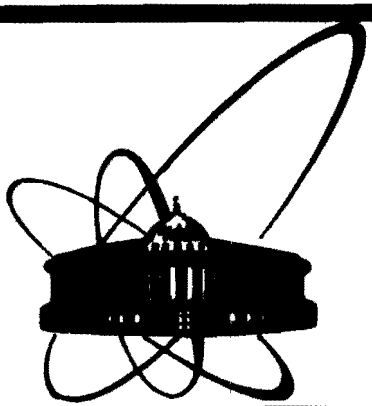

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RETARDED DEEXCITATION OF THE ATOM INDUCED BY EXTERNAL CURRENT

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1. INTRODUCTION

The principle of local commutativity states that two local field observables must commute if their supports are space-like separated. The consequence is that the corresponding physical observables (events) are independent of each other and cannot be the cause and effect (e.g., see[1]). This principle (named (I) in the following) is supplemented (and sometimes combined) with another causality principle (II):"only retarded solutions of the physical equations are realized in the Nature". This means that the cause can only be inside the past light cone of the effect but not inside its future cone.

It is natural to set the problem of verifying (II) in the same sence as one has verified (I) or conservational laws: one has to point out physical processes which are forbidden by them. It seems that stationary (S-matrix) processes do not suit this purpose. In order to verify (I) and (II), the following nonstationary problem is considered here, namely, the action of the external current on the deexcitation of the excited atom. The current J and atom are supposed to be localized in regions V_J and V separated by the distance R which is much greater than λ, the wavelength of the atom radiation. The problem is introductory to a more realistic (but more complex) problem: how one atom influences the deexcitation of another.

The inclusive probability of the deexcitation of the atom (accompanied by its recoil) is calculated, i.e., we suppose that only the atom states are measured, photons are not detected. The change of this probability induced by switching on of the current J is defined as the following difference: "the probability with J being present minus the same probability but without J (i.e., when $J = 0$ always)".

It is shown in section 2 that the recoil momentum of the atom which is due to the current action is directed towards the current, i.e. the current attracts the atom but does not exert "light pressure" upon it. This circumstance is important for the interpretation of the mechanism of the current action which is discussed in sect. 3.

The calculations in sect. 2 are performed using a "semi-Heisenberg picture" which is, in some sense, intermediate between the interaction and Heisenberg pictures. If the recoil is not measured, then the result coincides with the one obtained in the Heisenberg picture, see [2].

The calculation in standard interaction picture perturbation theory is sketched in sect. 3. Being cumbersome as compared with the "semi-Heisenberg" calculation it turns out to be important for the interpretation of the mechanism of the current action on the atom.

The results are discussed in Sect. 4. It is shown how to verify principle (II) along with (I) using switching on of the current in appropriate time intervals. It is demonstrated that the proposition "signal velocity cannot exceed C" is not equivalent to principle (I) and must be considered as a consequence of both (I) and (II).

2. SEMI-HEISENBERG PICTURE

The additional probability of the atom deexcitation induced by the external current J is defined as $\Delta W(t) = W_J(t) - W_0(t)$. Here $W_0(t)$ is the probability of the atom transition from the initial state having the energy ϵ_2 to the state having the energy $\epsilon_1 < \epsilon_2$, the atom being acquired the recoil momentum $\vec{P}_f$. $W_J(t)$ is the same probability but when J is present, being nonzero in a time interval (τ_1, τ_2).

The atomic nucleus is supposed to have a finite mass m_n. I use, in this section, the nonrelativistic description of the atom electron. The approximation is good enough for the considered problem and it allows one to escape troubles arising in the relativistic description of the system "electron + + nucleus".

The initial state is described by the vector $\psi_2 \cdot \Omega \equiv \Phi(\vec{X}) \cdot \phi_2(\vec{r})\Omega$: photons are absent (Ω is no-photon state); the atom is in the excited state 2; $\Phi(\vec{X})$ describes (in the center-of-mass-coordinate-$\vec{X}$ representation) the atom localized as a whole in a volume V, its dimensions being $\gg \lambda_2, \lambda_1 \sim (\epsilon_2 - \epsilon_1)^{-1}$. The final atom state is described by $\psi_1 = \exp(i\vec{P}_f\vec{X})\,\phi_1(\vec{r})$. The distance R between V and the current localization region V_J is much greater than V and V_J dimensions, see fig. 1.

The probabilities W_J and W_0 are inclusive ones:

$$W_J(t) = \Sigma_\Gamma |\langle \psi_1 \Gamma | U_J(t,0) | \psi_2 \Omega \rangle|^2, \quad W_0(t) = \Sigma_\Gamma |\langle \psi_1 \Gamma | U(t,0) | \psi_2 \Omega \rangle|^2. \tag{1}$$

Here Γ denotes the complete set of the electromagnetic field states; Ω, one-phonon states $c^+_{k\epsilon}\Omega$; $\forall \vec{k}, \epsilon$; two-photon states, etc. $U(t,0)$ can be considered as the evolution operator in the interaction picture when the current is absent:

$$e^{-iHt} = e^{-iH_0 t}\, U(t,0), \tag{2}$$

$$H = H_0 + H_I.$$

When J is present,

$$U_J(t,0) = T\exp[-i\int_0^t H_I^J(t')\,dt'], \tag{3}$$

where $H_I^J(t)$ is (the Coulomb gauge being chosen)

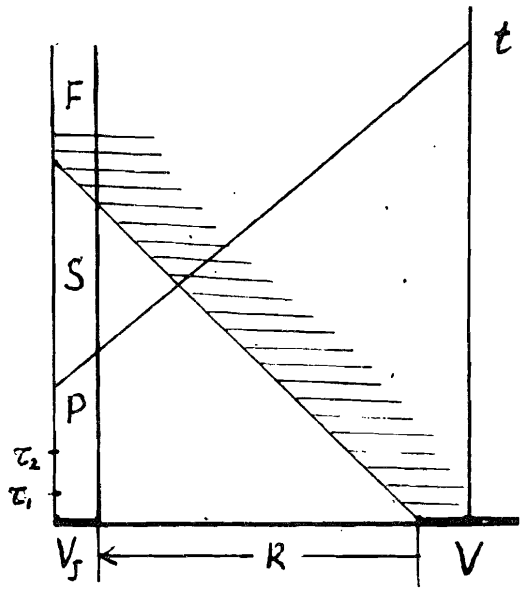

Fig. 1.

$$H_I^J(t) = \frac{e}{m_e}\vec{p}_e(t)\cdot\vec{A}_\perp(\vec{x}_e(t),t) - \frac{e}{m_n}\vec{p}_n(t)\cdot\vec{A}_\perp(\vec{x}_n(t),t) + \tag{4}$$

$$+ \frac{e^2}{2m_e}\vec{A}_\perp^2(\vec{x}_e(t),t) + \frac{e^2}{2m_n}\vec{A}_\perp^2(\vec{x}_n(t),t) +$$

$$+ \frac{1}{4\pi}\int d^3x\left[\frac{eJ_0(\vec{x},t)}{|\vec{x}-\vec{x}_e(t)|} - \frac{eJ_0(\vec{x},t)}{|\vec{x}-\vec{x}_n(t)|}\right] - \int d^3x\vec{J}(\vec{x},t)\vec{A}_\perp(\vec{x},t).$$

As usual, H_I^J is a function of the interaction picture operators, e.g.

$$\vec{x}_e(t) = e^{iH_0 t}\vec{x}_e e^{-iH_0 t} = e^{iH_{0a}t}\vec{x}_e e^{-iH_{0a}t}, \quad H_0 = H_{0a} + H_{0\gamma} \tag{5}$$

$$\vec{A}_\perp(\vec{x},t) = e^{-iH_{0\gamma}t}\vec{A}_\perp(\vec{x})e^{-iH_{0\gamma}t}.$$

2.1. The calculation of (1) can be considerably simplified using a special representation of the operator $U_J(t,0)$. Note the separation in eq. (2) of $\exp(-iHt)$ into the product of $\exp(-iH_0t)$ and $U(t,0)$. Let us separate analogously $U_J(t,0)$ the left multiplier being $T\exp i\int JA$, see the last term in H_I^J, eq. (4):

$$U_J(t,0) = T\exp\left[i\int_0^t dt' \int d^3x\, \vec{J}(\vec{x},t')\cdot\vec{A}_\perp(\vec{x},t')\right]\cdot U_J'(t,0). \qquad (6)$$

Using the equation $i\partial_t U_J = H_I^J U_J$ and the analogous eq. for $V_J \equiv T\exp[i\int JA]$ one obtains for U_J' the equation:

$$i\partial_t U_J'(t,0) = H_I'(t)\,U_J'(t,0) \qquad (7)$$

$$H_I'(t) = V_J^+(t)\,\{H_I^J \text{ without the last term in eq. (4)}\}\,V_J(t)$$

$$V_J(t) \equiv T\exp\left[i\int_0^t dx_0 \int d^3x\, \vec{J}(\vec{x},x_0)\cdot\vec{A}_\perp(\vec{x},x_0)\right].$$

The electron and nucleus operators commute with $\vec{J}\cdot\vec{A}_\perp$ and therefore H_I' is given by the expression {r.h.s. of eq. (4) without the last term} in which $A_\perp(x,t)$ is replaced by

$$A_{\perp i}(\vec{x},t) = V_J(t)A_{\perp i}(\vec{x},t)V_J(t) = \qquad (8)$$

$$= A_{\perp i}(\vec{x},t) - \int_0^t dy_0 \int d^3y \sum_j D^\perp_{ij}(\vec{x}-\vec{y},\,t-y_0)\,J_j(\vec{y},y_0).$$

The calculation of $V_J^+ A_\perp V_J$ is realized using formulae from § 20 of the book [3] (see, e.g., (20.2) and (20.14a)) or in § 21.2 of the first edition of [4]. The function $D^\perp$ is the transverse D-function which appears in the r.h.s. of the commutator which is used to calculate $V_J^+ A_\perp V_J$

$$[A_{\perp i}(\vec{x},x_0),\, A_{\perp j}(\vec{y},y_0)] = -iD^\perp_{ij}(\vec{x}-\vec{y},\,x_0-y_0).$$

The operator $\vec{A}_\perp^J$ satisfies the equation $\Box\vec{A}_\perp^J = -\vec{J}_\perp$. The electron and nucleus current are not present in the r.h.s. of this equation and therefore $\vec{A}_\perp^J$ is not the Heisenberg operator, it can be called the semi-Heisenberg operator.

The representation (6) for U_J where U_J' is the solution of eq. (7)

$$U_J'(t,0) = T\exp\left[-i\int_0^t H_I'(t')\,dt'\right] \qquad (9)$$

simplifies the calculation of W_J because $V_J \equiv T\exp[i\int JA]$ disappears from r.h.s. of (1) in the following manner. One has

$$W_J = \Sigma_\Gamma \langle\psi_2\Omega\,|\,U_J'^+ V_J^+\,|\,\psi_1\Gamma\rangle\langle\psi_1\Gamma\,|\,V_J U_J'\,|\,\psi_2\Omega\rangle =$$

$$= \Sigma_{n_1 n_2}\langle\psi_2\Omega\,|\,U_J'^+\,|\,n_1\rangle\langle n_1\,|\,V_J^+\,|\,\psi_1\Gamma\rangle\langle\psi_1\Gamma\,|\,V_J\,|\,n_2\rangle\langle n_2\,|\,U_J'\,|\,\psi_2\Omega\rangle.$$

Here I inserted the sums $\Sigma_n|n\rangle\langle n| = 1$ over the complete set of states $|n\rangle = |\Gamma\psi_n\rangle$ of the considered system "electron + nucleus + electromagnetic field". As V_J does not depend upon the electron and nucleus operators, one has

$$\langle\psi_1\Gamma\,|\,V_J\,|\,n_1\rangle = \langle\Gamma\,|\,V_J\,|\,\Gamma_2\rangle\langle\psi_1\,|\,\psi_n\rangle$$

and the sums over n_1 and n_2 reduce to the sums over Γ_1 and Γ_2. Using the V_J unitarity property

$$\Sigma_\Gamma\langle\Gamma_1\,|\,V_J^+\,|\,\Gamma\rangle\langle\Gamma\,|\,V_J\,|\,\Gamma_2\rangle = \delta_{\Gamma_1,\Gamma_2}$$

we obtain

$$W_J = \Sigma_{\Gamma_1}\langle\psi_2\Omega\,|\,U_J^+\,|\,\psi_1\Gamma_1\rangle\langle\psi_1\Gamma_1\,|\,U_J'\,|\,\psi_2\Omega\rangle = \qquad (10)$$

$$= \Sigma_\Gamma|\langle\psi_1\Gamma\,|\,U_J'(t,0)\,|\,\psi_2\Omega\rangle|^2.$$

2.2. Let us calculate (10) in the first nonvanishing approximation in e, the electron charge. The approximation turns out to be $\sim e^1$ and therefore the terms $e^2A_\perp^2$ from eq. (4) do not contribute.

Let the current $\vec{J}(\vec{x},t)$ be localized in a region V_J near the end of the vector $\vec{R}$, see fig. 1. Then one has $\vec{x} = \vec{R} + \vec{x}'$, where $|\vec{x}'| \ll R$. The equation

$$\int d^3x\, e\, J_0(x,t)\left[\frac{1}{|\vec{x}-\vec{x}_e|} - \frac{1}{|\vec{x}-\vec{x}_n|}\right] = \qquad (11)$$

$$= \frac{e}{R^3}\int d^3x'\, J_0(\vec{R}+\vec{x}',\,t)[-2r_z x'_z + r_x x'_x + r_y x'_y],\ \vec{r} \equiv \vec{x}_n - \vec{x}_e$$

can be deduced. We shall see that $\langle\psi_1\Gamma\,|\,U_J'\,|\,\psi_2\Omega\rangle$ is of the order $1/R$, so the contribution (11) to the matrix element can be discarded, as being of the order $1/R^3$.

Let us further use the dipole or long-wave approximation substituting $\vec{A}_\perp^J(\vec{X},t)$ for $\vec{A}_\perp^J(\vec{x}_e,t)$ and $\vec{A}_\perp^J(\vec{x}_n,t)$, $\vec{X} = (m_n\vec{x}_n + m_e\vec{x}_e)/(m_n+m_e)$.

After these approximations one has

$$U_J'(t,0) \cong 1 - i\int_0^t dt\, e^{+iH_0 a t'}\left[-\frac{e}{\mu}\vec{p}\cdot\vec{A}_\perp^J(\vec{X},t)\right]e^{-iH_0 a t'}, \qquad (12)$$

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_n};\quad \frac{\vec{p}}{\mu} = \frac{\vec{p}_n}{m_n} - \frac{\vec{p}_e}{m_e}.$$

I have represented here U_J' in terms of the Schroedinger electron and nucleus operators, using eqs. (5).

Now one can see that the operator (12) has nonvanishing elements $\langle\psi_1\Gamma|U_J'|\psi_2\Omega\rangle$ only if $\Gamma=\Omega$ or $\Gamma=c^+_{k\epsilon}\Omega$. If $\Gamma=c^+_{k\epsilon}\Omega$, then only the free part $A_\perp(x,t)$ of the operator $\vec{A}^J_\perp$, see eq. (8), contributes to $\langle\psi_1 c^+_{k\epsilon}\Omega|U_J'|\psi_2\Omega\rangle$. So the elements coincide with $\langle\psi_1 c^+_{k\epsilon}\Omega|U|\psi_2\Omega\rangle$, the square of modulus of which is equal to the "background" $W_0(t)$. We get

$$\Delta W(t) = W_J - W_0 \cong |\langle\psi_1\Omega|U_J'(t,0)|\psi_2\Omega\rangle| . \tag{13}$$

Using the equations

$$e^{-iH_0 a t} e^{i\vec{P}_f\vec{X}}\phi_1(\vec{r}) = \exp[-(\frac{P_f^2}{2M}+\epsilon_1)t]\, e^{i\vec{P}_f\vec{X}}\phi_1(\vec{r}); \tag{14}$$

$$e^{-iH_0 a t}\Phi(\vec{X})\phi_2(\vec{r}) = \frac{1}{(2\pi)^3}\int d^3P\, \exp[-i(\frac{P^2}{2M}+\epsilon_2)t]\cdot e^{iPX}\tilde{\Phi}(\vec{P})\,\phi_2(\vec{r}); \tag{15}$$

$$\tilde{\Phi}(\vec{P}) \equiv \int d^3X\, e^{-i\vec{P}\vec{X}}\Phi(X), \quad M = m_n + m_e$$

one gets for $\langle\psi_1\Omega|U_J'|\psi_2\Omega\rangle$

$$\langle\psi_1\Omega|U_J'(t,0)|\psi_2\Omega\rangle \cong -\frac{i}{(2\pi)^3}\int_0^t dt'\int d^3P\tilde{\Phi}(\vec{P})\, e^{-iE_{12}t'} \int d^3X e^{i(\vec{P}-\vec{P}_f)\vec{X}} \int d^3r\,\phi_1^*(\vec{r})\sum_{i,j}(-\frac{e}{\mu}p_i)\,\phi_2(\vec{r})\times \tag{16}$$

$$\times\int_0^t dy_0\int d^3y\, D^\perp_{ij}(\vec{X}-\vec{y},\, t'-y_0)\, J_j(\vec{y},\, y_0)$$

$$E_{12} \equiv (P_f^2 - P^2)/2M + \epsilon_1 - \epsilon_2. \tag{17}$$

If $\tilde{\Phi}(\vec{P})$ describes the state with a vanishing average momentum with the uncertainty $|\Delta P| \ll \epsilon_2 - \epsilon_1 \equiv \Delta$, then $P^2/2M \ll \Delta^2/2M$. Assume as an Anzatz (to be confirmed by the subsequent calculation) that $|\vec{P}_f|$ is of the order Δ. Then $P_f^2/2M \sim \Delta^2/2M$ which is $\ll\Delta$ because of $M \gg \Delta$. So $E_{12}(P)$, see eq. (17), is equal approximately to $-\Delta$. Let us replace $E_{12}(\vec{P})$ by the expression $-\Delta + \Delta^2/2M \equiv -\Delta'$ which does not depend upon P. One can show using some realistic values of the parameters R, $\tau_2 - \tau_1$, Δ that the resulting error is small.

To calculate (16), I begin with the time integrals in the r.h.s. of eq. (16)

$$Y_i = \sum_j \int_0^t dt' e^{-i\Delta' t'}\int_0^{t'} dy_0\, D^\perp_{ij}(\vec{X}-\vec{y},\, t'-y_0)\, J_j(\vec{y},\, y_0). \tag{18}$$

Let $J_j(\vec{y},\, y_0)$ be zero outside an interval (τ_1, τ_2) and

$$J_j(\vec{y},\, y_0) = K_j(\vec{y})\, e^{i\Delta' y_0} + K_j^*(\vec{y})\, e^{-i\Delta' y_0} \tag{19}$$

if $y_0 \in (\tau_1, \tau_2)$. Taking into account only the first term of the order $1/R$ in expression (A.4) for $D^\perp$, see Appendix, one gets (cf. subsect 2.3 in /2/)

$$Y_i = \frac{1}{4\pi\rho}[K_i - n_i(Kn)]\, e^{-i\rho\Delta} F_\rho(t);$$
$$\rho \equiv |\vec{X}-\vec{y}|; \quad \vec{n} \equiv (\vec{X}-\vec{y})/\rho \tag{20}$$

$$F_\rho(t) = \begin{cases} 0 & t < \tau_1 + \rho \\ t - (\tau_1+\rho) & \tau_1+\rho < t < \tau_2+\rho \\ \tau_2 - \tau_1 & t > \tau_2 + \rho \end{cases} \tag{21}$$

Here $\vec{X} \in V$; $\vec{y} \in V_J$ and therefore $\rho \cong R$. But if one lets simply $\rho = R$, the result will be $\vec{P}_f = \vec{P}$: in such an approximation the atom recoil is absent. One needs an approximate ρ which depends upon $\vec{X}$. I let the volume V_J to be much less than V. Then $\rho = |\vec{X}-\vec{y}| = |\vec{R}-\vec{X}|$, where $|\vec{X}| \ll R$. The main terms of Y_i expansion into the series over $\vec{X}$ (see, e.g., eq. (29) in /2/) have the order $1/R$. They originate from the $\exp(-i\rho\Delta)$ expansion, see eq. (20). The infinite series of these terms sums to the expression

$$Y_i = \frac{1}{4\pi R}[K_i - n_i(\vec{K}\vec{n})]\exp[-i(R - \vec{n}\vec{X})\Delta]\, F_R(t);\ \vec{n} = \vec{R}/R \tag{22}$$

(note that $|\vec{R}-\vec{X}| \cong R - \vec{n}\vec{X}$). Inserting this expression into the r.h.s. of eq. (16) we find out that integrals $\int d^3r \ldots$ and $\int d^3y \ldots$ turn into dipole moments of the atom $\vec{d}_{12}$ and the current $\vec{d}_J$ (see eq. (27) in /2/); $\int d^3X \ldots$ turns into

$$\int d^3X \exp[i(\vec{P}-\vec{P}_f)\vec{X}]\exp i\vec{n}\vec{X}\Delta = (2\pi)^3\delta^{(3)}(\vec{P}_f - \vec{P} - \vec{n}\Delta)$$

and then $\int d^3P \ldots$ becomes

$$\int d^3P\,\tilde{\Phi}(\vec{P})\,\delta^{(3)}(\vec{P}_f - \vec{P} - \vec{n}\Delta) = \tilde{\Phi}(\vec{P}_f - \vec{n}\Delta). \tag{23}$$

The final result is

$$\langle\psi_1\Omega|U'_J(t,0)\psi_2\Omega\rangle = \tilde{\Phi}(\vec{P}_f - \vec{n}\Delta)\,e^{-iR\Delta}\cdot\frac{(-ie\Delta^2)}{4\pi R}\cdot[(\vec{d}_{12}\cdot\vec{d}_J) - (\vec{d}_{12}\cdot\vec{n})(\vec{d}_J\cdot\vec{n})]\,F_R(t). \quad (24)$$

2.3. If one integrates the squared modulus of this expression (see eq. (13)) over $\vec{P}_f$, then one will get the change of the transition $2\to1$ probability induced by J. The result will coincide with expression (32) for the change calculated in [2] using the perturbation theory in the Heisenberg picture in the Lorentz gauge. Here I started with the Coulomb gauge, but I omitted the terms $\sim 1/R^3$ (see (11) and the second term in eq. (A.4)). The results obtained in the Coulomb and Lorentz gauges differ only by these small terms. It was argued in [5,6] that these terms must be considered as unobservable. So, the calculation in the Lorentz gauge (which has the defect discussed in subsect 2.6 in [2]) gives the same result as in the Coulomb gauge and I shall use, in the next section, the Lorentz gauge which is simpler.

2.4. The presence of $\tilde{\Phi}(\vec{P} - \vec{n}\Delta)$ in eq. (24) means that the distribution over $\vec{P}_f$ differs from the initial momemtum distribution by the simple displacement $\vec{n}\Delta$. In particular if $\tilde{\Phi}(\vec{P})$ has a zero average momentum $\langle\vec{P}\rangle = 0$, then $\langle\vec{P}_f\rangle = \Delta\vec{n} = \Delta\vec{R}/R$. $\vec{R}$ is directed from the atom towards the current and so does the recoil momentum: the current attracts the atom.

3. STANDARD PERTURBATION THEORY

Following the standard QED perturbation theory [4,7], I use, in this section, the Lorentz gauge and relativistic secondary-quantized description of the electron in the field of the atomic nucleus (now I let $m_n = \infty$). In the preceding section, the interaction $\int JA$ was not considered small and was treated nonperturbatively. Now I assume that the external current J contains a small constant e_J (analogous to the constant e in the electron current) and use perturbation theory to take interaction $\int JA$ into account. The starting equation is $\Delta W = W_J - W_0$, where

$$W_J(t) = \sum_\Gamma |\langle\phi_1\Gamma|\,U_J(t,0)\,|\phi_2\Omega_j\rangle|^2, \quad (25)$$

$$U_J(t,0) = T\exp i\int_0^t dt'\int d^3x'(j_\mu A_\mu + J_\mu A_\mu). \quad (26)$$

Here Ω_γ is the photon vacuum. Using the electron creation operators one can write $\phi_1 = a_1^+\Omega_e$, $\phi_2 = a_2^+\Omega_e$, Ω_e being the electron-positron vacuum, a_1^+ creates electron in the state 1*.

To calculate W_J in the first nonvanishing order, one has now to expand the T-exponential $U_J(t,0)$ up to the terms of the third order (it was sufficient to use the term $\sim e^1$ from $U'_J(t,0)$ in the preceding section):

$$W_J(t) = \Sigma_\Gamma|\langle\phi_1\Gamma|\,1 + U_J^{(2)} + U_J^{(2)} + U_J^{(3)}\,|\Phi_{in}\rangle|^2 \to$$
$$\to \Sigma_\Gamma|\langle\phi_1\Gamma|\,U_J^{(2)}\,|\Phi_{in}\rangle|^2 + \quad (27)$$
$$+\{\Sigma_\Gamma\langle\phi_1\Gamma|\,U_J^{(1)}\,|\Phi_{in}\rangle^*\langle\phi_1\Gamma|\,U_J^{(3)}\,|\Phi_{in}\rangle + \text{c.c.}\}.$$

Here I take into account that $\langle\phi_1\Gamma|\,1\,|\Phi_{in}\rangle = 0$ because of $\langle\phi_1|\phi_2\rangle = 0$. The terms $U_J^{(1)}U_J^{(1)}$ are equal to the corresponding terms of W_0 and therefore do not enter into ΔW. They are not written out in the r.h.s. of (27). The terms $U_J^{(1)}U_J^{(2)}$ are absent because the possible Γ in $\langle\phi_1\Gamma|\,U_J^{(1)}\,|\Phi_{in}\rangle$ are only the one-photon states whereas in $\langle\phi_1\Gamma|\,U_J^{(2)}\,|\Phi_{in}\rangle$ the states Γ are no-photon and two-photons ones.

To simplify the exposition of the calculation and comparison with the preceding section, I assume $e_J \gg e$ in what follows and omit the terms $\sim e^4$ and $\sim e^3 e_J$ in ΔW as being small compared to $e^2 e_J^2$.

3.1. Consider the matrix elements $\langle\phi_1\Gamma|\,U_J^{(2)}\,|\Phi_{in}\rangle$ from eq. (27). If $\Gamma = \Omega$, they are represented by diagrams "a" and "b" in fig. 2 (the term quadratic in J vanishes because of $\langle\phi_1|\phi_2\rangle = 0$). The diagram b is of the order e^2. The r.h.s. of eq. (27) contains $|b|^2$ and the product b*a which are of the order e^4 and $e^3 e_J$, respectively and are omitted. The diargam a is of

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* The probability ΔW differs from ΔN, defined in [2] though the difference is small numerically. ΔN was defined as the probability to find the electron in the bound state 1. The probability turns out to contain the contribution (described by the second item in eq. (18) from [2]) of the following origin. The current J emits a photon, it propagates to the atom and creates a pair, the electron of which is in state 1. The atomic electron remains in its initial state 2. Then the final state contains the negative ion, not the neutral atom. Here I assume that just the neutral atom in the bound state 1 is detected at the moment t.

a *Fig. 2.* b

the proper order ee_J and represents the expression (here and in the following the coordinate representation[7] of diagrams is implied)

$$(-1)\int_0^t dx_0 \int_0^t dy_0 \int d^3x \int d^3y\, j^{\mu}_{12}(x)\, D^c(x-y)\, J_{\mu}(y)$$

$$j^{\mu}_{12}(x) \equiv \langle \phi_1 | j^{\mu}(x) | \phi_2 \rangle, \quad x \equiv (\vec{x}, x_0) \tag{28}$$

(the notation is as in[4]).

If Γ are two-photon states $\Gamma_2 = 1/\sqrt{2}\, C^+_{k_1 f_1} C^+_{k_2 \epsilon_2} \Omega_\gamma$ then the part of the amplitude $\langle \phi_1 \Gamma_2 | U^{(2)}_J | \Phi_{in} \rangle$ which is of the order ee_J is

$$\frac{-1}{2\sqrt{2}(2\pi)^3} \int_0^t d^4x \int_0^t d^4y\, \frac{1}{\sqrt{k_1 k_2}} \{ j^{\mu}_{12}(x)\, e^{\epsilon_1}_{\mu}(\vec{k}_1) e^{-ik_1 x}\, J_{\nu}(y)\, e^{\epsilon_2}_{\nu}(\vec{k}_2) e^{-ik_2 y} + \tag{29}$$

$$+ j^{\mu}_{12}(x)\, e^{\epsilon_2}_{\mu}(\vec{k}_2) e^{-ik_2 x}\, J_{\mu}(y)\, e^{\epsilon_1}_{\nu}(\vec{k}_1) e^{-ik_1 y} \}.$$

The r.h.s. of eq. (27) contains the squares of the moduli of the first and second items in the curly brackets in (29) summed over Γ_2 (i.e. integrated over $\vec{k}_1$, $\vec{k}_2$ and summed over polarizations ϵ_1, ϵ_2). This contribution is the product of the probabilities of the atom one-photon emission and the probability of the current one-photon emission. It is the "crossed" product of the first item and the complex-conjugated second one summed over Γ_2 which turns out to be important for the interpretation of the mechanism of the current action upon the atom deexcitation (see the subsect 3.3 below). Let us describe it. The amplitude of the photon $\vec{k}_1, \epsilon_1$ emission by the current is multiplied by the c.c. of the amplitude of the emission by the atom of a photon with the same momentum and polarization $\vec{k}_1, \epsilon_1$. The result is integrated over $\vec{k}_1$ and summed over ϵ_1 and its modulus is squared.

Using the known equations

$$\Sigma_{\epsilon} e^{\epsilon}_{\mu_1}(\vec{k})\, e^{\epsilon}_{\mu_2}(\vec{k}) = \delta_{\mu_1 \mu_2}$$

$$D^+(x) = \frac{i}{2(2\pi)^3} \int \frac{d^3k}{k} e^{i(\vec{k}\vec{x} - k x_0)}, \quad x \equiv (\vec{x}, x_0) \tag{30}$$

one can introduce the D^+-functions instead of integrals over $\vec{k}_1$, $\vec{k}_2$. This allows us to represent the quantities $\Sigma_{\Gamma_2} |\langle \phi_1 \Gamma_2 | U^{(2)}_J | \Phi_{in} \rangle|^2$ described above, by the products D and B of diagrams, see fig. 3. "B" represents the "crossed" product (the vertex * represents $j^{\mu *}_{12}$).

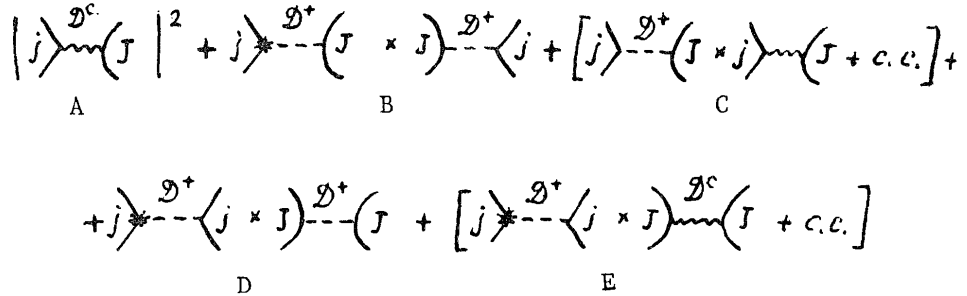

Fig. 3.

The contributions of the terms $U^{(1)} \times U^{(3)}$ in eq. (27) which are $\sim e^2 e_J^2$ are represented by the products of diagrams C and E, see fig. 3 (c.c. denotes complex-conjugated to them). Here the functions D^+ and D^c originate from the equation

$$\Sigma_{\Gamma} \langle \Omega_\gamma | A_a | \Gamma \rangle \langle \Gamma | T(A_\mu A_\nu A_\lambda) | \Omega_\gamma \rangle =$$

$$= \langle \Omega | A_a T(A_\mu A_\nu A_\lambda) | \Omega \rangle = \tag{31}$$

$$= -iD^+_{a\mu} D^c_{\nu\lambda} - iD^+_{a\nu} D^c_{\mu\nu} - iD^+_{a\lambda} D^c_{\mu\nu}$$

which follows from the completeness of the states Γ and from the known expansion of the T-product in the normal products.

3.2. It turns out that the sum A+B+C+C* is exactly equal to

$$\left| \int_0^t d^4x \int_0^t d^4y\, j^{\mu}_{12}(x)\, \theta(x_0 - y_0) D(x-y)\, J_{\mu}(y) \right|^2 \tag{32}$$

because of the equation

$$D^c(x-y) + iD^+(y-x) = -iD^R(x-y) \equiv -i\theta(x_0 - y_0) D(x-y). \tag{33}$$

It is possible also to prove that $D+E+E^* = 0$ using the equation

$$D^c(x-y) + D^{c*}(x-y) + i[D^+(x-y) - (D^+(x-y))^*] = 0. \qquad (34)$$

The equality $D+E+E^* = 0$ can be obtained also as a consequence of the unitarity of the operator $V_J = \mathrm{T}\exp i\int JA$, see sect. 2.

So, the cumbersome expression represented in fig. 3 given by the standard perturbation theory for ΔW can be reduced to the simple expression (32) which can be obtained far simpler in the semi-Heisenberg or Heisenberg pictures (see, e.g., eq. (19) in[2]).

3.3. Let the current J be switched on in such a time interval (τ_1, τ_2) that the points $(\vec{y}, y_0)$, $\vec{y} \in V_J$, $y_0 \in (\tau_1, \tau_2)$ are in the 4-volume P ("past", see fig.1) which is placed inside the past light cone having the vertex at the 4-point (0,t) (more exactly, one should speak about the intersection of the cones with vertices at the point $(\vec{x}, t)$, $\vec{x} \in V$). Let us show that the amplitude (28) corresponding to the diagram a, see fig.2, is small in this case.

The function $D^c(x-y)$, see (28), is small when $x \sim y$ (x is space-like relative to y). Therefore, only those x_0 which are greater than $\tau_1 + R/c$ contribute to the integral $\int_0^t dx_0$ (the current action reaches the atom with the delay R/c). Because of $y_0 \le \tau_2$ we have $x_0 > y_0$ assuming $\tau_2 < \tau_1 + R/c$. Then only the part $-i\theta(x_0 - y_0)D^+(x-y)$ of the function $D^c(x-y)$ (see eq. (17.31') in[4]) contributes to (28). Using the representation (30) for D^+ we get that the time integrals in (28) are equal to

$$\int_{\tau_1+R}^{t} dx_0 \int_{\tau_1}^{\tau_2} dy_0\, e^{i(\epsilon_1-\epsilon_2)x_0} e^{-ik(x_0-y_0)} [K_\mu e^{i\Delta y_0} + K_\mu^* e^{-i\Delta y_0}]. \qquad (35)$$

Equation (19) was used here (the atom mass M is set equal to infinity in this section and therefore $\Delta' = \Delta \equiv \epsilon_2 - \epsilon_1$). Only $k \sim \Delta$ can contribute to the integral over y_0 (let $\Delta(\tau_2 - \tau_1) \gg 1$) but for such k values the integral over x_0 is small. One can say that (35) is small because in the cases when the energy is conserved (approximately) at the J vertex, it cannot be conserved at j vertex.

Using the same line of reasoning one can conclude that the diagram in the product B, see fig.3, is not small. In this case we have (using that $D^+(x-y)$ is small at $x \sim y$ and that $j^{\mu *}_{12} \sim \exp(i x_0 \Delta)$)

$$\int_{\tau_1+R}^{t} dx_0 \int_{\tau_1}^{\tau_2} dy_0\, e^{i\Delta x_0} e^{-ik(x_0-y_0)} [K_\mu e^{i\Delta y_0} + K_\mu^* e^{-i\Delta y_0}] \qquad (36)$$

instead of (35) and the energy can be conserved at both vertices. The conservation also follows evidently from the description of the "crossed" product (see the text before eq.(30)) which B represents.

Now one can see that $A+B+C+C^* \cong B$, see fig.3, because A and C contain the small integral (28) (or (35)). We conclude that in the standard perturbation theory the main contribution to ΔW is brought by the two-photon states described above before eq.(30): J emits a photon $\vec{k}$ and the atom emits a photon with the same momentum $\vec{k}$. The presented calculation does not show that $\vec{k}$ is directed from J towards the atom. But this follows from calculations of sect.2: remind that the atom acquires the recoil in the direction towards current.

So one can interpret the retarded action of the current on the atom in the following manner: J emits a photon in the direction of the atom, this photon reaches the atom with the delay R/c and the atom then emits one more photon, exactly like the incident one*.

Note that the calculation of sect. 2 does not hint at this mechanism: according to eq.(13) it is no-photon state (but not two-photon ones) that contributes to (10). But we use summation over all photon states Γ (including two-photon ones) when obtaining (10) from (1).

3.4. Another interpretation was suggested in[2]. It was based on the inspection of the structure of the formulae of the Heisenberg picture perturbation theory (see, e.g., eq.(13) here and eq.(32) in[2]) and used the notion of the quantum with negative energy. The mechanism described in the preceding subsect 3.3 does not reject the interpretation (it can be considered as an alternative). For rejecting we must replace the external current by an unexcited atom which is unable to emit photons. The calculation of such a problem by means of the standard perturbation theory gives the following qualitative result (the derivation will be presented in another paper): the unexcited atom does not change the deexcitation of the excited atom. More exactly, the unexcited atom retarded action (represented by the curve with a bump in fig.1 from[2]) is unobservably small, its value being of the same order as the energy nonconserving contributions of A and C (fig.3) to

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* The mechanism of stimulated atom radiation in the presence of the current photon was suggested by V.I.Ritus and V.P.Frolov. I am grateful to them for discussions of the paper[2].

ΔW. In the semi-Heisenberg or Heisenberg pictures this fact turns out to follow from the mutual (looking accidental) annihilation of big contributions: the term of the type $U^{(2)} \times U^{(2)}$ (cf. eq. (27)) cancels the terms of the type $U^{(1)} \times U^{(3)}$.

So, one must conclude that the existence of negative energy quanta does not follows from QED.

4. DISCUSSION

Let us discuss two qualitative consequences of the equation ΔW = (32), see sect. 3. The first is that $\Delta W = 0$ if J is switched on in the time interval (τ_1, τ_2) which is in the 4-region S ("space", see fig.1), the points of which $(\vec{y}, y_0)$, $\vec{y} \in V_J$, $y_0 \in (\tau_1, \tau_2)$ are space-like relative to the points $(\vec{x}, x_0)$, $\vec{x} \in V$, $0 < x_0 < t$. This fact is not consequence of the details of QED theory but follows from the general principle of the local commutativity (principle I, see the Introduction): the function $D(x-y)$ which vanishes at $y \sim x$ originates from the commutator $[A_\mu(x), A_\nu(y)]$, cf. the text after eq. (8).

The second consequence is that $\Delta W = 0$ also in the case when J is turned on in the 4-region F ("future", see fig. 1) the points of which $(\vec{y}, y_0)$, $\vec{y} \in V_J$, $R/c < y_0 < t$ are located inside future cones with vertices at the points $(\vec{x}, x_0)$, $\vec{x} \in V$, $x_0 = 0$ (shaded in fig. 1). In such a case some 4-points $(\vec{y}, y_0)$ are time-like relative to $(\vec{x}, x_0)$, $\vec{x} \in V$, $0 < x_0 < t$ and vanishing of ΔW is now the consequence of the fact that just the function $D^R(x-y) = \theta(x_0 - y_0)\, D(x-y)$ enters into (32), i.e. the consequence both of rejecting of the advanced solutions (principle II) and of principle I. Let us stress that nonvanishing of ΔW in this case would mean that the velocity of the current action upon atom deexcitation is greater than C (though principle I may hold).

I am grateful to V.Ritus, V.Frolov, A.Nikishov, M.Markov, V.Petrunkin, V.Skarzhinsky and Yu.Golfand for useful discussions.

APPENDIX. THE FUNCTION $D^\perp$

One can obtain for $D^\perp$ a representation analogous to

$$D(\vec{x}, x_0) = \frac{1}{4\pi R}[\delta(r - x_0) - \delta(r + x_0)], \quad r = |\vec{x}| \qquad (A.1)$$

and in the same manner (see § 17.1 in [4]). One integrates

$$D^\perp_{ij}(\vec{x}, x_0) = \frac{1}{(2\pi)^3}\int d^3k \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) e^{i\vec{k}\vec{x}} \frac{\sin k x_0}{k} \qquad (A.2)$$

over the $\vec{k}$ angles using the formula

$$\frac{1}{4\pi}\int d\Omega_k \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) e^{i\vec{k}\vec{x}} = (\delta_{ij} - n_i n_j)\frac{\sin kr}{kr} + \qquad (A.3)$$

$$+ (\delta_{ij} - 3n_i n_j)\left[\frac{\cos kr}{(kr)^2} - \frac{\sin kr}{(kr)^3}\right], \quad n_i = x_i / r.$$

Further, the integrals of the type

$$\int_0^\infty dk \frac{\sin k x_0}{k^n} \left\{ \begin{matrix} \sin kr \\ \cos kr \end{matrix} \right\} \qquad n = 0, 1, 2$$

can be reduced to $\delta(r \pm x_0)$ for n = 0 and to the functions $\theta(r \pm x_0)$ for n = 1,2, see, e.g., 3.721.1 and 3.784.3 in [8]. ($\theta(\alpha) = 0$ if $\alpha < 0$ and $\theta(\alpha) = 1$ if $\alpha > 0$). For $x_0 > 0$ the result is (the retarded transverse function)

$$D^\perp_{ij}(\vec{x}, x_0) = \frac{1}{4\pi r}(\delta_{ij} - n_i n_j)\delta(r - x_0) - \frac{x_0 \theta(r - x_0)}{4\pi r^3}(\delta_{ij} - 3n_i n_j). \qquad (A.4)$$

The last item does not vanish outside the light cone, i.e. when $x_0 < r$.

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Широков М.И. E4-87-686

Запаздывающее высвечивание атома, индуцированное внешним током

Исследуется изменение временного закона высвечивания атома, индуцированное внешним током. Показано, что запаздывающее действие тока на атом является качественным следствием не только принципа локальной коммутативности, но и принципа отбрасывания опережающих решений. Предложен способ вычисления этого действия, использующий картину, промежуточную между картиной взаимодействия и гейзенберговской. Он оказывается более простым, чем стандартная теория возмущений в картине взаимодействия. Обсуждается механизм влияния тока на возбужденный атом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Shirokov M.I. E4-87-686

Retarded Deexcitation of the Atom Induced by External Current

The change in the time law of the atom deexcitation induced by the remote external current is investigated. It is shown that the retarded current action on the atom is a qualitative consequence of both the principle of local commutativity and the principle of discarding the advanced solutions. A method of the calculation of the action is suggested which uses a picture intermediate between the interaction and the Heisenberg pictures. It turns out to be simpler than standard perturbation theory in the interaction picture. The mechanism of the current action on the excited atom is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1987