilon(0d_{5/2}) = -4.15$ MeV, $\varepsilon(1s_{1/2}) = 3.28$ MeV, $\varepsilon(0d_{3/2}) = 0.93$ MeV. As for the nucleon-nucleon interaction, we combine a phenomenological form [24]

$$V_{res}(i,j) = V_0 \frac{\delta(r_i - r_j)}{r_i r_j} \sum_L \vec{Y}_L(i) \cdot \vec{Y}_L(j) \times \tag{32}$$

$$\times \{1 - \eta + \eta \vec{\sigma}(i) \cdot \vec{\sigma}(j)\} \{1 - \mu + \mu \vec{\tau}(i) \cdot \vec{\tau}(j)\}$$

with a two-body spin-orbit interaction /25/

$$V_{s.o.}(i,j) = \frac{\xi_\ell}{A} \{(\vec{r}_i - \vec{r}_j) \times (\vec{p}_i - \vec{p}_j)\}(\vec{\sigma}(i) + \vec{\sigma}(j)) \tag{33}$$

to get the correct spin-orbit splitting of the single-particle levels. The parameters V_0, η, μ, ξ_ℓ, as well as the oscillator length parameter b, were determined so that the diagonal elements of the HF matrix $h^{HF}_{\alpha\alpha} = T_{\alpha\alpha} + 1/4 \sum_\beta V_{\alpha\beta\alpha\beta}$ provide the

best fit to the above single-particle energies and the GS energy calculated in the present model reproduces the experimental binding energy of a given nucleus. The resulting values are listed in the Table.

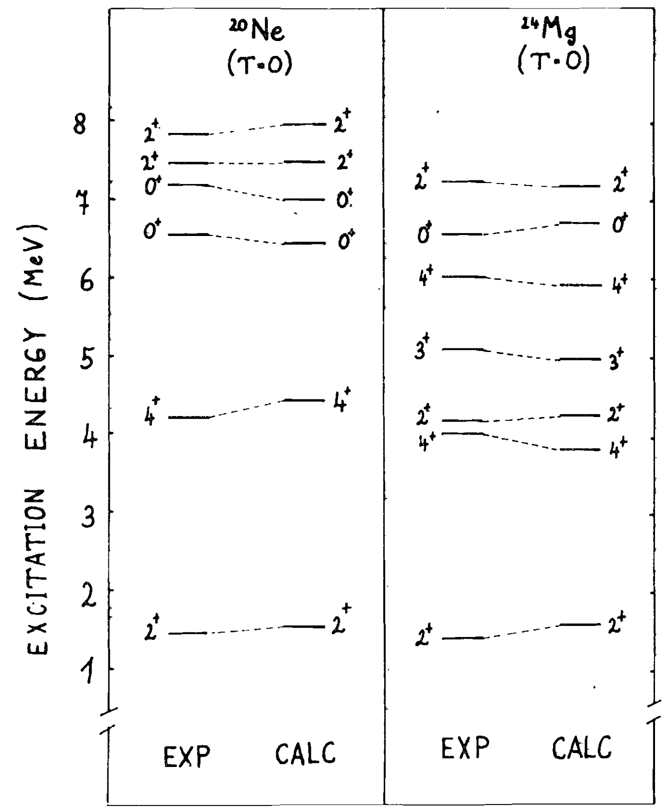
Table

Calculated parameters characterizing the nucleon-nucleon interaction (32), (33) and the single-particle oscillator wave functions ($b = m\omega/h$)

Nucleus	V_0 (MeV fm^2)	η	μ	ξ_1	ξ_2	b (fm)
^{20}Ne	73.6	0.254	0.509	-1.26	0.16	1.65
^{24}Mg	74.1	0.257	0.490	-1.26	0.16	1.68

Since the oscillator basis is not translationally invariant, spurious centre-of-mass (c.m.) excitations may enter the spectrum. In order to avoid them we use the transformed shell-model Hamiltonian /26/

$$\tilde{H} = \frac{A-1}{A} \sum_{i=1}^{A} \epsilon_i^{osc} + \frac{1}{2} \sum_{i \neq j}^{A} \{V_{res}(i,j) + V_{s.o.}(i,j) + V_{c.m.}(i,j)\}, \tag{34}$$



Positive-parity T = 0 *levels of* ^{20}Ne *and* ^{24}Mg. *Experimental values are from ref.* /27/.

where

$$V_{c.m.}(i,j) = -\frac{1}{A}\{\frac{1}{2m}\vec{p}_i \cdot \vec{p}_j + \frac{1}{2} m\omega^2 \vec{r}_i \cdot \vec{r}_j\}. \tag{35}$$

This transformation is expected to separate the c.m. excitations from the intrinsic ones and it has indeed proved to be quite successful in some realistic calculations /5/.

In the figure we compare the experimental positive-parity T = 0 levels of ^{20}Ne and ^{24}Mg with those calculated in the present boson approach. As is seen, the agreement is quite good, both for ^{20}Ne and for ^{24}Mg. This suggests the potential power and utility of our approach, at least for the description of the energy spectra in light nuclei.

6. SUMMARY AND DISCUSSION

In this paper we have proposed a new method for studying the energy spectra of some sd-shell nuclei using a second quantized boson representation of the Hamiltonian. The key ingredients of our approach are the Dyson boson mapping applied to the particle-hole form of the fermion Hamiltonian and the subsequent approximate treatment of the resulting boson Hamiltonian in the framework of the mean field (MF) techniques. The boson states arising from the calculation were shown to be free from spurious components due to the violation of the Pauli principle. The angular momentum and isospin eigenstates were projected out of them and used to calculating the spectra. The results obtained in this way agree well with the experimental data and they indicate that the MF approach applied to a suitable boson image of the nuclear Hamiltonian provides a promising tool for the investigation of nuclear structure. The main advantage of the MF approach consists in that it is able to include many different degrees of freedom on an equal footing with no drastic increase in the numerical work involved, which is certainly not the case in an exact treatment. Although the MF approach is not new in nuclear physics, its applications to boson problems have appeared only recently /2,19-21/ as a reasonable alternative of approximately solving the Schrödinger equation appropriate to the interacting boson model (IBM)/1/. All these MF calculations, however, have been performed at the level of the pure IBM phenomenology, i.e. with no direct connection to the underlying fermion problem. The present study thus appears to be the first report on the *microscopic boson MF Theory*, in the sense that it

i) starts from a shell-model Hamiltonian with realistic single-particle energies and an effective nucleon-nucleon interaction,

ii) maps exactly the hermitian one-plus two-body fermion Hamiltonian onto a hermitian one-plus two-body boson Hamiltonian,

iii) solves the resulting boson Hamiltonian approximately within the MF approach, extracting all the necessary physical information exclusively from the boson picture,

iv) needs no additional procedure to deal with spurious boson solutions.

ACKNOWLEDGEMENT

The author thanks Dr.M.Gmitro for his stimulating interest in this work.

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Received by Publishing Department
on October 14, 1987.

SUBJECT CATEGORIES
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Кухта Р. E4-87-747
Микроскопический бозонный подход к описанию ядер sd-оболочки

Предлагается микроскопический метод для изучения легких ядер с одинаковым числом протонов и нейтронов в модели многих взаимодействующих бозонов. Построен точный бозонный образ фермионного гамильтониана и динамика фермионной системы изучается непосредственно в бозонном представлении в приближении среднего поля. Показано, что полученные бозонные состояния не содержат духовых компонент, связанных с нарушением принципа Паули, так что не надо строить сложные физические бозонные состояния. Метод применяется к изучению энергетических спектров ^{20}Ne, ^{24}Mg и получается хорошее согласие с экспериментальными данными

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1987

Kuchta R. E4-87-747
Microscopic Boson Approach to the Description of sd-Shell Nuclei

A microscopic method is proposed for analyzing the properties of light nuclei with an equal number of protons and neutrons in terms of many interacting bosons. An exact boson image of the underlying shell-model Hamiltonian is derived and the dynamical behaviour of the original fermion system is studied directly in the boson picture using the mean field approximation. The resulting boson states are shown to be free from spurious components, so that the cumbersome procedure of constructing the physical boson states can be avoided. The method is applied to calculating the energy spectra of ^{20}Ne, ^{24}Mg and a satisfactory agreement with experimental data is found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1987