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DYNAMICAL INEQUIVALENCE OF THE STRUCTURE OF THE COLLECTIVE SUBSPACE IN THE FERMION AND BOSON REPRESENTATIONS

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1. Introduction

Recent years have witnessed a considerable revival of interest in the bosonic description of nuclei [1], especially due to the success of the Interacting Boson Model (IBM) [2]. In order for such a description to be useful in practice, there must exist a relatively simple boson subspace (hereafter referred to as the boson collective subspace) which is to a large extent decoupled from the rest of the whole boson space [3]. A typical example of the boson collective subspace is the IBM-space consisting of bosons with angular momenta $J = 0\,(s)$ and $J = 2\,(d)$.

In view of the fact that nuclei are basically fermion systems, it is highly desirable to find a microscopic interpretation of the relevant bosons, as well as to derive the IBM-Hamiltonian from the underlying shell-model Hamiltonian. Most of the suggested microscopic approaches [4] have pictured the s- and d-bosons of the IBM as representing pairs of identical nucleons coupled to angular momentum $J = 0$ ($\mathcal{S}$-pair) and $J = 2$ ($\mathcal{D}$-pair), respectively. This interpretation is certainly justifiable in spherical nuclei [5] where the ground state is built to a large extent by correlated $\mathcal{S}$-pairs and the low-lying excited states contain at most two (one proton- and one neutron-) $\mathcal{D}$-pairs. In transitional and deformed nuclei, however, the situation is much more complicated. Recently, a large number of investigations have been carried out [6] which suggest that for a microscopic understanding of the properties of such nuclei one has to take into account not only the $\mathcal{S}$- and $\mathcal{D}$-pairs but also the higher-multipole ($J>2$) ones, among which the $\mathcal{G}$-pair ($J = 4$) seems to play the most essential role. Nevertheless, this fact does not necessarily mean the incapability of the IBM to explain the same physics in terms of s- and d-bosons only. Indeed, it is well known that the IBM works well even in deformed nuclei [2]. The above discrepancy between the fermion and boson descriptions merely indicates that a straightforward identification of the IBM-states with the corresponding shell-model states built of S- and D-pairs is oversimplified. In fact, the bosons may in general represent rather complicated fermion configurations [7], and therefore, there is no a priori reason why the fermion collective subspace should have the same structure as the boson collective subspace. The main purpose of the present paper is to shed some light on this subject.

2. The Theoretical Background

2.1 THE BOSON IMAGE OF THE FERMION HAMILTONIAN

We consider a system of n identical nucleons moving in several non-degenerate j-shells and interacting through the pairing plus quadrupole-quadrupole (P + QQ) interaction. The Hamiltonian of such a system has the form

$$\hat{H}_F = \sum_{am_a} \varepsilon_a c^+_{am_a} c_{am_a} - G P^+ P - \tfrac{1}{2}\chi \sum_M (-)^M Q_{2M} Q_{2-M}\,, \qquad (1)$$

where $c^+_{am_a}$ (c_{am_a}) is the creation (annihilation) operator of a nucleon in the single-particle state (am_a), $a = (n_a, l_a, j_a)$; ε_a is the corresponding single-particle energy, G and χ are the strengths of the pairing and quadrupole-quadrupole interactions, respectively, and the operators P^+, Q_{2M} are defined as

$$P^+ = \tfrac{1}{2}\sum_a \sqrt{2j_a+1}\,[c^+_a \times c^+_a]^0_0\,,$$
$$Q_{2M} = 5^{-1/2}\sum_{ab} q_{ab}\,[c^+_a \times \tilde{c}_b]^2_M\,, \qquad (2)$$
$$q_{ab} = \langle a \| r^2 Y_2 \| b\rangle\,, \quad \tilde{c}_{bm_b} = (-)^{j_b+m_b} c_{b-m_b}\,.$$

The symbol $[\ \times\ ]^L_M$ in (2) means the standard angular momentum coupling. The Hamiltonian (1) is conveniently rewritten in the form

$$\hat{H}_F = \sum_a \tilde{\varepsilon}_a \sqrt{2j_a+1}\, U_{00}(aa) - \tfrac{1}{4}\sum_{abcd}\sum_K V^K_{abcd}\,[U_K(ab) \times U_K(cd)]^0_0\,, \qquad (3)$$

where

$$\tilde{\varepsilon}_a = \varepsilon_a - \tfrac{1}{4}G\,; \qquad (4)$$
$$V^K_{abcd} = \sqrt{2K+1}\left\{\tfrac{2}{5}\chi q_{ab} q_{cd}\,\delta_{K2} - G\,\delta_{ac}\,\delta_{bd}\right\}, \qquad (5)$$
$$U_{KM}(ab) = [c^+_a \times \tilde{c}_b]^K_M\,. \qquad (6)$$

Now we introduce the spherical boson creation and annihilation operators $B^+_{JM}(ab)$, $B_{JM}(ab)$ which satisfy the following commutation relations

$$[B_{JM}(ab), B_{J'M'}(a'b')] = [B^+_{JM}(ab), B^+_{J'M'}(a'b')] = 0\,,$$
$$[B_{JM}(ab), B^+_{J'M'}(a'b')] = \tfrac{1}{2}\delta_{JJ'}\delta_{MM'}\left\{\delta_{aa'}\delta_{bb'} - (-)^{j_a+j_b+J}\delta_{ab'}\delta_{ba'}\right\}. \qquad (7)$$

We also define the boson vacuum $|0)_B$ by the condition

$$B_{JM}(ab)\,|0)_B = 0\,. \qquad (8)$$

In the Belyaev-Zelevinsky-Marshalek (BZM) mapping scheme [8], the boson images $\mathcal{U}_{KM}(ab)$ of the fermion operators $U_{KM}(ab)$ given by (6) can be expressed as [9]

$$\mathcal{U}_{KM}(ab) = -2\sum_{J_1J_2}(-)^{J_1+J_2+K}\hat{J}_1\hat{J}_2\begin{Bmatrix} J_1 & J_2 & K \\ j_b & j_a & j_c\end{Bmatrix}[B^+_{J_1}(ca) \times \tilde{B}_{J_2}(bc)]^K_M\,, \qquad (9)$$

where $\hat{J} = \sqrt{2J+1}$, $\tilde{B}_{JM}(ab) = (-)^{J-M} B_{J-M}(ab)$ and $\{\ \dots\ \}$ stands for the usual 6j-symbol. The boson image $\mathcal{H}_B$ of the fermion Hamiltonian (3) is then simply

$$\mathcal{H}_B = \sum_a \tilde{\varepsilon}_a\sqrt{2j_a+1}\,\mathcal{U}_{00}(aa) - \tfrac{1}{4}\sum_{abcd}\sum_K V^K_{abcd}\,[\mathcal{U}_K(ab) \times \mathcal{U}_K(cd)]^0_0\,. \qquad (10)$$

Since the physical bosons are associated with certain kind of collectivity, we introduce a unitary transformation to new bosons,

$$B^+_{\sigma JM} = \sum_{ab}\beta^\sigma_{abJ}\,B^+_{JM}(ab)\,, \qquad (11)$$

where the index σ labels different bosons with the same JM. Unitarity of the transformation (11) means that the coefficients β^σ_{abJ} satisfy

$$\sum_{ab}\beta^\sigma_{abJ}\beta^{\sigma'}_{abJ} = \delta_{\sigma\sigma'}\,, \qquad (12)$$
$$\sum_\sigma \beta^\sigma_{abJ}\beta^\sigma_{a'b'J} = \tfrac{1}{2}\left\{\delta_{aa'}\delta_{bb'} - (-)^{j_a+j_b+J}\delta_{ab'}\delta_{ba'}\right\}. \qquad (13)$$

The relation (12) guarantees that the proper boson commutator

$$[B_{\sigma JM}, B^+_{\sigma'J'M'}] = \delta_{\sigma\sigma'}\delta_{JJ'}\delta_{MM'} \qquad (14)$$

holds. The most important consequence of (13) is that the relation (11) can be inverted to yield

$$B^+_{JM}(ab) = \sum_\sigma \beta^\sigma_{abJ}\,B^+_{\sigma JM}\,. \qquad (15)$$

The coefficients β^σ_{abJ} are determined by the requirement that the one-boson states $B^{+}_{\sigma JM}|0\rangle_B$, where $|0\rangle_B$ is the boson vacuum defined by (8) , be eigenstates of the boson Hamiltonian (10) with the corresponding eigenvalues E^{σ}_{J} . This leads to the following system of linear eigenvalue equations for β^{σ}_{abJ} :

$$\sum_{a'b'} h^{J}_{aba'b'}\,\beta^{\sigma}_{a'b'J} = E^{\sigma}_{J}\,\beta^{\sigma}_{abJ}\,,$$

$$\begin{aligned} h^{J}_{aba'b'} = {} & \Big\{ \varepsilon_a + \varepsilon_b - \tfrac{1}{2}\chi\Big[(2j_a+1)^{-1}\sum_c (q_{ac})^2 + (2j_b+1)^{-1}\sum_c (q_{bc})^2\Big]\Big\}\delta_{aa'}\delta_{bb'} \\ & + (-)^{j_b+j_{a'}+J}\,\chi\, q_{aa'}\,q_{bb'}\begin{Bmatrix} j_{a'} & j_{b'} & J \\ j_b & j_a & 2 \end{Bmatrix} \\ & - \tfrac{1}{2}G\,\delta_{ab}\,\delta_{a'b'}\,\delta_{J0}\sqrt{(2j_a+1)(2j_{a'}+1)}\,, \end{aligned} \qquad (16)$$

where the relation (5) was explicitly introduced. The procedure for determining the coefficients β^{σ}_{abJ} is essentially the two-particle Tamm-Dancoff approximation (TDA) . We shall therefore refer to these coefficients as TDA-amplitudes and to the associated eigenvalues E^{σ}_{J} as TDA-energies. Correspondingly, the objects created by the operators $B^{+}_{\sigma JM}$ will be called TDA-bosons. The different TDA-energies E^{σ}_{J} for a given J can be used for ordering the set $\{\sigma = 0,1,2,\ldots\}$ in such a way that E^{σ}_{J} increases with σ . The bosons $B^{+}_{\sigma JM}$ with $\sigma = 0$ will then have the lowest energy (for a given J) , and therefore, they will be called "collective", while those with $\sigma = 1,2,\ldots$, having a higher energy, will be identified with the "noncollective" bosons.

Having fixed the TDA-amplitudes β^{σ}_{abJ} and the TDA-energies E^{σ}_{J} , we use (9) , (10) and (15) to express the boson Hamiltonian $\mathcal{H}_B$ in terms of the TDA-bosons,

$$\begin{aligned} \mathcal{H}_B = {} & \sum_{\sigma JM} E^{\sigma}_{J}\,B^{+}_{\sigma JM}B_{\sigma JM} \\ & + \sum_{\substack{\sigma_1\sigma_2\sigma_3\sigma_4 \\ J_1J_2J_3J_4}}\sum_K W^{K}_{\sigma_1J_1\sigma_2J_2\sigma_3J_3\sigma_4J_4}\Big[\big[B^{+}_{\sigma_1J_1}\times B^{+}_{\sigma_2J_2}\big]^K\times\big[\tilde{B}_{\sigma_3J_3}\times\tilde{B}_{\sigma_4J_4}\big]^K\Big]^0_0\,, \end{aligned} \qquad (17)$$

where

$$\begin{aligned} W^{K}_{\sigma_1J_1\sigma_2J_2\sigma_3J_3\sigma_4J_4} = {} & (-)^{J_2+J_4}\,\hat{J}_1\hat{J}_2\hat{J}_3\hat{J}_4\sum_{abcdpq}\sum_L (-)^{j_p+j_q+j_a+j_b}(2L+1)\,V^{K}_{adcb} \\ & \times\begin{Bmatrix} J_1 & J_2 & K \\ J_4 & J_3 & L \end{Bmatrix}\begin{Bmatrix} j_d & j_p & J_3 \\ J_1 & L & j_a \end{Bmatrix}\begin{Bmatrix} j_b & j_q & J_2 \\ J_4 & L & j_c \end{Bmatrix}\beta^{\sigma_1}_{apJ_1}\beta^{\sigma_2}_{bqJ_2}\beta^{\sigma_3}_{dpJ_3}\beta^{\sigma_4}_{cqJ_4}\,. \end{aligned} \qquad (18)$$

In obtaining (17) , the commutation relations (14) have been used to arrange $\mathcal{H}_B$ into the normal order with respect to the boson vacuum $|0\rangle_B$. Using (18) and (5) , one can easily verify that

$$W^{K}_{\sigma_1J_1\sigma_2J_2\sigma_3J_3\sigma_4J_4} = (-)^{J_1+J_2+J_3+J_4}\,W^{K}_{\sigma_4J_4\sigma_3J_3\sigma_2J_2\sigma_1J_1}\,,$$

which means that $\mathcal{H}_B$ is hermitian. It is also worthwhile to point out that provided <u>all</u> the TDA-bosons are included in (17), the boson Hamiltonian $\mathcal{H}_B$ is an exact image of the original fermion Hamiltonian $\hat{H}_F$.

2.2 REMOVAL OF SPURIOUS BOSON STATES

It is well known that the diagonalization of any boson image of a fermion Hamiltonian in the boson basis generated by the states

$$\begin{aligned} & |N;\alpha\rangle_B = \frac{1}{\sqrt{N!}}\,B^{+}_{\alpha_1}B^{+}_{\alpha_2}\cdots B^{+}_{\alpha_N}|0\rangle_B\,, \\ & \alpha_i \equiv (\sigma_i J_i M_i)\,;\quad \alpha = \{\alpha_1,\alpha_2,\ldots,\alpha_N\}\,,\quad N = \frac{n}{2} \end{aligned} \qquad (19)$$

produces not only the eigenvectors corresponding to actual states of the underlying nucleon system, but also the spurious ones which have no physical meaning and are associated with the overcompleteness of the basis (19) . Since the operators (9) commute with the projector onto the physical subspace [10,11] , it is easily seen from (10) that the present form of $\mathcal{H}_B$ does not mix the physical and spurious states. Consequently, these two types of boson states are strictly separated from each other and the only task one is left with is to identify which states are physical and which are spurious. This can most easily be done by means of the method first proposed by Janssen et al. [11] and recently elaborated by Park [12]. This method exploits the non-unitary character of the Dyson mapping [13] , namely the fact that the images of fermion operators have, in general, different properties in the physical and non-physical subspaces. The starting point of the method is the fermion operator

$$\hat{K} = \hat{N}^2_{I} - \hat{N}^2_{II}\,, \qquad (20a)$$

where

$$\hat{N}_{I}^{2} = \sum_{abm_a m_b} c^{+}_{am_a} c_{am_a'} c^{+}_{bm_b} c_{bm_b} \ , \qquad (20b)$$

$$\hat{N}_{II}^{2} = \sum_{am_a} c^{+}_{am_a} c_{am_a} - \sum_{abm_a m_b} c^{+}_{am_a} c^{+}_{bm_b} c_{am_a} c_{bm_b} \ . \qquad (20c)$$

The operator $\hat{K}$ is clearly equal to zero because $\hat{N}_{II}^{2}$ is nothing but $\hat{N}_{I}^{2}$ written in normal ordered form. Using the Dyson transformation [10-13]

$$\begin{aligned} c^{+}_{\alpha} c^{+}_{\beta} &\to (c^{+}_{\alpha} c^{+}_{\beta})_D = b^{+}_{\alpha\beta} - \sum_{\gamma\delta} b^{+}_{\alpha\gamma} b^{+}_{\beta\delta} b_{\gamma\delta} \\ c_{\beta} c_{\alpha} &\to (c_{\beta} c_{\alpha})_D = b_{\alpha\beta} \\ c^{+}_{\alpha} c_{\beta} &\to (c^{+}_{\alpha} c_{\beta})_D = \sum_{\gamma} b^{+}_{\alpha\gamma} b_{\beta\gamma} \end{aligned} \qquad (21)$$

and introducing the TDA-bosons (11) with $B^{+}_{JM}(ab) = \frac{1}{\sqrt{2}} \sum_{m_a m_b} \langle j_a m_a j_b m_b | JM \rangle b^{+}_{\alpha\beta}$, we get the Dyson image $\hat{K}_D$ of (20) in the form

$$\hat{K}_D = \hat{N}_D^{2} - \hat{N}_D - \hat{F}_D \ , \qquad (22a)$$

where

$$\hat{N}_D = 2 \sum_{\sigma JM} B^{+}_{\sigma JM} B_{\sigma JM} \ , \qquad (22b)$$

$$\hat{F}_D = \sum_{\sigma JM} A^{+}_{\sigma JM} B_{\sigma JM} \qquad (22c)$$

with

$$A^{+}_{\sigma JM} = B^{+}_{\sigma JM} - \sum_{\substack{\sigma_1\sigma_2\sigma_3 \\ J_1 J_2 J_3}} \sum_{K} \xi^{(\sigma J)}_{\sigma_1 J_1 \sigma_2 J_2 \sigma_3 J_3}(K) \left[\left[B^{+}_{\sigma_1 J_1} \times B^{+}_{\sigma_2 J_2} \right]^{K} \times \tilde{B}_{\sigma_3 J_3} \right]^{J}_{M} \ , \qquad (22d)$$

$$\xi^{(\sigma J)}_{\sigma_1 J_1 \sigma_2 J_2 \sigma_3 J_3}(K) = 2 \sum_{abcd} \hat{K} \hat{J}_1 \hat{J}_2 \hat{J}_3 \begin{Bmatrix} j_b & j_a & J \\ j_d & j_c & J_3 \\ J_2 & J_1 & K \end{Bmatrix} \beta^{\sigma}_{abJ} \beta^{\sigma_1}_{caJ_1} \beta^{\sigma_2}_{dbJ_2} \beta^{\sigma_3}_{cdJ_3} \ . \qquad (22e)$$

The quantity $\begin{Bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{Bmatrix}$ in (22e) is the usual 9j-symbol. As is shown in [12], $\hat{K}_D$ has zero expectation value in all the physical boson states but positive expectation value in all the spurious boson states. This property allows one to distinguish between the physical and spurious eigenstates of the boson Hamiltonian.

Of course, in cases when some truncation of $\mathcal{H}_B$ is made, the described procedure cannot be used because the physical and spurious boson states are no longer well separated (i.e. <u>all</u> eigenstates of the truncated boson Hamiltonian may contain both physical and spurious components). It is therefore necessary to exclude the occurrence of spurious states a priori, e.g. by constructing a suitable boson basis which can be put into one-to-one correspondence with the fermion basis. To this end, let us consider the following fermion states

$$|N;\lambda\rangle_F = \frac{1}{\sqrt{N!}} \Gamma^{+}_{\lambda_1} \Gamma^{+}_{\lambda_2} \cdots \Gamma^{+}_{\lambda_N} |0\rangle_F \ , \qquad (23)$$

where $|0\rangle_F$ is the fermion vacuum ($c_\alpha |0\rangle_F = 0$) and

$$\Gamma^{+}_{\lambda_i} \equiv \Gamma^{+}_{\sigma_i J_i M_i} = \frac{1}{\sqrt{2}} \sum_{ab} \beta^{\sigma_i}_{abJ_i} \left[c^{+}_{a} \times c^{+}_{b} \right]^{J_i}_{M_i} \qquad (24)$$

with the $\beta^{\sigma_i}_{abJ_i}$ determined from (16). The states (23) form an overcomplete non-orthogonal basis in the fermion space. By diagonalizing the norm matrix $\langle N;\lambda | N;\lambda' \rangle_F$,

$$\sum_{\lambda'} \langle N;\lambda | N;\lambda' \rangle_F \, u^{(\alpha)}_{\lambda'} = \mathcal{N}_\alpha u^{(\alpha)}_{\lambda} \ ; \ \sum_{\lambda} u^{(\alpha)}_{\lambda} u^{(\alpha')}_{\lambda} = \delta_{\alpha\alpha'} \qquad (25)$$

and excluding the zero-eigenvalue solutions $u^{(\alpha_0)}_{\lambda}$, $\mathcal{N}_{\alpha_0} = 0$, we construct a complete orthonormal basis in the fermion space

$$|N;\alpha\rangle\rangle_F = \frac{1}{\sqrt{\mathcal{N}_\alpha}} \sum_{\lambda} u^{(\alpha)}_{\lambda} |N;\lambda\rangle_F \ ; \quad \alpha \neq \alpha_0 \ . \qquad (26)$$

The corresponding boson basis in which the actual calculations have to be carried out in order to avoid spurious solutions is then given by

$$|N;\alpha\rangle\rangle_B = \sum_{\lambda'} u^{(\alpha)}_{\lambda} |N;\lambda\rangle_B \ ; \quad \alpha \neq \alpha_0 \ , \qquad (27)$$

where $|N;\lambda\rangle_B$ are the boson states (19).

2.3 CHOICE OF THE COLLECTIVE SUBSPACE

Having established an exact boson image of the fermion Hamiltonian, as well as the method for dealing with spurious boson solutions, we are ready to examine the "goodness" of both the boson and fermion collective subspaces. According to current thinking [14], this can be done by selecting certain part of the whole Hilbert space as collective, disregarding the rest and checking whether the important physical observables such as the energies and the transition matrix elements remain essentially unchanged by enlarging the chosen collective subspace. If this is indeed so, one may be reasonably sure that the selected collective subspace is to a good approximation decoupled from the rest of the whole space. Being inspired by the IBM [2], we suppose that the boson collective subspace is generated by the most collective ($\sigma = 0$) TDA-bosons $B^+_{\sigma JM}$ with $J = 0$ and $J = 2$ ($B^+_{000} \equiv s^+$, $B^+_{02M} \equiv d^+_M$). The corresponding fermion collective subspace is assumed to be that composed of the $\Gamma^+_{000} \equiv S^+$ and $\Gamma^+_{02M} \equiv D^+_M$ fermion pairs. The above boson and fermion subspaces will be referred to as the sd- and SD-subspaces, respectively. Since there are strong indication that the $J = 4$ bosons (nucleon pairs) may also play an important role in the low-lying collective states of nuclei [6], we take as the enlarged space the sdg-(SDG) subspace which contains additional $g^+_M \equiv B^+_{04M}$-bosons ($G^+_M \equiv \Gamma^+_{04M}$-pairs).

3. Results

For actual calculations we consider a system of $2N = 6$ identical nucleons distributed over 3 non-degenerate j-shells $j_1 = 1/2$, $j_2 = 3/2$, $j_3 = 5/2$ ($\varepsilon_{j_1} = 1\,\mathrm{MeV}$, $\varepsilon_{j_2} = 3\,\mathrm{MeV}$, $\varepsilon_{j_3} = 0\,\mathrm{MeV}$) and interacting through the P + QQ Hamiltonian (1) with $G = 0.1\,\mathrm{MeV}$, $\chi = 0.2\,\mathrm{MeV}\,(m\omega/\hbar)^2$. This choice is complex enough to simulate some real situations in nuclei, but at the same time, it is sufficiently simple as to allow for an exact solution of the Hamiltonian (1). This enables one to estimate not only the relative "goodness" of the truncated collective subspace as described above but also its absolute adequacy with respect to the exact solution.

In Fig. 1 we compare the low-lying levels of the boson and fermion spectra obtained in various approximations. First of all, Figs. 1c) and 1d) show the spectrum of the boson Hamiltonian (17) and the exact spectrum of the fermion Hamiltonian (1), respectively. The boson spectrum is seen to be much richer than the fermion one, as a consequence of the overcompleteness of the boson basis (19) with respect to the space available for fermions. However, by com-

puting the expectation value of the operator (22) in all the boson states one can easily find that the states marked by full lines are physical (zero expectation value), while those represented by dashed lines are spurious (positive expectation value). By simply ignoring the latter we immediately observe that the remaining (i.e. physical) boson eigenstates coincide with the exact fermion eigenstates given in Fig. 1d. Thus, the diagonalization of the boson Hamiltonian (17) in the whole boson space correctly reproduces all the physical eigenenergies.

In Figs. 1a) and 1b) we display the energy spectra obtained by diagonalizing the boson Hamiltonian (17) in the sd- and sdg-subspace respectively. Possible spurious boson states are removed before diagonalization by excluding the zero-eigenvalue eigenstates of the fermion norm matrix. The finiteness of the model space is responsible for the fact that only some energy levels of the exact spectrum can be reproduced in the sd-subspace. Nevertheless, the energies of these levels remain essentially unchanged when the boson space is enlarged to include the g^+-bosons, which means that the sd-truncation provides a relatively good subspace, at least for the description of the energetically lowest states. By comparing the sd-levels of Fig. 1a) with the exact ones (full lines in Fig. 1c) we can conclude that the sd-subspace is well decoupled not only from the sdg-subspace but also from the whole rest of the boson space (which includes the noncollective ($\sigma \neq 0$) TDA-bosons as well). Comparison of Figs. 1b) and 1c) further shows that the sdg-subspace is a good subspace for the whole part of the exact spectrum displayed in the figure.

However, the same conclusions cannot be made for the results obtained in the fermion SD- and SDG-subspaces (see Figs. 1d, e,f). First, even the lowest SD-levels differ considerably from the corresponding SDG-ones, which means that the fermion SD-subspace is not at all a good subspace. This is confirmed

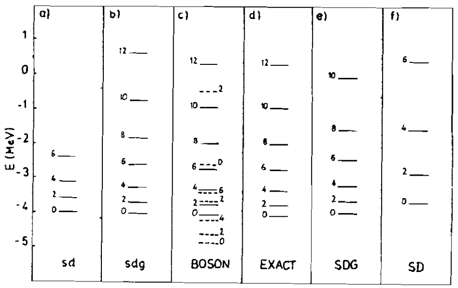

Fig. 1. Energy spectra associated with 6 identical nucleons moving in 3 non-degenerate j-shells $j_1 = 1/2, j_2 = 3/2, j_3 = 5/2$ and interacting through the P + QQ Hamiltonian (1). For the parameters of the Hamiltonian as well as for the description of individual approximations in a) - f) see the text.

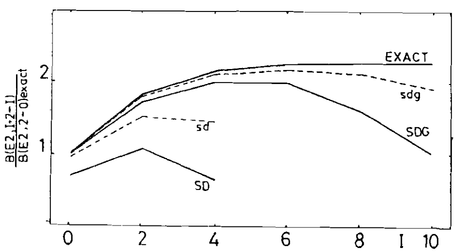

Fig. 2. Calculated values of the ratio $B(E2; I+2 \to I)/B(E2; 2 \to 0)_{exo}$ for the states shown in Fig. 1. Various approximations are explained in the text.

by the observation that the SD-subspace does not provide a good approximation to the exact spectrum given in Fig. 1d). Second, the SDG-subspace works much better but still worse than the corresponding boson sdg-subspace (figs. 1b, 1e).

These results indicate that analogous truncations in the fermion and boson spaces are not equivalent and that a boson trun-

cation may provide a better approximation to the exact fermion problem than the corresponding fermion truncation. However, the energy spectra alone do not tell much about the structure of the wave functions. Spectroscopic quantities such as the electromagnetic transition rates are generally considered to provide a more detailed information about this structure. We have therefore calculated the B(E2) values in different approximations as well. The results are shown in Fig. 2 and they support the idea that the boson sd- and sdg-subspaces are better for the description of low-lying states than the corresponding fermion SD- and SDG-subspaces, respectively.

This observation seems to contradict the commonly accepted opinion [4-6] that the corresponding fermion and boson approximations should be equivalent in the sense that, for example, the success or failure of the sd-boson truncation is determined by the success or failure of the SD-fermion truncation, respectively. This opinion, however, originates from the presupposed correspondence between fermion and boson states [9, 15, 16]. On the other hand, we have carried out the fermion and boson calculations without specifying in advance the character of correspondence between these states. The present boson-fermion correspondence is guaranteed to be unambiguos (due to the proper exclusion of spurious solutions) but it need not be "simple" in the sense discussed by Ginocchio and Talmi [15], because we have worked with a hermitian boson image of the fermion Hamiltonian, while the "simple" boson-fermion correspondence requires the boson Hamiltonian to be non-hermitian, in general. Keeping in mind that the fermion pairs are not real bosons, it is quite natural to expect that the states of a given boson subspace correspond to certain complicated fermion states in which pairs of higher multipolarity may play an important role. The strongest indication for this is the fact that the phenomenological IBM with s- and d-bosons works well even in deformed nuclei [2], where a correct microscopic theory requires an explicit inclusion of $\mathcal{G}$-pairs [6]. However, a detailed understanding of the above-mentioned boson-fermion correspondence is still far from clear and deserves further investigation.

4. Conclusion

The results of the present paper show that for a system of identical nucleons moving in several non-degenerate j-shells and interacting through the P + QQ force, the boson and fermion collective subspaces with the same multipole structure are dynamically inequivalent. In particular, the boson space restricted to s- and d-bosons is a much better invariant subspace of the Hamiltonian than the corresponding fermion space restricted to $\mathcal{S}$- and $\mathcal{D}$- pairs. This means that the boson collective subspace is dominated by s- and d- bosons, while its fermion counterpart comprises not only the $\mathcal{S}$- and $\mathcal{D}$- pairs but also the higher-multipole ones ($\mathcal{G}$) . A similar finding has recently been made by Dukelsky et al. [17] in the framework of the mean field approach [10].

Of course, the validity of the above assertion depends in general on the boson mapping chosen (kinematical aspect) as well as on the Hamiltonian (dynamical aspect). It is well known that the seniority conserving mapping (SCM) applied to the single-j-shell Hamiltonian with the pure QQ interaction provides a very bad decoupling of the sd-subspace from the rest of the whole space [16]. In the present paper we have considered the case of 3 nondegenerate j-shells and we have included the pairing force into the Hamiltonian. In addition, the dynamics of the boson and fermion systems has been studied independently using the BZM mapping scheme instead of the SCM. As a result, the coupling of the collective sd-subspace with the rest of the boson space has proved to be considerably weakened.

However, the exact form of the realistic effective interaction in actual nuclei with protons and neutrons is not well established so far and it is by no means clear that a P + QQ force is adequate. Moreover, peculiarities of the subshell structure in various nuclei are expected to play a non-negligible role as well. Further investigations in this direction are therefore very desirable.

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Динамическая неэквивалентность структуры коллективного подпространства в фермионном и бозонном представлениях

Построен точный бозонный образ ядерного гамильтониана, содержащего спаривательное и квадруполь-квадрупольное взаимодействия и действующего в пространстве нескольких невырожденных j-оболочек. Показано, что бозонный гамильтониан, действующий на подпространстве s- и d-бозонов, описывает фермионный энергетический спектр и вероятности электромагнитных переходов лучше, чем оригинальный фермионный гамильтониан на подпространстве S- и D-пар. Такая же ситуация встречается в рамках sdg-SDG приближения. Это значит, что бозонные и фермионные подпространства с одной и той же мультипольной структурой являются динамически неэквивалентными.

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Dynamical Inequivalence of the Structure of the Collective Subspace in the Fermion and Boson Representations

An exact boson mapping of the multi-j-shell pairing-plus-quadrupole Hamiltonian onto a Hermitian boson image with at most two-body terms is performed. The resulting boson Hamiltonian truncated to s- and d-boson is shown to be capable of describing the exact energy spectrum and electromagnetic transition rates better than the original fermion Hamiltonian restricted to the space of S- and D-pairs. This situation persists within the sdg-SDG truncation as well. It can thus be concluded that the boson and fermion subspaces with the same multipole structure are dynamically inequivalent.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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