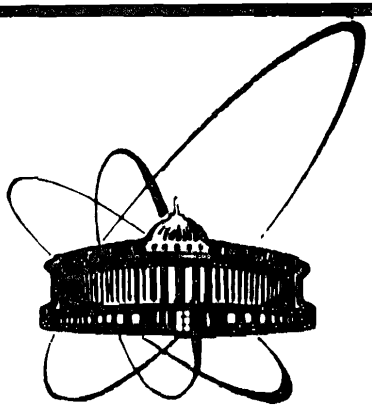

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MULTISTEP ADIABATIC REPRESENTATION

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1. The adiabatic approximation is a well-known method for effective study of few-body systems in nuclear and atomic physics. Originated by the pioneer works of Born and Oppenheimer the method was developed in various directions following the actual necessities of the physicists, but the main idea of separation of "fast" variables from the "slow" ones remains a guiding one.

Recently the application of adiabatic techniques in quantum mechanics allowed one to emphasize the geometrical structure appearing in the external field problems [2]. Dirac was the first who stressed the role of the geometrical phase factor [1] multiplying the wave function in presence of nontrivial monopole field. More recently it turned out [3] that the wave function for some topologically nontrivial base manifolds acquires a geometrical phase-factor during an "adiabatic excursion" in the space of slow variables. In other terms when some physical process can be interpreted as an evolution over a closed path in the space of slow variables, then a nontrivial holonomy phase factor may appear besides the dynamical one. This fact visualizes the complex line-bundle structure of quantum mechanics (see, e.g. [4]).

These theoretical concepts were extensively used for explanation of anomalies in gauge theories and current algebra [5], in interpretation of Bohm-Aharonov experiments [6], in geometrical analysis of the two-dimensional three-body problem [7], etc. Moreover an experimental test recently performed with polarized light spreading in optical fibre perfectly confirmed the theoretical predictions for the input-output polarization angle difference [8].

The purpose of this letter is to generalize the standard adiabatic Ansatz for the multi-channel wave function case when all variables are treated dynamically. For this reason one introduces an ordering of the configuration space degrees of freedom. Then, in a complete analogy with the usual adiabatics, one averages step by step eliminating consequently new fast variables. We find a sufficient conditions which assure us that the induced external field after ℓ sequential averagings equals (up to gauge transformation) to the sum of external fields induced at every step.

A holomorphic vector-bundle construction is proposed for description of the underlying geometrical structure. For illustration of ideas discussed a simplified "N-body" potential example is studied all along. The self-consistency condition for the applicability of multistep procedure implies specific relations between the energy terms and the matrix elements of the corresponding potentials entering into the Hamiltonian.

The more realistic physical examples will be published elsewhere.

2. Let us formulate the problems we are interested in on the example of quantum mechanical system evolving in (generally not trivial topologically) N-dimensional configuration space. The Hamiltonian of the system is written in the following form

$$H = -\sum_{j=1}^{k} \nabla_j^2 + \sum_{i=1}^{k-1} V_i + W \equiv -\sum_{j=1}^{k} \Delta_j + V = \sum_{i=1}^{k} H^{(i)} + W, \quad (I)$$

where

$$V_i = V_i(r_i, r_{i+1}, \dots, r_k), \quad H^{(k)} \equiv -\Delta_k,$$

$$\dim r_i = n_i, \quad \sum_{i=1}^{k} n_i = N$$

and W is such part of the potential which cannot be presented in the form of V_i. All parameters and external matrix structure that are irrelevant for the exposition will be suppressed in the sequal.

In order to solve eigenvalue problem in

$$H\Psi = E\Psi \leftrightarrow H(E)\Psi = 0 \quad (2)$$

one introduces the standard adiabatic representation of two types of variables r_1, r_2 called fast and slow respectively. Moreover one assumes tacitly that they do not interchange, i.e. none of the fast variables could become slow in some region of configuration space and vice versa. Here the notions fast and slow have to be interpreted in quasiclassical limit only. Namely for $\hbar \to 0$ this means the existence of angle-momentum variables such that momentum J varies slowly when the slow variables $\vec{r}_2$ change, while the angle variable φ /9/ changes fastly under variation of the fast variables $\vec{r}_1$.

For such ordering of the variables in configuration space we

introduce special notation $r_1 \succ r_2$ and say that "one-step adiabatic ladder" is given. Now recast (1) into the form

$$-W + H = H^{(1)} + H^{(2)} = -\Delta_1 - \Delta_2 + V_1. \quad (3)$$

It is convenient to introduce the adiabatic Ansatz:

$$\Psi(r_1 r_2) = \sum_n \Phi_n^{(1)}(r_1; r_2)\, \chi_n(r_2), \quad (4)$$

where $\{\Phi_n^{(1)}\}$ is an orthonormal set of functions satisfying the "fast" eigenvalue problem

$$H^{(1)}\Phi_n^{(1)} = \mathcal{E}_n^{(1)}(r_2)\, \Phi_n^{(1)} \quad (5)$$

with eigenvalues $\mathcal{E}_n^{(1)}(r_2)$ depending on the slow variables as parameters. (In the spectroscopic literature $\mathcal{E}_n^{(1)}$ are known as energy terms). Notice that the summation in (4) might generally contain integration corresponding to the income from the continuous, but by appropriate choice of variables, in practice only discrete part of the spectrum is relevant.

Replacing (4) in (2) the eigenvalue problem reduces to the system of coupled partial differential equations for the slow amplitude in the following manner:

$$\langle \Phi_n^{(1)} (H - E) \Psi \rangle = 0. \quad (6)$$

Using orthogonality and assuming completeness for the solutions of (5) it is easy to rewrite (6) into the form:

$$\left[-(\nabla_2 + A)^2_{nm} + W_{nm} + (\mathcal{E}_m - E)\delta_{nm}\right]\chi_m \equiv H_{nm}(E) = 0, \quad (7)$$

where the external field A by definition equals the matrix element

$$A_{nm} = \langle n | \nabla_2 | m \rangle \equiv \int \bar{\Phi}_n^{(1)} \nabla_2 \Phi_m^{(1)}\, dr_1. \quad (8)$$

In writing (7),(8) an integration by parts and differentiability of solutions $\{\Phi_n^{(1)}\}$ with respect to the slow variables r_2 are used.

Proceeding here formally one can introduce a "gauge transformation" acting on the amplitude χ_m according to the rule:

$$\chi_m \to \mathcal{U}_{mn}\chi_n\,, \tag{9}$$

i.e. $\mathcal{U}$ is some unitary operator in the space of solutions of (5). By compensating this unitary transformation by appropriate unitary change of the basis in the Hilbert space $\mathcal{H}_1$: $\Phi_n \to \Phi_m \mathcal{U}^+_{mn}$ nothing is changed because (4) is "gauge" invariant by construction. Besides this, our external field A possesses the right transformation properties under (9):

$$A^{\mathcal{U}} = A + \overset{+}{\mathcal{U}}\nabla_2\mathcal{U}\,. \tag{10}$$

This interesting observation was the main reason for trying to interprete the gauge field dynamically – as induced by some underlying substructure. Unfortunately it is not difficult to observe that the corresponding field strength F_A vanishes:

$$F_A = dA + A\wedge A = d(\langle n|)dm\rangle + \langle n|dk\rangle\wedge\langle k|dm\rangle = \tag{11}$$
$$= \langle dn|k\rangle\wedge\langle k|dm\rangle - \langle dn|k\rangle\wedge\langle k|dm\rangle \equiv 0\,,$$

i.e. it seems that induced field A is purely longitudinal. What is a matter ? In fact (11) is obtained by using extensively the completeness of the states $|\Phi^{(i)}_n\rangle$. Moreover, it was assumed that the connection form has to be nonabelian. Indeed, for abelian curvature form one gets:

$$F = dA = \langle dn|\wedge|dm\rangle \neq 0\,.$$

This simple argument means that in every nondegenerate energy level $\mathcal{E}^{(i)}_n$ the graphic (χ_n, r_2) constitutes a (local) coordinates for complex line-bundle having its first Chern class $\oint_C F_A = 2\pi C_1$. It is well known that C_1 is the unique obstacle for global existing of potential A and the integral $C_1 \in \mathbb{Z}$ ennumerates the nonequivalent line bundles [4]. So, in order to generalize the construction (11) to nonabelian case one needs to introduce degenerate energy levels and its wave functions (see,e.g. [10]). Comparing with (4) this implies in fact a multichannel generalization of the initial, scalar valued quantum mechanical problem. In the rest of the paper we shall assume always this case. (This way

in the example of diatomic molecule [11] a monopole external field appears).

3. We want to extend the construction (4) introducing an ordered set of k-variables

$$r_1 \succ r_2 \succ r_3 \ldots \succ r_k \tag{12}$$

for our N-dimensional system. It is worthwhile to note here that ordering (12) might correspond to the natural velocity scale governed by the mass parameters in the theory.

The standard Ansatz (4) suggests the following generalization

$$\Psi = \sum_n \Phi^{(1)}_n(r_1; r_2 r_3 \check{r})\chi_n(r_2 r_3 \check{r}) = \sum_{nk} \Phi^{(1)}_n(r_1; r_2 r_3 \check{r})\chi_{nk}(r_3\check{r})\Phi^{(2)}_k(r_2; r_3\check{r}) = \tag{13}$$
$$= \ldots\,,$$

where a (k-1)-step recurrence formula for slow amplitudes is assumed

$$\chi_{m_1\ldots m_s}(r_{i_{s+1}}, \ldots, r_k) = \int \chi_{m_1\ldots m_s}(r_{i_s}, r_{i_{s+1}}, \ldots, r_k)\,\bar{\Phi}^{(i_s)}_{m_s}(r_{i_s}; r_{i_{s+1}}, \ldots)\,dr_{i_s}\,. \tag{13'}$$

Now substituting (13') into (13) consequently l-times ($l \leq k-1$) one gets the l-th adiabatic ansatz corresponding to the l-th step of the ladder (12). Here a complete set of orthogonal vectors $\{\Phi^{(i)}_n\}_{n=1}^{\infty}; \forall i$ (in the Hilbert spaces $\mathcal{H}^{(i)}$) is introduced as eigenstate vectors for the self-adjoint operators $H^{(i)}$:

$$H^{(i)}\Phi^{(i)}_n = \mathcal{E}^{(i)}_n\Phi^{(i)}_n\,, \quad H^S\Phi^S_n = \mathcal{E}^S_n\Phi^S_n\,. \tag{14}$$

Note that the index i corresponds to simple Hamiltonians ($H^{(i)}$, i=1, k-1); while S, to the sum of such Hamiltonians:

$$H^S = \sum_{j\in S} H^{(j)}\,, \quad S = \{i_1, \ldots, i_s\}\,, \quad i_n \in [1, k-1]\,, \quad i_l \neq i_m \;\forall\, l \neq m\,, \; n = \overline{1,s}\,. \tag{14'}$$

At each step "l" in the ladder (12) a new group of slow variables r_{l-1} is treated as fast ones and the scalar product becomes

$$\langle\Phi^{(l)}_p\Phi^{(l)}_q\rangle = \int dr_1\ldots dr_{l-1}\,\bar{\Phi}^{(l)}_p\Phi^{(l)}_q\,, \quad \langle\Phi^S_p\Phi^S_q\rangle = \int dr_{i_1}\ldots dr_{i_s}\,\bar{\Phi}^S_p\Phi^S_q\,.$$

Substituting further Ψ from (13) and (13') into (2) and averaging the obtained equation on the left or with $\langle \Phi^{(1)} \otimes \Phi^{(2)} \dots \Phi^{(l)} |$ or with $\langle \Phi^{\mathcal{S}} |$, one gets the set of generalized slow equations:

$$\sum_{M} H_{MM'}(E)\, \chi_{M'} = 0\,, \quad M \equiv \{m_1,\dots,m_{\bar{k}}\},\ m_i \in \mathbb{Z}_+,\ i=\overline{1,k-1}\,. \tag{15}$$

The averaging procedure just described can be reformulated in a more compact form in terms of the following maps, see (7):

$$\begin{aligned} \mathcal{J}^{(l)}&: \ H \to H^{(l)}\,, \\ \mathcal{J}^{\mathcal{S}}&: \ H \to H^{\mathcal{S}}\,, \end{aligned} \tag{16}$$

where l and $\mathcal{S}$ run over the sets of simple and composite indices respectively.

Looking on the adiabatic scheme (13),(13') a natural question for self-consistency emerges: whether maps $\mathcal{J}^{(l)}, \mathcal{J}^{\mathcal{S}}$ satisfy the intuitively obvious associative law:

$$\mathcal{J}^{(i)} \cdot \mathcal{J}^{(i+1)} = \mathcal{J}^{\{i\,i+1\}} \tag{17}$$

and what kind of additional conditions imposed on H leads to (17)?

In answering this question it is more convenient to simplify the notations. That is why we shall limit ourselves with two-step adiabatic ladder. The consideration of the general case follows by induction.

Assertion: Let $\Phi^{(1)}$, $\Phi^{(2)}$ and $\Phi^{\{12\}}$ are complete orthonormal sets inseparable Hilbert space $\mathcal{H}$, where operators (14) are self-adjoint. If $A^{(1)}, A^{(2)}, A^{\{12\}}$ denote the induced external fields with matrix elements $A^{\cdot}_{nm} =: \langle \Phi^{\cdot}_n \nabla_2 \Phi^{\cdot}_m \rangle$, then

$$(i)\ A^{\{12\}} = C^{+}(A^{(1)} + A^{(2)})C + C^{+}\nabla_2 C\,, \tag{18}$$

$$\begin{aligned} (ii)\ C^{n'm'}_{q}\,\mathcal{E}^{12}_{q} = C^{nm}_{q}\{&\delta_{nn'}(\mathcal{E}^{1}_{n})_{n'm} - \langle n'm'|V_1|nm\rangle + \delta_{mm'}\delta_{nn'}\mathcal{E}^{2}_{m} - \\ &- \delta_{nn'}\langle m'|V_2|m\rangle - \langle m'[A_1]^{2}_{n'n}m\rangle - 2\langle m'[A_1]_{n'n}m\rangle\}\,, \end{aligned} \tag{19}$$

where the quantities C are defined by

$$C^{nm}_{q}(r_3,\check{r}) = \int \Phi^{\{12\}}_{q}(r_1 r_2; r_3\check{r})\,\bar{\Phi}^{(1)}_{n}(r_1; r_2 r_3\check{r})\,\bar{\Phi}^{(2)}_{m}(r_2; r_3\check{r})\,dr_1 dr_2\,. \tag{20}$$

The proof is straightforward but in order to simplify it we shall prove the following.

Lemma

Let $\nabla = (\nabla_1, \nabla_2, \dots, \nabla_k)$ is the standard vector field in $\mathbb{R}^N$. Then the action of operators $\mathcal{J}$ on ∇ is given by

$$[\mathcal{J}^{(1)}\mathcal{J}^{(2)}\circ\dots\circ\mathcal{J}^{(j)}\nabla]_m = \sum_{\substack{J=\overline{1,\min(m,j)} \\ 0<\alpha^{k}_{J}<\alpha^{k+1}_{J}\le j;\ k=\overline{J,j}}} \nabla_m^{\mathcal{A}_J} + \nabla_m\,\theta(m-j);\quad \theta(\rho) = \begin{cases} 1, & \rho>0 \\ 0, & \rho\le 0, \end{cases} \tag{21}$$

where $\mathcal{A}_J$ is a collection of at most j indices where

$$\mathcal{A}_J =: \{\alpha^{J}_{J}, \alpha^{J+1}_{J}, \dots, \alpha^{j}_{J}\} \tag{22}$$

and $\nabla^{\mathcal{A}_J}$ means matrix element of the gradient between basic functions $\Phi^{\mathcal{A}_J}$.

The proof is inductive and the first steps in the iteration scheme start with

$$\begin{aligned} &[\mathcal{J}^{1}(\nabla_1,\dots,\nabla_k)]_{mn} = \frac{\delta}{\delta\chi_n}\langle 1m|(\nabla_1,\dots,\nabla_k)\Psi\rangle = \big(\nabla^{(1)}_{nm}, \nabla_2 + (\nabla^{(1)}_{2})_{nm}, \dots, \\ &\dots, \nabla_k + (\nabla^{(1)}_{k})_{nm} \equiv \big(\nabla^{(1)}, \nabla_2 + \nabla^{(1)}_{2}, \dots, \nabla_k + \nabla^{(1)}_{k}\big)_{nm}; \\ &[\mathcal{J}^{2}(\nabla_1,\dots,\nabla_k)]_{mm',nn'} = \frac{\delta}{\delta\chi_{nn'}}\langle 2m'1m|(\nabla_1,\dots,\nabla_k)\Psi\rangle = \\ &= \big(\nabla^{\{12\}}_{1}, \nabla^{(2)}_{2} + \nabla^{\{12\}}_{2}, \nabla_3 + \nabla^{(2)}_{3} + \nabla^{\{12\}}_{3;\cdot}, \nabla_k + \nabla^{\{12\}}_{k} + \nabla^{(2)}_{k}\big)_{mm',nn'}\,. \end{aligned} \tag{23}$$

The jth steps directly reproduce the wanted formula (21).

Now let us go back to the proof of Assertion. Completeness of the basis $\Phi^{(1)}, \Phi^{(2)}, \Phi^{\{12\}}$ guarantees existence of C for almost all values of the slow variables. Then, computing as in the lemma the action of operator $\mathcal{J}^{(12)}$ on ∇ (see (17)):

$$\left(\mathcal{J}^{\{12\}}\nabla\right)=\left(\nabla_1^{\{12\}},\ \nabla_2^{\{12\}},\ \nabla_3\otimes\mathbb{1}+\nabla_3^{\{12\}},\ldots,\nabla_k\otimes\mathbb{1}+\nabla_k^{\{12\}}\right) \quad (24)$$

and taking into account the representation for the H^S:

$$H^{\{12\}} = -\Delta_1 - \Delta_2 + V - W \quad (25)$$

one sees that

$$\begin{aligned}
\nabla_1^{\{12\}} &= \overset{+}{C}\circ\nabla_1^{\{12\}}\circ C\ , \\
\nabla_2^{\{12\}} &= \overset{+}{C}\circ\mathbb{1}\otimes\nabla_2^{(2)}\circ C + \overset{+}{C}\circ\nabla^{\{12\}}\circ C\ , \\
\overset{\vee}{\nabla}{}^{\{12\}} &= \overset{+}{C}\circ\mathbb{1}\otimes\overset{\vee}{\nabla}{}^{(2)}\circ C + \overset{+}{C}\circ\overset{\vee}{\nabla}{}^{\{12\}}\circ C + \overset{+}{C}\circ\overset{\vee}{\nabla}\circ C
\end{aligned} \quad (26)$$

in which we recognize (i). In order to obtain (ii) it is sufficient to note that C's are Clebshes for the decomposition of the basis $\Phi^{\{12\}}$ in terms of the tensor product $\Phi^{(1)}\otimes\Phi^{(2)}$, and to remember that we supposed $V_{12}=V_1+V_2$, $W=0$. If the last condition fails, then additional terms of the form $\langle\Phi^{\{12\}}_{\cdot}(V_{12}-V_1-V_2+W)\Phi^{\{12\}}_{\cdot}\rangle$ will enter into (19).

4. Now we like to look for an appropriate geometrical interpretation on the example of two-step procedure. The existing experience with complex line-bundles strongly suggests the complex vector bundles as a good candidate for explaining the adiabatic procedure. For this reason let us consider a holomorphic vector-bundle $E\xrightarrow{\pi}B$, $\dim E = N+\nu\ (\equiv 2m)$ with canonical complex structure $\mathcal{J}$, $\mathcal{J}^2=-\mathbb{1}$, acting as $\mathcal{J}:(P_1P_2)\to(P_2P_1)$ on the bundle considered as a real manifold /12/.

Every complex vector bundle admits hermitian metric defined via hermitian scalar product in every fibre $\pi^{-1}(B)$:

$$\langle\xi_1\xi_2\rangle(r_1,r_2;\overset{\vee}{r}) = \sum_\Gamma\left(\overset{+}{\xi}_1\xi_2\right)_\Gamma g_\Gamma\ , \quad (27)$$

where g_Γ is partition of unity adapted to the covering of the manifold, i.e.

$$g_\Gamma(P)=\begin{cases}0 & \text{if } P\bar{\in}\,\mathcal{U}_\Gamma\\ a_\Gamma & \text{if } P\in\mathcal{U}_\Gamma\end{cases}\quad a_\Gamma>0,\ \sum_\Gamma a_\Gamma=1,\ \bigcup_\Gamma\mathcal{U}_\Gamma\supset B.$$

In natural way every hermitean metric $h(\xi)=\langle\xi\xi\rangle$ can be continued to (pseudo)metric on the space Γ of sufficiently smooth sections $\Psi(\cdot)$:

$$\langle\Psi_1\Psi_2\rangle=\int\sum_\Gamma d\mu_\Gamma\left(\overset{+}{\Psi}_1\right)_\Gamma g_\Gamma\left(\Psi_2\right)_\Gamma\ . \quad (28)$$

The measure $d\mu_\Gamma$ and basis functions $\{\Phi^{\cdot}_n\}_\Gamma\equiv\Phi^{\cdot}_\Gamma$ are defined only locally and on the chart crossover they have to satisfy the usual consistency conditions

$$\varphi_\Gamma(\cdot,\Phi_{\Gamma'})=\varphi_{\Gamma'}(\cdot,\Phi_\Gamma)\ ,$$

where φ_Γ are local trivialisations: $\varphi_\Gamma:\mathcal{U}_\Gamma\times\mathbb{R}^\nu\to\pi^{-1}(\mathcal{U}_\Gamma)$.

Note that the measure $d\mu_\Gamma$ is not concentrated in the whole manifold but may have support only on subsets in the base B obtained by some homomorphism $:B\to B$ consistent with the complex structure J.

Consider the pseudo-metrics generated by the set of functions

$$h^{(i)}(\Phi)=\sum_\Gamma\int\bar{\Phi}^{(i)}_\Gamma\otimes g_\Gamma\Phi^{(i)}_\Gamma d\mu^{(i)}_\Gamma,\quad h^S=\sum_\Gamma\int\bar{\Phi}^S_\Gamma\otimes g_\Gamma\Phi^S_\Gamma d\mu^S_\Gamma, \quad (29)$$

where $\Phi^{(i)}_\Gamma$ and Φ^S_Γ are defined in (14) as local eigenvalues of the elliptic operators $\sum g_\Gamma H^{(i)}_\Gamma$, $\sum g_\Gamma H^S_\Gamma$ (see e.g. /12/). Then the canonical connection form Θ and corresponding curvature form F are fixed for holomorphic bundles by the requirement to be consistent with the metrics h:

$$\Theta^S=(h^S)^{-1}\partial h^S,\quad \Theta^{(i)}=(h^{(i)})^{-1}\partial h^{(i)},\quad F=\bar{\partial}\Theta. \quad (30)$$

where $\partial,\bar{\partial}$ are $\mathcal{J}$- conjugate differentials. Afterwards, replacing h from (29) and using the representation $(h)^{-1}=\int\bar{\Phi}\tilde{g}\otimes\Phi d\mu$, where $\tilde{g}_\Gamma g_\Gamma=1$, $\operatorname{supp}\tilde{g}_\Gamma\subset\mathcal{U}_\Gamma$, one obtains all results from 3. For example, consider the proof of gauge equivalence between $A^{\{12\}}$ and $A^{(1)}+A^{(2)}$:

$$A^{\{12\}} = (h^{\{12\}})^{-1}\partial h^{\{12\}},\ A^2=(h^{(2)})^{-1}\partial h^{(2)},\ A^1=(h^{(1)})^{-1}\partial h^{(1)}.$$

Expressing $\Phi^{\{12\}}$ in terms of $\Phi^{(1)}$ and $\Phi^{(2)}$ in the formula for $A^{\{12\}}$ one obtains

$$A^{\{12\}} = \sum_\Gamma \int g_\Gamma \overline{C(\Phi^{(1)}\otimes\Phi^{(2)})_\Gamma}\,\check{\nabla}\otimes C(\Phi^{(1)}\otimes\Phi^{(2)})_\Gamma\, dr_1 dr_2 =$$

$$= \sum_\Gamma ((\bar{C}^T\otimes C)(<\Phi^{(1)}\otimes\check{\nabla}\Phi^{(1)}>\otimes 1 + 1\otimes<\Phi^{(2)}\otimes\check{\nabla}\Phi^{(2)}>)) + \overset{+}{C}\check{\nabla}C,$$

which obviously reproduces (26).

5. The proposed here geometrical framework for interpretation of the adiabatic procedure is incomplete. A more rigorous and sofisticated study of the problem is in progress. The use of the powerfull technique based on K-theory, would provide a new effective instrument for constructing physically interesting examples possessing prescribed geometrical structure.

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Многоступенчатое адиабатическое представление

Для изучения индуцированного калибровочного поля вводится многоступенчатое адиабатическое приближение в квантовой задаче N-тел. Формализм комплексных векторных расслоений дает естественное описание эффективной многоканальной динамики.

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Multistep Adiabatic Representation

A step-by-step adiabatic approximation for the N-particle quantum mechanical problem is introduced in order to study the induced gauge field. The complex-vector-bundle formalism provides a natural framework for description of the multi-channel effective dynamics.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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