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ON THE QUASIPARTICLE-PHONON NUCLEAR MODEL AT FINITE TEMPERATURE

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1. INTRODUCTION

The investigation of thermal effects in a finite nuclear system has been the subject of many recent publications $\frac{1}{1}$. Among them the influence of temperature-dependent effects has been investigated in connection with the collapse of shell corrections /2/ and with the collapse of pairing correlations $\frac{3}{3}$. The explicit treatment of level densities shows the temperature dependence of the level-density parameters $^{/4/}$. The recent discovery of giant resonances built on states above the yrast line of highly excited nuclei in heavy ion fusion/6/ and deep inelastic reactions/6/ has caused a great interest in studying the properties of collective states in system with a large intrinsic energy and high spin. Thus, several approaches have been advanced in the framework of the finite temperature BCS and RPA to investigate the temperature dependence of such nuclear characteristics as the location of giant multipole resonances $\frac{7}{7}$, the energy weighted sum rules /8/. Also, the finite temperature HFB Cranking equations had been derived and applied '9/ This theory provides a frameinvestigation of nuclear properties above the yrast work for line. In deep inelastic reactions between ions large amounts of the energy and angular momentum are transferred from the relative motion to the internal degrees of freedom. The excitation energy which is not involved in deformation of the dinuclear system is distributed among many degrees of freedom and may be interpreted in terms of nuclear temperature.

It is the aim of this paper to develop a method that takes into account the effects of the excitation energy, statistically distributed among the nucleons, on the nuclear structure. After statistical averaging over canonical ensemble of many nuclei all being in different configurations we define grand canonical potential. The basic system of equations for parameters describing quasiparticle and collective excitations in a self-consistent manner is just the set of necessary conditions for grand canonical potential to be minimum. Although many interesting questions are posed by the interplay between deformation and temperature, we restrict the discussion to the nonrotational part of the spectrum.

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The Hamiltonian under consideration is that of the shell model with pairing and multipole-multipole residual interactions.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATION

Starting with a many-body Hamiltonian of the spherically symmetric nucleus

$$H = H_{shell} + H_P + H_{QQ}, \qquad (1)$$

where

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$$H_{shell} = \sum_{jm} (\epsilon_j - \lambda) a_{jm}^{\dagger} a_{jm}, \qquad (2)$$

$$H_{\mathbf{p}} = -G \mathcal{P}^{+} \mathcal{P}, \qquad (3)$$

$$H_{QQ} = -\frac{1}{2} \sum_{LM} \kappa_L Q_{LM}^+ Q_{LM}, \qquad (4)$$

 $\boldsymbol{\epsilon}_j$ - shell model one-particle energies, λ - chemical potential,

$$\mathcal{P}^{+} = \sum_{jm} (-1)^{j-m} a^{+}_{jm} a^{+}_{j-m}, \qquad (5)$$

$$Q_{LM}^{+} = \sum_{1,2} (1 | q_{LM} | 2) a_{j_1 m_1}^{+} a_{j_2 m_2}, \qquad (6)$$

$$(1 | q_{LM} | 2) = \int \phi_1^* (x) q_{LM}(x) \phi_2(x) dx,$$
 (7)

$$\mathbf{1} = \mathbf{j}_1 \mathbf{m}_1; \qquad \mathbf{2} = \mathbf{j}_2 \mathbf{m}_2, \tag{8}$$

$$q_{LM}(x) \equiv r^2 Y_{LM}(\theta, \phi), \qquad (9)$$

we perform the u-v transformation

$$a_{jm}^{+} = u_{j} a_{jm}^{+} + (-1)^{j-m} a_{j-m},$$
 (10)

$$(u_j^2 + v_j^2 = 1).$$
 (11)

In the quasiparticle basis the Hamiltonian can be written as

$$H_{shell} = \sum_{1} (\epsilon_{1} - \lambda) [2\Omega_{1}v_{1}^{2} + \sqrt{2\Omega_{1}}(u_{1}^{2} - v_{1}^{2})B_{1} + \sqrt{4\Omega_{1}}u_{1}v_{1}(A_{1}^{+} + A_{1})], \qquad (12)$$

$$\begin{split} H_{p} &= -G\{(\sum_{1} 2\Omega_{1}u_{1}v_{1}(1-\frac{2B}{\sqrt{2\Omega}}))^{2} + \sum_{11} 2\Omega_{1}u_{1}v_{1}\sqrt{4\Omega_{1}}(u_{1}^{2}-v_{1}^{2})(A_{1}^{+}+A_{1}^{-}) - \\ &= 2\sum_{11} \sqrt{2\Omega_{1}}\sqrt{4\Omega_{1}}\sqrt{4\Omega_{1}}(u_{1}v_{1}B_{1}(u_{1}^{2}A_{1}^{-}-v_{1}^{2}A_{1}^{+})) - \\ &= 2\sum_{11} \sqrt{4\Omega_{1}}\sqrt{2\Omega_{1}}(u_{1}^{2}A_{1}^{+}-v_{1}^{2}A_{1})u_{1^{\prime}}v_{1^{\prime}}B_{1^{\prime}} + \\ &+ \sum_{11^{\prime}}\sqrt{4\Omega_{1}}\sqrt{4\Omega_{1^{\prime}}}(u_{1}^{2}A_{1}^{+}-v_{1}^{2}A_{1})(u_{1^{\prime}}^{2}A_{1^{\prime}}-v_{1}^{2}A_{1^{\prime}}^{+})\}, \end{split}$$

$$\begin{aligned} H_{QQ} &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}q_{L}(11^{\prime})q_{L}(22^{\prime})(A_{LM}^{+}(11^{\prime}) + \\ &+ (-1)^{L-M}A_{L-M}(11^{\prime}))(A_{LM}(22^{\prime}) + (-1)^{L-M}A_{L-M}^{+}(22^{\prime})) - \\ &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}q_{L}(11^{\prime})p_{L}(22^{\prime})(A_{LM}^{+}(11^{\prime}) + (-1)^{L-M}A_{L-M}(11^{\prime}))B_{LM}(22^{\prime}) - \\ &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}p_{L}(11^{\prime})q_{L}(22^{\prime})B_{LM}(11^{\prime})(A_{LM}(22^{\prime}) + (-1)^{L-M}A_{L-M}^{+}(22^{\prime})) - \\ &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}p_{L}(11^{\prime})q_{L}(22^{\prime})B_{LM}(11^{\prime})B_{LM}(22^{\prime}) + (-1)^{L-M}A_{L-M}^{+}(22^{\prime})) - \\ &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}p_{L}(11^{\prime})q_{L}(22^{\prime})B_{LM}(11^{\prime})B_{LM}(22^{\prime}) + (-1)^{L-M}A_{L-M}^{+}(22^{\prime})) - \\ &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}p_{L}(11^{\prime})p_{L}(22^{\prime})B_{LM}(11^{\prime})B_{LM}(22^{\prime}) + (-1)^{L-M}A_{L-M}(22^{\prime})) - \\ &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}p_{L}(11^{\prime})p_{L}(22^{\prime})B_{LM}(11^{\prime})B_{LM}(22^{\prime}) + (-1)^{L-M}A_{L-M}(22^{\prime}) + (-1)^{L-M}A_{L-M}(22^{\prime})) - \\ &= -\frac{1}{2}\sum_{LM|11^{\prime}|22^{\prime}}\kappa_{L}p_{L}(11^{\prime})p_{L}(22^{\prime})B_{LM}(11^{\prime})B_{LM}(22^{\prime}) + (-1)^{L-M}A_{L-M}(22^{\prime}) +$$

where

$$A_{j}^{+} = \frac{1}{\sqrt{4\Omega_{j}}} \sum_{m} (-1)^{j-m} a_{jm}^{+} a_{j-m}^{+}, \qquad (A_{j} = (A_{j}^{+})^{+}, \qquad (15)$$

$$B_{j} = \frac{1}{\sqrt{2\Omega_{j}}} \sum_{m} a_{jm}^{+} a_{jm}, \qquad 2\Omega_{j} = 2j' + 1, \qquad (16)$$

 $-j \leq m \leq j,$ (17)

$$A_{LM}^{+}(12) = \frac{1}{\sqrt{2}} \sum_{m_{1}m_{2}} (12 \mid LM) a_{j_{1}m_{1}}^{+} a_{j_{2}m_{2}}^{+}, \qquad (18)$$

(12 LM) - Clebsh coefficient, (19)

$$B_{LM}(12) = \sum_{\substack{m_1m_2}} (1-2 | LM) (-1)^{j_2 - m_2} a_{j_1m_1}^{+} a_{j_2m_2}^{+}, \qquad (20)$$

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$$q_{L}(12) = \frac{1}{\sqrt{2}} (u_{1}v_{2} + v_{1}u_{2}) \frac{(1||q_{L}||2)}{\sqrt{2\Omega_{L}}},$$
 (21)

$$\mathbf{p}_{\mathrm{L}}(12) = (\mathbf{u}_{1}\mathbf{u}_{2} - \mathbf{v}_{1}\mathbf{v}_{2}) \frac{(1||\mathbf{q}_{\mathrm{L}}||2)}{\sqrt{2\Omega_{\mathrm{L}}}}, \qquad (22)$$

$$(1 | q_{LM} | 2) = \frac{(1 || q_{L} || 2)}{\sqrt{2\Omega_{L}}} (-1) (1 - 2 | LM), \qquad (23)$$

(Wigner - Eckart theorem).

For system being at a constant temperature and constant chemical potential the equilibrium state minimizes grand canonical potential

$$\Omega = \langle H \rangle - TS - \mu \langle N \rangle, \qquad (24)$$

where $\langle \dots \rangle$ means the statistical averaging

$$\langle O_{\mathbf{p}} \rangle \equiv \operatorname{Tr}(\rho O_{\mathbf{p}})$$
 (25)

and the trace implies a sum over a complete set of states,

$$S = -k < \ln \rho > - \text{ entropy of the system,}$$
(26)

$$\beta = \frac{1}{kT}, \quad T - \text{temperature,}$$
(27)

$$z = \frac{T^{-1} \exp \left[-\Omega \left(|\mathbf{U}|_{\infty} \right) N \right]}{2}$$
(28)

 $\rho = \mathbf{Z} - \exp[-\beta(\mathbf{H} - \lambda \mathbf{N})], \qquad (28)$

$$Z = \operatorname{Tr}(\rho). \tag{29}$$

In general shell-model parameters defining one-particle energies ϵ_j could depend on T. We neglect the effect of the temperature on ϵ_j following the result of $^{/10./}$. For a system of interacting particles we have to exploit the inequality

$$\Omega \leq \Omega_{0}$$
, (30)

where Ω_0 is an approximate value of Ω obtained with an approximate density operator ρ_0 . Looking for the minimum of Ω_0 with respect to all parameters we shall find the equilibrium state approximation. In the following we will separate the Hamiltonian into the free quasiparticle part, free phonon part, and the interaction term describing quasiparticlephonon interaction:

$$H = H_{o}^{f} + H_{o}^{b} + H^{fb}.$$
(31)

With this connections we suppose ρ_0 in the following form

$$\rho_{o} = \rho_{o}^{f} \cdot \rho_{o}^{b} , \qquad (32)$$

where

$$\rho_{0}^{f} = \frac{-\beta H_{0}^{f}}{Z_{0}^{f}}, \qquad \rho_{0}^{b} = \frac{-\beta H_{0}^{b}}{Z_{0}^{b}}. \qquad (33)$$

(It is a crude approximation because in that case e

 $\neq e^{-\beta H_0^f} e^{-\beta H_0^b}$). According to this choice of the density operator we will calculate the trace in the basis constructed as a simple product of n-quasiparticle and m-phonons states:

$$| \rangle \equiv |\mathbf{n}\rangle^{\mathbf{f}} |\mathbf{m}\rangle^{\mathbf{b}} . \tag{34}$$

Now, we construct the boson-like phonon operators

$$Q_{n} = \sum_{1} (\psi_{n1} A_{1}^{+} - \phi_{n1} A_{1}), \qquad (35)$$

$$Q_{\lambda\mu N} = \sum_{11'} \left(\psi_{N\lambda}(11') A^{\dagger}_{\lambda\mu}(11') - (-1)^{\lambda-\mu} \phi_{N\lambda}(11') A_{\lambda-\mu}(11') \right), \quad (36)$$

with the symmetry of the coefficients as follows $\psi_{N\lambda}(11') = -(-1)^{j_1+j_1'+\lambda} \psi_{N\lambda}(1'1),$ $\phi_{N\lambda}(11') = -(-1)^{j_1+j_1'+\lambda} \phi_{N\lambda}(1'1).$ (37)

We require the following commutation relation for Q_n , $Q_{\lambda\mu\,N}$:

$$[\mathbf{Q}_{n}, \mathbf{G}_{n'}^{+}] = \delta_{nn'}, \qquad (38)$$

$$[Q_{\lambda\mu N}, Q^{+}_{\lambda'\mu'N'}] = \delta_{\lambda\lambda'} \delta_{\mu\mu'} \delta_{NN'}$$
(39)

The operators A_j^+ , A_j , $A_{\lambda\mu}^+$ (jj'), $A_{\lambda\mu}$ (jj'), could be expressed linearly through Q_n , Q_n^+ , $Q_{\lambda\mu N}$, $Q_{\lambda\mu N}^+$, only in that case if we replace the operators B_j and B_{LM} (jj') on the right-hand side of the commutation relations

$$[A_{j}, A_{j'}^{\dagger}] = \delta_{jj'} (1 - \frac{2B_{j}}{\sqrt{2\Omega_{i}}}), \qquad (40)$$

$$\begin{bmatrix} \Sigma & q_{L}(22') A_{LM}(22'), \Sigma & \psi_{N\lambda}(11') A_{\lambda\mu}^{\dagger}(11') \end{bmatrix} = \delta_{L\lambda} \delta_{M\mu} \sum_{11'} q_{L}(11') \psi_{N\lambda}(11') = \delta_{L\lambda} \sum_{11'} q_{L}(11') \psi_{N\lambda}(11') = \delta_{L\lambda} \sum_{11'} q_{L}(11') \psi_{N\lambda}(11') = \delta_{L\lambda} \sum_{11'} q_{$$

$$G_{N\lambda L}(11', L') = \sum_{2} \psi_{N\lambda}(12) q_{L}(21') W(11', \lambda L; L'2),$$

and $W(11', \lambda L; L'2)$ represents a Racah coefficient, by the statistical averages in the chosen approximation. Then forward and backward amplitudes fulfil the orthonormality conditions

$$\begin{split} &\sum_{1} (1-2f_{1})(\psi_{n1} \ \psi_{m1} - \phi_{n1} \phi_{m1}) = \delta_{mn} \sum_{n} (1-2f_{1})(\psi_{n1} \psi_{n2} - \phi_{n1} \ \phi_{n2}) = \delta_{12} \\ &\sum_{1} (1-2f_{1})(\psi_{n1} \ \phi_{m1} - \psi_{m1} \phi_{n1}) = 0; \qquad \sum_{n} (1-2f_{1})(\psi_{n1} \ \phi_{n2} - \phi_{n1} \ \psi_{m2}) = 0, \\ &\sum_{11} (1-f_{1} - f_{1} \cdot)(\psi_{N\lambda} (11') \phi_{N'\lambda} (11') - \psi_{N'\lambda} (11') \phi_{N\lambda} (11')) = 0, \\ &\sum_{11'} (1-f_{1} - f_{1'})(\psi_{N\lambda} (11') \psi_{N'\lambda} (11') - \phi_{N\lambda} (11') \phi_{N'\lambda} (11')) = \delta_{NN'}, \quad (42) \\ &\sum_{11'} (1-f_{1} - f_{1'})(\psi_{N\lambda} (11') \psi_{N\lambda} (22') - \phi_{N\lambda} (11') \phi_{N\lambda} (22')) = \frac{1}{2} (\delta_{12} \delta_{1'2'} - (-1)^{j_{1} + j_{2} + \lambda} \delta_{12'} \delta_{1'2}), \\ &\sum_{N} (1-f_{1} - f_{1'})(\psi_{N\lambda} (11') \phi_{N\lambda} (22') - \phi_{N\lambda} (11') \psi_{N\lambda} (22')) = 0, \end{split}$$

where

$$f_{j} = |\langle | \frac{B_{j}}{\sqrt{2} \Omega_{j}} | \rangle = [1 + \exp(\beta E_{j})]^{-1}$$
(43)

means the quasiparticle occupation number. Therefore the transformed Hamiltonian depends on two sets of parameters $\{u, v\}$ and $\{\psi, \phi\}$ and consists the free quasiparticle term, plus

quasiparticle interaction term, the bilinear in bosons part and the fermion-boson interaction terms linear in Q, Q^+ operators. This last terms do not contribute to trace in the boson basis.

The necessary conditions of the statistical equilibrium are the following

$$\frac{\partial \Phi}{\partial u_k} = 0, \qquad \frac{\partial \Phi}{\partial v_k} = 0, \qquad (44)$$

$$\frac{\partial \Phi}{\partial \psi_{nk}} = 0, \qquad \frac{\partial \Phi}{\partial \phi_{nk}} = 0, \qquad (45)$$

$$\frac{\partial \Phi}{\partial \psi_{N\lambda}(kk')} = 0, \qquad \frac{\partial \Phi}{\partial \phi_{N\lambda}(kk')} = 0, \qquad (46)$$

where the functional Φ has the form

$$\Phi = \langle H \rangle - \sum_{1} \mu_{1} ((u_{1}^{2} + v_{1}^{2}) - 1) - \sum_{n1} \omega_{n} [(1 - 2f_{1})(\psi_{n1}^{2} - \phi_{n1}^{2}) - 1] - \sum_{N\lambda_{11}'} \omega_{N\lambda} \{(1 - f_{1} - f_{1'})[(\psi_{N\lambda} (11'))^{2} - (\phi_{N\lambda} (11'))^{2}] - 1\},$$
(47)

and $\mu_{\rm j}\,,\omega_{\rm NL}\,,\,\omega_{\rm n}$ are the Lagrange multipliers. Finally we can write

$$\langle \mathbf{H} \rangle = \sum_{\mathbf{l}} (\epsilon_{1} - \lambda) 2 \Omega_{1} (\mathbf{u}_{1}^{2} \mathbf{f}_{1} + \mathbf{v}_{1}^{2} (\mathbf{1} - \mathbf{f}_{1})) - \mathbf{G} \{ (\sum_{\mathbf{l}} 2 \Omega_{1} (\mathbf{u}_{1} \mathbf{v}_{1}) (\mathbf{1} - 2 \mathbf{f}_{1}))^{2} + \sum_{\mathbf{n}, \mathbf{11}'} \sqrt{4 \Omega_{1}} \sqrt{4 \Omega_{1}'} \frac{1}{4} (\mathbf{1} - 2 \mathbf{f}_{1}) (\mathbf{1} - 2 \mathbf{f}_{1'}) (\mathbf{1} + 2 \mathbf{g}_{n}) [(\mathbf{u}_{1}^{2} - \mathbf{v}_{1}^{2}) (\mathbf{u}_{1'}^{2} - \mathbf{v}_{1'}^{2}) \times \\ \times (\psi_{\mathbf{n}1} + \phi_{\mathbf{n}1}) (\psi_{\mathbf{n}1'} + \phi_{\mathbf{n}1'}) - (\psi_{\mathbf{n}1} - \phi_{\mathbf{n}1}) (\psi_{\mathbf{n}1'} - \phi_{\mathbf{n}1'})] - \\ - \frac{1}{2} \sum_{\mathbf{LN}} \sum_{\mathbf{11}' 2 \mathbf{2}'} \kappa_{\mathbf{L}} 2 \Omega_{\mathbf{L}} \mathbf{q}_{\mathbf{L}} (\mathbf{11}') \mathbf{q}_{\mathbf{L}} (2 \mathbf{2}') (\mathbf{1} - \mathbf{f}_{1} - \mathbf{f}_{1'}) (\mathbf{1} - \mathbf{f}_{2} - \mathbf{f}_{2'}) \times \\ \times (\mathbf{1} + 2 \mathbf{g}_{\mathbf{NL}}) (\psi_{\mathbf{NL}} (\mathbf{11}') + \phi_{\mathbf{NL}} (\mathbf{11}') (\psi_{\mathbf{NL}} (2 \mathbf{2}') + \phi_{\mathbf{NL}} (2 \mathbf{2}')) , \\ \text{where } \mathbf{g}_{\mathbf{n}} \text{ and } \mathbf{g}_{\mathbf{NL}} \text{ are the boson occupation numbers:} \\ \mathbf{g}_{\mathbf{n}(\mathbf{NL})} = \left[\exp \left(\beta \omega_{\mathbf{n}(\mathbf{NL})} - \mathbf{1} \right]^{-1}. \end{cases}$$

The system of equations (44) can be reduced by the usual method to

$$2R_{k}u_{k}v_{k} - (u_{k}^{2} - v_{k}^{2})r_{k} = 0, \qquad (49)$$

where

$$R_{k} = 2\Omega_{k} (\epsilon_{k} - \lambda) + G \sqrt{4\Omega_{k}} \sum_{n,1} \sqrt{4\Omega_{1}} (1 - 2f_{1})(u_{1}^{2} - v_{1}^{2}) \times (\psi_{n1} + \phi_{n1})(\psi_{nk} + \phi_{nk})(1 + 2g_{n}),$$
(50)

$$r_{k} = 2 \Omega_{k} 2G \sum_{1}^{*} 2\Omega_{1} u_{1} v_{1} (1 - 2f_{1}) + 2 \sum_{LN 122'} \kappa_{L} (1 || q_{L} || k) (2 || q_{L} || 2') \times$$

$$\times (u_{2} v_{2'} + v_{2} u_{2'}) (1 - f_{k} - f_{1}) (1 - f_{2} - f_{2'}) (1 + 2g_{NL}) \times$$

$$\times (\psi_{NL} (1k) + \phi_{NL} (1k)) (\psi_{NL} (22') + \phi_{NL} (22')) .$$
(51)

The quantities R_k play the role of the one particle energies. The renormalization, as we see, being due to pairing vibrations. The quantities r_k are the renormalized energy gaps, the renormalization being due to the multipole vibrations. The equations for amplitudes of pairing-vibrations (45) in linear approximation (equivalent to RPA) have the form:

$$\psi_{nj} + \phi_{nj} = G\sqrt{4\Omega_{j}} \frac{X \cdot \omega_{n}}{4E_{j}^{2} - \omega_{n}^{2}} + G\sqrt{4\Omega_{j}} \frac{Z \cdot 2E_{j}}{4E_{j}^{2} - \omega_{n}^{2}},$$
(52)

$$\psi_{nj} - \phi_{nj} = G\sqrt{4\Omega_j} \frac{X \cdot 2E_j}{4E_j^2 - \omega_n^2} + G\sqrt{4\Omega_j} \frac{Z \cdot \omega_n}{4E_j^2 - \omega_n^2},$$

where

$$\begin{split} X &= \sum_{j'} \sqrt{4\Omega_{j'}} (1 - 2f_{j'}) (\psi_{nj'} - \phi_{nj'}) \frac{1}{2} (1 + 2g_n); \\ Z &= \sum_{j'} \sqrt{4\Omega_{j'}} (1 - 2f_{j'}) (u_{j'}^2 - v_{j'}^2) (\psi_{nj'} + \phi_{nj'}) \frac{1}{2} (1 + 2g_n). \end{split}$$

This system of equations is the same as $in^{/11/}$ and would be solved in the analogic way, but iteratively, including the equations (49). The equations for ψ (kk') and ϕ (kk') are

formally identical with the analogous system obtained for example in $^{\prime\,12\prime}$ if we identify

$$\mathbf{E}_{j} = (\epsilon_{j} - \lambda)(\mathbf{u}_{j}^{2} - \mathbf{v}_{j}^{2}) + 4\mathbf{u}_{j}\mathbf{v}_{j} \mathbf{G} \sum_{j'} 2\Omega_{j'} \mathbf{u}_{j'}\mathbf{v}_{j'} (1 - 2f_{j'}), \qquad (53)$$

with the quasi-particle energy. But in our case the {u, v} parameters have to be corrected by interation procedure. From equations (45) and (46) the secular equation follows for the collective excitation energies ω_n and $\omega_{\rm NL}$:

$$\frac{1}{\kappa_{\rm L}} = \sum_{jj'} \frac{(E_{\rm j} + E_{\rm j'})(q_{\rm L}(jj'))^2 (1 - f_{\rm j'} - f_{\rm j})}{(E_{\rm j} + E_{\rm j'})^2 - \omega_{\rm NL}^2} (1 + 2g_{\rm NL}), \quad (54)$$

and

$$\begin{vmatrix} -1 + G\sum_{j} \frac{4\Omega_{j}E_{j}(1-2f_{j})}{4E_{j}^{2}-\omega_{n}^{2}}(1+2g_{n}) & G_{\omega}\sum_{j} \frac{2\Omega_{j}(1-2f_{j})}{4E_{j}^{2}-\omega_{n}^{2}}(1+2g_{n}) \\ \omega_{n}G\sum_{j} \frac{2\Omega_{j}(1-2f_{j})k_{j}}{4E_{j}^{2}-\omega_{n}^{2}}(1+2g_{n}) & -1 + G\sum_{j} \frac{4\Omega_{j}E_{j}(1-2f_{j})k_{j}}{4E_{j}^{2}-\omega_{n}^{2}}(1+2g_{n}) \end{vmatrix} = 0$$
(55)

where $k_j = u_j^2 - v_j^2$. Therefore we can say that even in that simple approximation proposed by us the system of equations for parameters has to be solved simultaneously by the iteration procedure. This should not be additional computing complication because in all existing approaches to the $T \neq 0$ case the quasiparticle occupation numbers f_j (depending on the quasiparticle energies E_k) are treated as given numbers. This means that in each realistic calculations the iteration procedures have to be performed.

3. PERTURBATIVE CORRECTION FROM THE QUASIPARTICLE-PHONON INTERACTION

We can apply the second order perturbation theory approximation to the calculation of the $\Omega^{\,\rm fb}$ (see for example $^{/13/}$). In the general case

$$\Omega_{2} = -\frac{1}{2} e^{\beta \Omega_{0}} \sum_{r,s} \frac{e^{-\beta W_{r}} - e^{-\beta W_{s}}}{W_{s} - W_{r}} (\langle r | H_{int} | s \rangle)^{2}.$$
(56)

We shown how this method works in the simple case with pairing forces only. In this case

$$H_{int} = H^{fb} = \sum_{n,11'} (C_1 \delta_{11'} + GD_{11'})(\psi_{n1} + \phi_{n1})(Q_n + Q_n^+), \quad (57)$$

where

$$C_{j} = \sqrt{4\Omega_{j}} (\epsilon_{j} - \lambda) (1 - 2f_{j}) u_{j} v_{j}, \qquad (58)$$
$$D_{jj'} = 2\Omega_{j} u_{j} v_{j} (1 - 2f_{j}) \sqrt{4\Omega_{j'}} (1 - 2f_{j'}) (u_{j'}^{2} - v_{j'}^{2}).$$

In abbreviabe notation

$$H^{fb} = \sum_{n} \mathcal{K}_{n} \left(\mathbf{Q}_{n}^{+} + \mathbf{Q}_{n} \right) .$$
⁽⁵⁹⁾

The vectors of basis |s| r > 1 ook like

$$|s\rangle = \prod_{n} |s_{n}\rangle, \tag{60}$$

n means type of phonon; ${\rm s}_{\rm n}^{}$, number of phonons in this state. The matrix element

$$<\mathbf{r} \mid \sum_{n} \mathcal{K}_{n} (\mathbf{Q}_{n}^{+} + \mathbf{Q}_{n}) \mid \mathbf{s} > -$$

$$= \sum_{n} \prod_{i} \delta_{\mathbf{s}_{i}} \mathbf{r}_{i} \mathcal{K}_{n} (\sqrt{\mathbf{r}_{n}} \delta_{\mathbf{s}_{n}} \mathbf{r}_{n-1}^{+} + \sqrt{\mathbf{r}_{n+1}} \delta_{\mathbf{s}_{n}} \mathbf{r}_{n+1}^{-}).$$
(61)

where $\prod_{i} = \prod_{i}$ excluding i = n, W_r , W_s are the energies of H_o^b and can be written as

$$\mathbf{W}_{\mathbf{r}} = \sum_{n} \mathbf{r}_{n} \boldsymbol{\omega}_{n}, \quad \mathbf{W}_{s} = \sum_{n} \mathbf{s}_{n} \boldsymbol{\omega}_{n}.$$
 (62)

Finally

$$\Omega_{2}^{fb} = -\frac{1}{2} e^{\beta \Omega_{b}} e^{-\beta \sum_{m} r_{m} \omega_{m}} \sum_{n} \frac{(1-e)^{-\beta \omega_{n}}}{\omega_{n}} K_{n}^{2} (r_{n}+1) +$$
(63)

$$+ \frac{1}{2} e^{\beta \Omega_{o}} e^{-\beta \sum_{m} r_{m} \omega_{m}} \sum_{n} \frac{(1-e^{\beta \omega_{n}})}{\omega_{n}} K_{n}^{2} r_{n}.$$

This correction can play some role at the specific heat calculations $^{\prime\,11\prime}\!.$

4. CONCLUDING REMARKS

In this paper, one more method has been proposed for a consistent consideration of quasiparticle, collective, and thermal degrees of freedom proceeding from the unique variational principle for the thermodynamic potential. The obtained equations include the equations of other papers as particular cases. It is clear that even if the form of the approximate density operator is estimated very roughly, crossing terms do appear in the equations.

One should be very careful with the rigorous statistical approach as one deals with a small number of particles for which only a limited amount of one-particle-spectrum levels is accessible. The proposed methos will be numerically tested by a suitable model.

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Назмитдинов Р.Г., Рыбарска-Навроцка В. Е4-87-579 О квазичастично-фононной модели ядра при конечных температурах

С помощью единого вариационного принципа для термодинамического потенциала получена система уравнений, позволяющая описать квазичастичные и фононные возбуждения атомного ядра. Квазичастично-фононное взаимодействие учтено по теории возмущений.

Работа выполнена в Лаборатории теоретической физики, ОИЯИ.

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Nazmitdinov R.G., Rybarska-Nawrocka W. On the Quasiparticle-Phonon Nuclear Model at Finite Temperature

The system of equations for the parameters describing the quasiparticle and phonon excitations in nucleus is obtained from the unique variational principle for the thermodynamic potential. Quasiparticle-phonon interaction is taken into account perturbatively in the first nonzero approximation.

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