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ON THE PION-NUCLEUS DYNAMICS AT LOW ENERGIES

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Low-energy pion-nucleus scattering (below 70 MeV) has intensively been studied in recent years both experimentally and theoretically [1,2]. In this paper we present the results of the analysis of the pion elastic scattering on p-shell nuclei such as ^{12}C and ^{16}O in the framework of unitary scattering theory (UST) based on the Kirzhnits method of evolution in coupling constant [2-4]. It is shown that the differential cross sections at energies around 50 MeV arises as a result of a strong interference between the absorption channel and the pure potential ones. It makes the scattering data to be very sensitive to the mechanism of the pion-nucleus interaction.

Experiment. At present there is an almost complete set of data for the pion scattering on ^{12}C and ^{16}O. at energies below 50 MeV. The measured differential cross sections (mainly with π^+) cover the energy range 14-50 MeV [5] (see also the review [2]). Recently [6] accurate measurements of the 1s and 2p-energy-level shifts and widths in the π^--atoms of ^{12}C and ^{16}O have been performed. These data provide the information on the pion-nucleus interaction at the threshold.

We restrict our consideration by scattering data below 50 MeV because in this energy range the parameters of the absorption correction defined from the pion-atomic data are approximately constant. This has been established in [4] for the π-^4He scattering and the calculations presented here confirm that conclusion.

Theory. Referring for the details to refs. [2,4], we outline only the basic elements of the UST-approach. This approach has certain advantages over the optical model. Firstly, the consistency theory with the unitarity condition provides a correct separation of the potential effects from the nonpotential (real absorption) ones. Secondly, the equations are formulated for the calculation of the pion-nucleus phase shifts. It makes it possible to search for the dynamics of its formation in a straightforward way. Besides, the unitarization method takes into account higher order corrections, thus leading to a rapidly convergent iterative series.

The general formula for the calculation of the π- nucleus phase shifts is of the form

$$\delta_{\pi A} = \delta_{\pi A}^{pot} + \delta_{\pi A}^{abs} . \tag{I}$$

Here $\delta_{\pi A}^{pot}$ is the pure potential part. A multiple scattering -like series for the calculation of $\delta_{\pi A}^{pot}$ in terms of the πN -phase shifts, nuclear form factors and correlation functions has been presented in [2]. At low energies (below $\sim$ 70 MeV) only two terms in the series are sufficient to get a good approximation for $\delta_{\pi A}^{pot}$.

In calculating the second order terms for $\delta_{\pi A}^{pot}$ we utilize the completeness approximation replacing the nuclear excitation energy by a certain mean excitation energy Δ . This is the only adjustable parameter in the theory. Unlike $Re\,\delta_{\pi A}^{pot}$ the imaginary parts $Im\,\delta_{\pi A}^{pot}$ depend rather strongly on Δ at low energies.

The absorption term $\delta_{\pi A}^{abs}$ takes into account all processes involving pure nucleonic intermediate states. Assuming the two-nucleon mechanism for the pion absorption, one can obtain [2,4] the following expression for $\delta_{\pi A}^{abs}$ for nuclei with both zero spin and isospin:

$$\delta_{\pi A,\ell}^{abs}(k) = A\cdot(A-1)\cdot k\cdot(1+\epsilon)\{\hat{\rho}_{\ell}^{2}(k)[\tilde{B}_0(k) + \alpha\cdot k^2\cdot\tilde{C}_0(k)] + \tag{2}$$
$$+\beta\cdot k^2\cdot\tilde{C}_0(k)\cdot((\ell+1)\cdot\hat{\rho}_{\ell+1}^{2}(k) + \ell\cdot\hat{\rho}_{\ell-1}^{2}(k))/(2\ell+1)\},\ \ell=0,1,2,\ldots,$$

where $\epsilon = \omega_{\pi}(k)/M$, ω_{π} is the pion energy, M is the nucleon mass, $\hat{\rho}_{\ell}^{2}$ is the partial wave of the Fourier transform of $\rho^2(r)$ ($\rho(r)$ is the nuclear density), the angular transformation parameters α and β are: $\alpha = -\epsilon(1+\epsilon)^{-2}$ and $\beta = (1+\epsilon)^{-1}$. Here, we neglect the terms containing the factor 1/A.

The complex quantities $\tilde{B}_0$ and $\tilde{C}_0$ are considered to be approximately constant in the low-energy (0 - 50 MeV) region. Their values are derived from the experimental data on the π -nucleus scattering length (a_0) and volume (a_1):

$$a_0^{exp} - a_0^{pot} = \gamma\cdot\hat{\rho}_0^{2}(0)\cdot\tilde{B}_0\ ,\quad \gamma = A\cdot(A-1)\cdot(1+\epsilon)\ , \tag{3}$$
$$a_1^{exp} - a_1^{pot} = \gamma\cdot[\delta\cdot\tilde{B}_0 + \beta\cdot\tilde{C}_0\cdot\hat{\rho}_0^{2}(0)/3]\ ,$$

where $\delta = \lim \hat{\rho}_1^{2}(k)/k^2,\ k\to 0$. The quantities $a_{0,1}^{pot}$ are calculated using expression for δ^{pot}: $a_\ell = \lim \delta_\ell(k)/k^{2\ell+1},\ k\to 0$. The experimental values for a_0^{exp} are extracted from the pion atomic data. It should be noted that the parameters $\tilde{B}_0$ and $\tilde{C}_0$ include effects caused by the potential distortion of the pion wave.

The low energy π -nucleus scattering is highly influenced by the Coulomb interaction. The Coulomb effects are taken into account following the procedure developed in ref.[7]. In refs.[8] the phase shift analysis (PSA) for the π -^{12}C and -^{16}O scattering has been carried out by using this procedure.

Parameters. In the calculations of the π -nucleus phase shifts $\delta_{\pi A}^{pot}$ we use the πN-phase shifts from ref.[9]. The parameters of nuclear form factors and the correlation functions calculated in the shell model are taken from the electron scattering data [10].

The parameters $\tilde{B}_0$ and $\tilde{C}_0$ of $\delta_{\pi A}^{abs}$ are determined from the pion atomic data [6] by using eq. (3). It is found that these parameters are approximately the same both for ^{12}C and ^{16}O:

$$\tilde{B}_0 = (-0.025 + i\,0.025)\mu^{-4}, \quad \tilde{C}_0 = (-0.35 + i\,0.13)\mu^{-6},$$

where μ is the pion mass. Practically the same values for $\tilde{B}_0$ and $\tilde{C}_0$ have been obtained in [2] (see table 5 therein) for the $\pi\,^4He$ scattering. Calculating $\delta^{pot}_{\pi A}$ we do not extract the pole

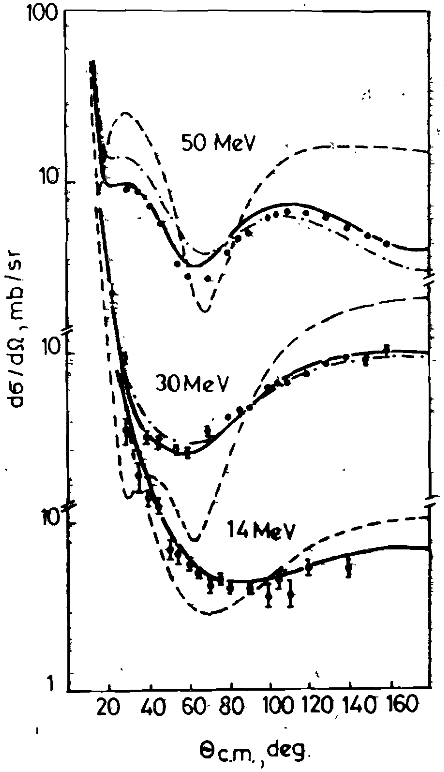

Fig.1. Comparison of the data [5] for $\pi^+\,^{12}C$ elastic scattering at 14, 30 and 50 MeV. The dashed curves represent the result of pure potential calculations ($\delta^{abs}=0$), while the solid lines include the effect of absorption and calculated with Δ = 25 MeV. The dash-dotted lines correspond to the full calculations with Δ = 5 MeV.

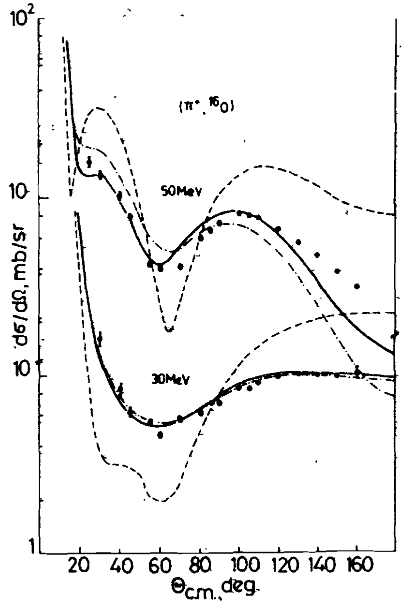

Fig.2. Differential cross sections for $\pi^+ - ^{16}O$ at 30 and 50 MeV. The sence of the curves are the same as in Fig. 1 . The data are from refs. [5].

part of the πN P_{11}-wave. The diagram analysis of the iterative series for (I) shows that the double counting problem does not arise in that case. It should be noted that $\sigma^{pot}_{0,1}$ (and hence $Re\,\tilde{B}_0$ and $Re\,\tilde{C}_0$) are rather sensitive to the πN phase shift data at the threshold. Unlike this, the quantities $Im\,\tilde{B}_0$ and $Im\,\tilde{C}_0$ are determined by eq. (5) through the 1s and 2p-energy level widths almost in the model independent way. In calculating the differential cross sections the absorptive parameters are supposed to be constant.

The only parameter which is not still defined is the nuclear mean excitation energy Δ . Below, we shall try to estimate its value from the scattering data.

Results. The calculated differential cross sections of the π^+ scattering on ^{12}C and ^{16}O along with the scattering data [5] are presented in Figs.1 and 2. The difference between the solid and dash-dotted lines shows the sensitivity to Δ : the dash-dotted lines correspond to Δ = 5 MeV; and the solid ones, to Δ = 25 MeV. The sensitivity to Δ becomes more pronounced at energies around 50 MeV. The best fit of the data is provided by Δ = 25 MeV. The strong effect of the pion absorption channel can be seen by comparing the solid lines with the dashed ones describing the pure potential scattering ($\delta^{abs}_{\pi A} = 0$). At lower energies the sensitivity to Δ becomes weaker because the inelasticity parameters are formed mainly by the absorption channel.

The total cross sections ($\bar{\sigma} = (\sigma^+ + \sigma^-)/2$) at 50 MeV (see table 1) are also very sensitive to Δ . Up to still now there are no experimental data for σ_{tot} . We can estimate its values using an interpolating formula obtained in ref.[11] : $\bar{\sigma}_{tot} \simeq$ 250 mb for ^{12}C and $\bar{\sigma}_{tot} \simeq$ 303 mb for ^{16}O at $T_\pi = 50$ MeV. Comparing these values with those in table I we find out that the appropriate range for Δ = 15-25 MeV.

Table 2 shows a more detailed picture of the $\pi\,^{12}C$-scattering at 50 MeV where the total ($\bar{\sigma}_{tot}$), total elastic ($\bar{\sigma}_{el}$) and reaction ($\bar{\sigma}_R = \bar{\sigma}_{tot} - \bar{\sigma}_{el}$) cross sections are presented for Δ = 25 MeV. The result of the full calculation is given in the first column. The second and the third columns show the results with the switching off $\delta^{abs}_{\pi A}$ or $\delta^{pot}_{\pi A}$, respectively. One can observe that switching on the absorption decreases σ_{el} twice. The total inelastic (σ_{inel}) and the total absorption (σ_{abs}) cross sections are estimated by σ_R in the second and third columns. We observe that these values are comparable. Thus, we conclude that the differential cross sections at energies around 50 MeV arise as a result of the strong interference between the absorptive channel and the potential ones. The same effect is observed also for the $\pi\,^{16}O$-scattering.

Table I. Total cross section $\bar{\sigma}_{tot} = (\sigma^+_{tot} + \sigma^-_{tot})/2$, mb of pion-^{12}C and ^{16}O scattering at 50 MeV as function of

Δ , MeV	5	10	15	20	25	30
^{12}C	288	274	262	250	240	233
^{16}O	352	332	317	303	290	278

Table 2. The $\pi\,^{12}C$ total ($\bar{\sigma}_{tot}$), total elastic ($\bar{\sigma}_{el}$) and reaction ($\bar{\sigma}_R = \bar{\sigma}_{tot} - \bar{\sigma}_{el}$) cross sections at 50 MeV

σ, mb	$\delta^{pot} + \delta^{abs}$	δ^{pot}	δ^{abs}	PSA [8]
$\bar{\sigma}$ tot	250	276	138	276
$\bar{\sigma}$ el	104	215	39	92
$\bar{\sigma}_R$	146	60	99	174

Table 3 Pure hadronic phase shifts (δ_ℓ, in degrees) and inelasticity parameters (η_ℓ) for $\pi\,^{12}C$-elastic scattering at 50 MeV

	δ^{pot}	$\delta^{pot} + \delta^{abs}$	FA [8]	OM [14]
δ_0	-12.34	-17.19	-17.06	-6.6
η_0	0.904	0.82	0.65	0.86
δ_1	24.51	14.04	12.70	15.5
η_1	0.942	0.83	0.86	0.8
δ_2	7.96	6.15	6.50	6.1
η_2	0.987	0.968	0.89	0.95

In table 3 we present the pure hadronic phase shifts (δ_ℓ) and the inelasticity parameters (η_ℓ) for the $\pi-{}^{12}C$ scattering at 50 MeV in comparison with the PSA data from ref. [8] (O.Dumbrais et al.) and the results of the optical model calculations from ref. [14]. There is a clear contradiction between the PSA data and the optical model results. The total cross sections corresponding to the PSA data are shown in table 2.

Thus, it is found that the best description of the scattering data is provided by the mean excitation energy parameter Δ = = 15 - 25 MeV at the pion energy $\sim$ 50 MeV. It is known that in the p-shell nuclei (see ref. [12]) the excitation spectra show a pronounced giant resonance structure concentrated around 20–30 MeV. Therefore, it is natural to suppose that the $\sim$ 20 MeV value for Δ reflects the dominant role of the resonance mechanism in the formation of the pion-nucleus inelasticity parameters. The importance of the virtual nuclear excitations with the isospin $I = 1$ in the description of the low-energy pion-nucleus scattering has been pointed out in ref. [13].

It is known that in the framework of the optical model (see for ex. ref. [14]) the best description of the low-energy data is achieved with optical potentials calculated in terms of the elementary πN- amplitudes the collision energy of which is shifted down by $\sim$ 20 MeV compared to its "two-body" variant. In [14] such a shift is explained in the framework of the three-body model ($\pi + N$ + "core"), thus suggesting the direct mechanism of the pion-nucleus interaction. Taking into account that the applicability of the three-body model to spin and isospin saturated nuclei such as ^{12}C and ^{16}O is questionable, the origin of the 20 MeV scale that reflects the resonance mechanism of pion-nucleus interaction at low energies seems to be more adequate.

In this respect we notice the paper [15] where the inelastic pion ^{12}C scattering at 67.5 MeV has been analysed. It was shown

that below 30 MeV excitation energy the nuclear structure effects are very important. The three-body continuum dominates at higher energies. In [15] the value 75 ± 25 mb has been obtained for the total inelastic cross section. Our estimation of σ_{inel} at 67.5 MeV is 100 mb, which reasonably reproduces this value.

In conclusion, it has been shown that the UST permits a quantitative description of the low-energy scattering data of pions by light nuclei. The strong influence of the absorption channel on the elastic scattering is demonstrated. The parameters of the absorption correction are approximately the same for ^4He, ^{12}C and ^{16}O. The constancy of these parameters defined from the pion atomic data in the energy range 0 — 50 MeV confirms the dominance of the two-nucleon mechanism of the pion absorption.

The differential cross sections at the pion energies around 50 MeV arise as a strong interference effect between the absorption channel and the pure potential ones. The sensitivity of the cross sections to Δ makes it possible to conclude that the resonance mechanism plays an important role in the low-energy pion-nucleus interaction.

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К динамике пион-ядерного взаимодействия
при низких энергиях

Получено количественное описание данных по рассеянию низкоэнергетических пионов (до 50 МэВ) на ^{12}C и ^{16}O в рамках унитарного подхода, основанного на использовании метода эволюции по константе связи. Показано, что для пионов с энергией ~50 МэВ дифференциальное сечение является результатом сильной интерференции канала поглощения и чисто потенциального рассеяния. Обсуждается роль ядерно-структурных эффектов в формировании полного сечения рассеяния.

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On the Pion-Nucleus Dynamics at Low Energies

The low-energy pion scattering (up to 50 MeV) on ^{12}C and ^{16}O is described quantitatively in the framework of the UST-approach. It is shown that at T_{π} ~ 50 MeV the differential cross sections arise as a result of a strong interference between the pure potential and absorption channels. The importance of nuclear structure effects in the pion-nucleus dynamics at low energies is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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