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ALPHA-TYPE CORRELATIONS IN ATOMIC NUCLEI AT FINITE TEMPERATURE

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I. INTRODUCTION

The influence of the temperature on the structure of the atomic nucleus has been taken into account in describing the heavy ion reactions whose exit channels contain at least one nucleus excited above the yrast line [1,2]. Since the time necessary to get the thermal equilibrium seems to be less than the deexcitation time [2], it is expected that above the yrast line the atomic nucleus could be described by mean field models at finite temperature. At the same time, the temperature seems to be the right parameter to distinguish phase transitions specific to the mean field. The competition between the long range correlation leading to the static deformation [3] of the average nuclear field and the thermal fluctuations and their contribution to the structure of the giant resonances [4] have already been studied. The possibility of superfluid - normal fluid phase transition using a model with pairing correlations has also been studied as well [5]. Taking into account the fluctuations, the sharp phase transition predicted by the mean field theory, as for infinite systems, is dramatically changed. In some cases no phase transition is observed.

In the present work the superfluid-normal fluid and superfluid-superfluid phase transitions described by the mean field model with pairing and alpha-type correlations [6-8] have been analysed with respect to the temperature parameter.

The paper is organized as follows. In section II we formulate the model. The discussion of the gap equations and the phase

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diagrams within a schematic single-particle model having equidistant twice degenerated energy levels at zero temperature is given in section III. The temperature dependence of the superfluid enhancement factors of the favoured α-clusterization processes (α-decay and α-transfer reactions) and two-nucleon transfer reactions is presented in Section IV. Section V shows the conclusions.

II. FORMULATION OF THE MODEL

We consider a nucleus formed as a system of nucleons moving in a certain single-particle self-consistent field generated by a deformed diffuse-edge potential.

The free (Gibbs) energy operator of such a system of interacting nucleons is

$$\hat{\mathcal{G}} = \hat{H} - \sum_{i=n,p} \lambda_i \hat{N}_i - K_B T S, \qquad (1)$$

where the Hamiltonian $\hat{H}$ has the form[7,8]

$$\hat{H} = \sum_{i=n,p} \left(\hat{H}_i^{(0)} - G_i \hat{P}_i^+ \hat{P}_i \right) - G_4 \hat{P}_p^+ \hat{P}_n^+ \hat{P}_n \hat{P}_p \qquad (2)$$

in which

$$\hat{H}_i^{(0)} = \sum_{s_i \sigma_i} \varepsilon_{s_i} a^+_{s_i \sigma_i} a_{s_i \sigma_i} \qquad (3)$$

is the single particle part,

$$\hat{P}_i = \sum_{s_i} a_{s_i -} a_{s_i +} \qquad (4)$$

is the pairing operator,

$$\hat{N}_i = \sum_{s_i \sigma_i} a^+_{s_i \sigma_i} a_{s_i \sigma_i} \qquad (5)$$

is the particle number operator and[1]

$$S = 2 \sum_{i=n,p} \sum_{s_i} \ln \left\{ (1-\bar{n}_{s_i})^{\bar{n}_{s_i}-1} \cdot \bar{n}_{s_i}^{-\bar{n}_{s_i}} \right\}$$

$$= 2 \sum_{i=n,p} \sum_{s_i} \left\{ \frac{E_{s_i}}{K_B T} \bar{n}_{s_i} - \ln (1-\bar{n}_{s_i}) \right\} \qquad (6)$$

is the entropy.

Here

$$\bar{n}_{s_i} = \left(1 + \exp \frac{E_{s_i}}{K_B T} \right)^{-1} \qquad (7)$$

is the occupation number of the quasiparticle states, ε_{s_i} and E_{s_i} are the particle and quasiparticle energies, λ_i are the chemical potentials, K_B is Boltzmai's constant and T is the absolute temperature. In the above formulas $n(p)$ stands for neutrons (protons).

The transformation from particle Fermi operators $a^+_{s\sigma}$ $(a_{s\sigma})$ to the quasiparticle Fermi operators $\alpha^+_{s\sigma}$ $(\alpha_{s\sigma})$ is done by the well-known **Bogolubov-Valatin** equation[9]:

$$a^+_{s\sigma} = u_s \alpha^+_{s-\sigma} + \sigma v_s \alpha_{s\sigma}. \qquad (8)$$

By introducing the two-quasiparticle operators[9]

$$A^+(ss') = \frac{1}{\sqrt{2}} \sum_{\sigma} \sigma \alpha^+_{s-\sigma} \alpha^+_{s\sigma} \qquad (9)$$

we have

$$H_i^{(0)} = \sum_{s\sigma} \varepsilon_s (u_s^2 - v_s^2) \alpha^+_{s\sigma} \alpha_{s\sigma} + \sqrt{2} \sum_s \varepsilon_s u_s v_s \left(A(ss) + A^+(ss) \right) + 2 \sum_s \varepsilon_s v_s^2 \qquad (10)$$

$$\hat{P}_i = \sum_s u_s v_s - \sum_{s\sigma} u_s v_s \alpha^+_{s\sigma} \alpha_{s\sigma} + \frac{1}{\sqrt{2}} \sum_s \left(u_s^2 A(ss) - v_s^2 A^+(ss) \right) \qquad (11)$$

and $$\hat{N}_i = \sum_{s\sigma} (u_s^2 - V_s^2)\, \alpha_{s\sigma}^+ \alpha_{s\sigma} + \sqrt{2} \sum_s u_s V_s \left(A(ss) + A^+(ss)\right) + 2\sum_s V_s^2 . \qquad (12)$$

Defining the single quasiparticle Hamiltonian by[1]

$$\tilde{H} = U_0 + \sum_{i=p,n} \sum_{s_i\sigma_i} E_{s_i} \alpha_{s_i\sigma_i}^+ \alpha_{s_i\sigma_i} \qquad (13)$$

and the corresponding thermodynamic average by

$$\langle \hat{O} \rangle_0 = Tr\{\hat{O} \exp(-\frac{\tilde{H}}{K_B T})\} / Tr\{\exp(-\frac{\tilde{H}}{K_B T})\} \qquad (14)$$

we have

$$\langle \sum_{\sigma_i} \alpha_{s_i\sigma_i}^+ \alpha_{s_i\sigma_i} \rangle_0 = 2\bar{n}_{s_i} \qquad (15)$$

$$\langle A(ss') \rangle_0 = 0 \qquad (16)$$

$$P_i \equiv \langle \hat{P}_i \rangle_0 = \sum_{s_i} u_{s_i} V_{s_i} (1 - 2\bar{n}_{s_i}). \qquad (17)$$

The Gibbs energy at finite temperature has the following form:

$$\mathcal{G} \equiv \langle \hat{\mathcal{G}} \rangle_0 = \sum_{i=n,p} \{2\sum_{s_i} (\tilde{\mathcal{E}}_{s_i} - \lambda_i)[\bar{n}_{s_i} + V_{s_i}^2 (1 - 2\bar{n}_{s_i})] - G_i P_i^2 - 2K_B T \sum_{s_i} [\frac{\bar{n}_{s_i}}{K_B T} E_{s_i} - \ln(1 - \bar{n}_{s_i})]\} - G_4 P_p^2 P_n^2 . \qquad (18)$$

As a remark we see that for $G_4 = 0$ the Gibbs energy $\mathcal{G}$ coincides with the expression given elsewere[1,2], if one includes the pairing interaction term in the Hamiltonian of the nucleus. By $\tilde{\mathcal{E}}_{s_i}$ we denote the following expression

$$\tilde{\mathcal{E}}_{s_{p(n)}} = \mathcal{E}_{s_{p(n)}} - \frac{1}{2}(G_{p(n)} + G_4 P_{n(p)}^2) V_{s_{p(n)}}^2 - \frac{1}{4} G_4 V_{s_{p(n)}}^2 \sum_{s_{n(p)}} V_{s_{n(p)}}^2 . \qquad (19)$$

In the following we shall neglect[9] these self-consistent field corrections, i.e. $\tilde{\mathcal{E}}_{s_i} \cong \mathcal{E}_{s_i}$.

Applying the variational principle

$$\delta\mathcal{G} = \sum_{s_i} \left(\frac{\partial \mathcal{G}}{\partial V_{s_i}} \delta V_{s_i} + \frac{\partial \mathcal{G}}{\partial \bar{n}_{s_i}} \delta \bar{n}_{s_i} \right) = 0 \qquad (20)$$

with the condition $u_{s_i}^2 + V_{s_i}^2 = 1$, we obtain

$$\frac{\partial \mathcal{G}}{\partial V_{s_i}} = \frac{2(1 - 2\bar{n}_{s_i})}{u_{s_i}} \{(\mathcal{E}_{s_i} - \lambda_i) 2u_{s_i} V_{s_i} - (u_{s_i}^2 - V_{s_i}^2)\Delta_i\} = 0 \qquad (21)$$

where

$$\Delta_i = P_i (G_i + G_4 P_j^2) \quad ; i = n,p \; ; j = p,n \qquad (22)$$

and

$$\frac{\partial \mathcal{G}}{\partial \bar{n}_{s_i}} = 2\{(\mathcal{E}_{s_i} - \lambda_i)(u_{s_i}^2 - V_{s_i}^2) + 2u_{s_i} V_{s_i} \Delta_i - E_{s_i}\} = 0. \qquad (23)$$

Combining eqs.(21) and (23), we obtain

$$E_{s_i} = \sqrt{(\mathcal{E}_{s_i} - \lambda_i)^2 + \Delta_i^2} \qquad (24)$$

$$\begin{pmatrix} u_{s_i}^2 \\ V_{s_i}^2 \end{pmatrix} = \frac{1}{2}\left(1 \pm \frac{\mathcal{E}_{s_i} - \lambda_i}{E_{s_i}}\right) \qquad (25)$$

for the nontrivial solution, i.e. $\Delta_i \neq 0$. This solution is obtained from eq.(22) and the condition that the number of nucleons should be conserved on average, i.e.,

$$\frac{1}{2} \sum_{s_i} \frac{1 - 2\bar{n}_{s_i}}{E_{s_i}} \{G_i + G_4 P_j^2\} = 1 \qquad (26)$$

$$\sum_{s_i} \{1 - \frac{\mathcal{E}_{s_i} - \lambda_i}{E_{s_i}} (1 - 2\bar{n}_{s_i})\} = N_i . \qquad (27)$$

where N_i is the number of the nucleons of the sort i (n or p) participating in the superfluidity.

III. SOLUTIONS OF THE GAP EQUATIONS AND THE PHASE DIAGRAM

In the following we shall proceed in the frame of a schematic model with equidistant twice degenerated single-particle energy levels at zero temperature

$$\varepsilon_{s_i} = \varepsilon_F(i) + \frac{k}{\rho_i} \tag{28}$$

$$\lambda_i = \varepsilon_F(i) + \frac{\sigma_i}{\rho_i} \tag{29}$$

$$g_i = \rho_i G_i \tag{30}$$

$$x_i = (\rho_i \Delta_i)^2 \tag{31}$$

$$g_4 = \rho G_4 \quad ; \quad \rho = \frac{1}{2}(\rho_p + \rho_n). \tag{32}$$

Here k are integers belonging to the interval $k \in (-20,21)$ reflecting the fact that we included in our treatment at zero temperature 42 single-particle levels. This seems to be the necessary number of levels[9] included in the superfluidity problem. The quantities $\varepsilon_F(i)$ and λ_i are, respectively, the Fermi energies for noninteracting and interacting fermions of the type i, and $i = p, n$.

Let us analyse, as in the previous papers[7,8], the symmetric situation where the protons and neutrons have the same following properties:

$$\begin{aligned} \rho_p &= \rho_n = \rho & \qquad g_p &= g_n = g_2 \\ \lambda_p &= \lambda_n = \lambda & \qquad \sigma_p &= \sigma_n = \tfrac{1}{2} \end{aligned} \tag{33}$$

for which the gap equations have symmetric solutions

$$x_p = x_n = x. \tag{34}$$

Introducing the following notation

$$t = \rho k_B T \tag{35}$$

$$S_{\pm 1}(t,x) = \frac{1}{2}\sum_{k=-20}^{21} \left(E_k(x)\right)^{\pm 1} \left(1 - 2\bar{n}_k(t,x)\right) \tag{36}$$

$$\bar{n}_k(t,x) = \left(1 + \exp\frac{E_k(x)}{t}\right)^{-1} \tag{37}$$

$$E_k(x) = \sqrt{x + \left(k - \tfrac{1}{2}\right)^2} \tag{38}$$

the correlated Gibbs energy (18) has the expression

$$\begin{aligned} E(g_2, g_4, t; x) &\equiv \rho\left(\mathcal{G}(g_2, g_4, t; x) - \mathcal{G}(g_2, g_4, t; 0)\right) = \\ &= 4 S_1(t,0) - 4 S_1(t,x) + 4x\, S_{-1}(t,x) - \\ &\quad - 2 g_2 x\, S_{-1}^2(t,x) - g_4 x^2 S_{-1}^4(t,x) - \\ &\quad - 4t \sum_{k=-20}^{21} \left\{ t^{-1} E_k(x)\, \bar{n}_k(t,x) - \ln\left(1 - \bar{n}_k(t,x)\right)\right\}. \end{aligned} \tag{39}$$

Here g_2, g_4 and t are the model parameters, and x replaces the gap variable.

Denoting now the first two derivatives of expression (39)

$$F(g_2, g_4, t; x) = \frac{d}{dx} E(g_2, g_4, t; x) \tag{40}$$

and

$$H(g_2, g_4, t; x) = \frac{d}{dx} F(g_2, g_4, t; x) \tag{41}$$

we can define the important curves (fig.1) of the phase diagram as in refs.[7,8]

$$F(g_2, g_4, t; x) = 0 \qquad H(g_2, g_4, t; x) = 0 \tag{42}$$

$$F(g_2, g_4, t; 0) = 0 \qquad E(g_2, g_4, t; 0) = 0 \tag{43}$$

$$F(g_2, g_4, t; x) = 0 \qquad E(g_2, g_4, t; x) = 0 \tag{44}$$

and

$$\begin{cases} F(g_2, g_4, t; x_1) = 0 \qquad F(g_2, g_4, t; x_2) = 0 \\ E(g_2, g_4, t; x_1) = E(g_2, g_4, t; x_2) < 0 . \end{cases} \tag{45}$$

The curve (42) separates regions in which the number of solutions of the gap equation

$$F(g_2, g_4, t; x) = 0 \tag{46}$$

differs by two.

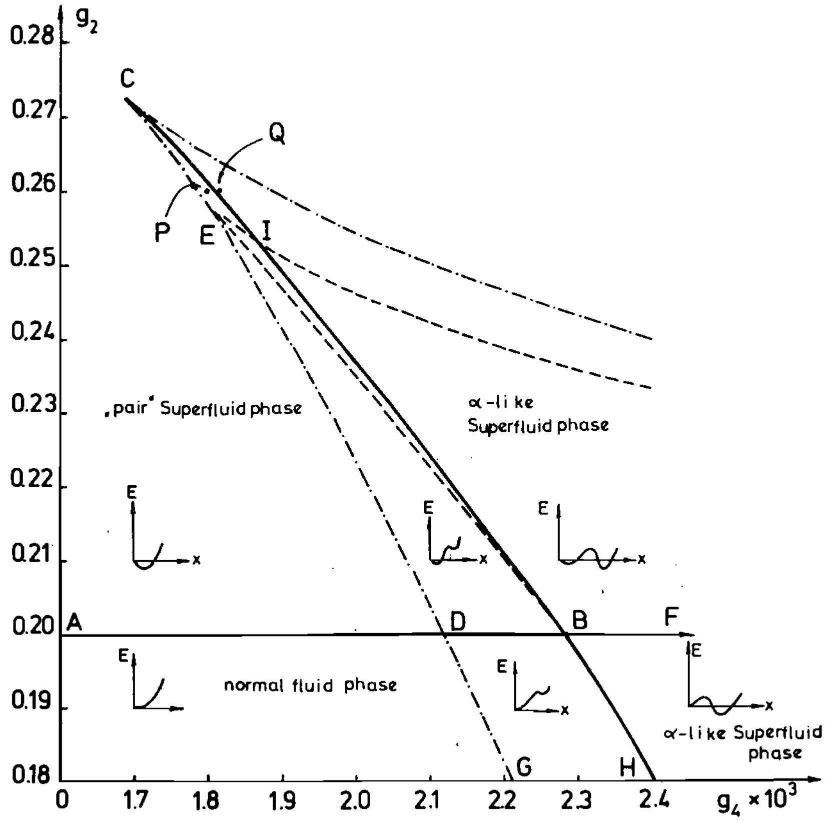

Fig. 1. Phase diagram for t = 0 as discussed in Section III. The curves GDECF, ADBF, HB EIF and CIB are given by eqs. (42), (43), (44) and (45), respectively.

Curve (43) separates regions in which the number of solutions of the gap equation (46) differs by one. In the case g_4 = 0 this curve reduces to a point representing the critical value given by Belyaev's condition /7,8,10/ for zero temperature. The crossing of curve (44) for $x \neq 0$ changes the sign of the correlated Gibbs energy (39) for one solution of the gap equation (46).

The region of the phase diagram delimited by the curves (42) for $x > 0$ and (43) is broken up into two regions by the curve (45). In these regions, the correlated Gibbs energy has two minima. The deepest minimum corresponds to the ground state of the system and the otherone corresponds to the metastable state. These two minima define two superfluid phases of the system. The smallest solution defines the "pair" superfluid phase (the region on the left side of the curve (45) in fig.1) and the largest one defined the α-type superfluid phase (the region on the right side of the curve (45) in fig.1) /7,8/. The jump from one minimum to the other occurs on curve (45). The crossing of this curve corresponds to a first order phase transition, while the crossing of the curve (43) corresponds to a second order phase transition as in the usual pairing case (g_4 = 0).

Further on we have some temperature dependent properties of two hypothetical nuclei defined by the following coordinates:

$$P \; (g_2 = 0.26; \quad g_4 = 0.00180)$$

and

$$Q \; (g_2 = 0.26; \quad g_4 = 0.00183)$$

in the phase diagram from fig.1. Such nuclei could lie in the region of Sn isotopes having the number of neutrons $\approx$ 66. The P -nucleus lies in the region of "pair" superfluid phase, while the Q -nucleus lies in the "α-like" superfluid phase.

Figs. 2 and 3 show the correlated Gibbs energy (39) as a function of x for different temperatures. The nucleus P

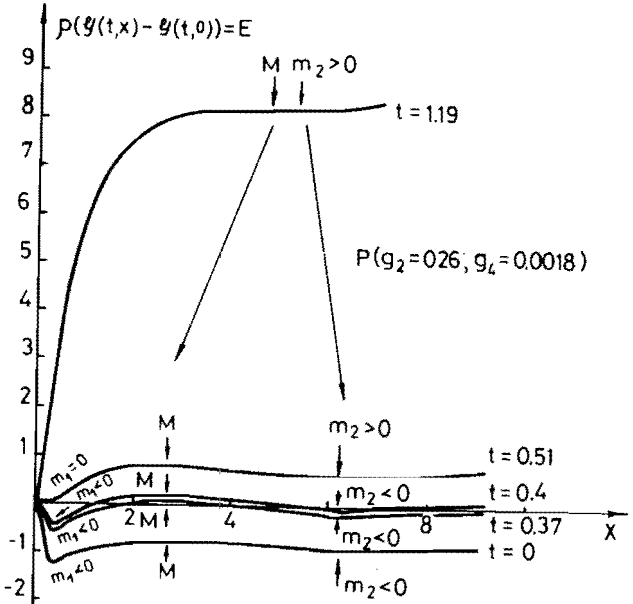

Fig. 2. The correlated Gibbs energy (39) versus $x = (\varsigma\Delta)^2$ for different temperatures in the case of a hypothetical nucleus P (g_2 = 0.26, g_4 = 0.00180).

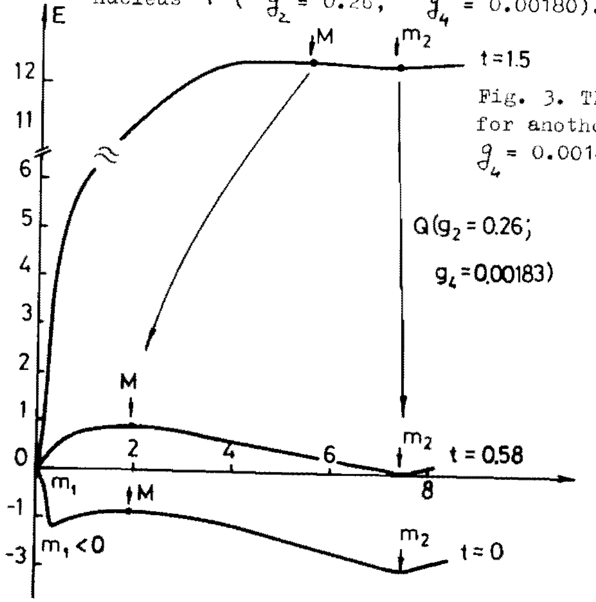

Fig. 3. The same as in fig.2 for another nucleus Q (g_2 = 0.26, g_4 = 0.00183).

suffers a second order phase transition for t = 0.51 for the ground state and its α-like superfluid metastable state disappears at t = 1.19. The nucleus Q suffers a first order phase transition at t = 0.58, the ground state becomes a metastable α-like superfluid state which desappears at t = 1.5.

Figs. 4-7 show the gaps and the ground and metastable state energies as a function of temperature.

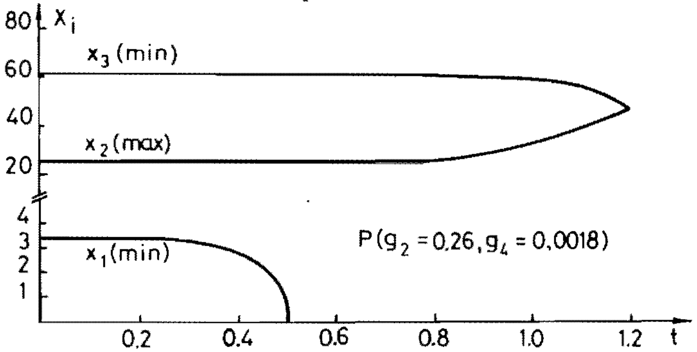

Fig. 4. The solutions of the gap eq.(46) for the P -nucleus versus temperature.

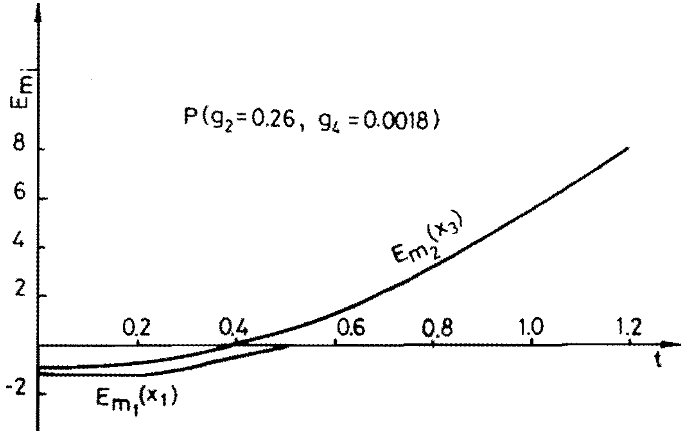

Fig. 5. The values of the correlated Gibbs energies (39) corresponding to the two minima of the P -nucleus versus temperature.

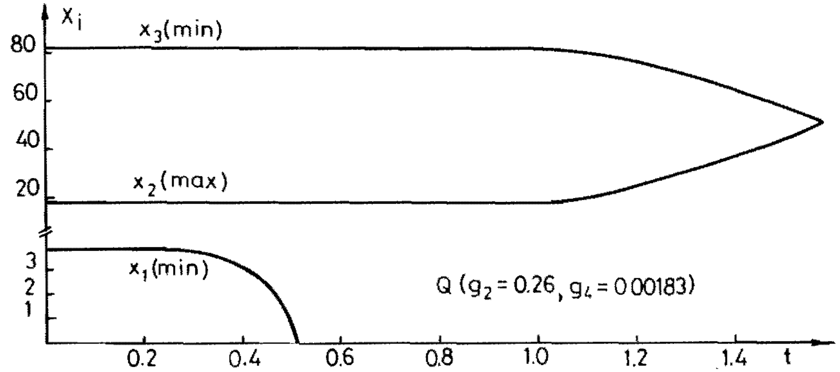

Fig. 6. The same as in fig.4 for the Q-nucleus.

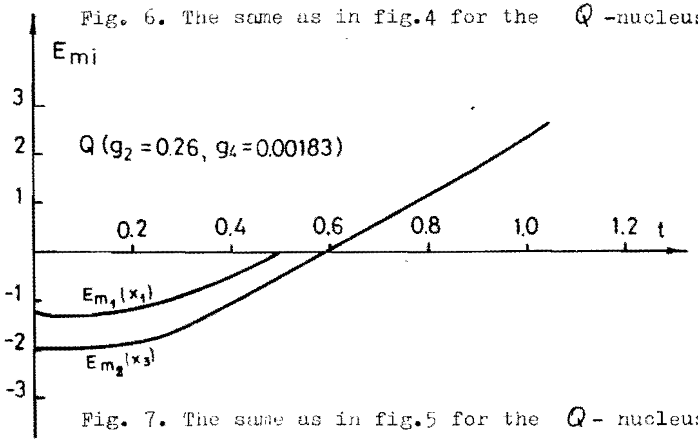

Fig. 7. The same as in fig.5 for the Q- nucleus.

IV. SUPERFLUID ENHANCEMENT FACTOR FOR THE α-CLUSTERIZATION PROBABILITIES AND TWO-NUCLEON TRANSFER REACTION PROBABILITIES

The α-clusterization probabilities (P_α) and the two-nucleon transfer reaction cross sections (σ_2) for the favoured processes at zero temperature can be estimated/7-9/ as follows:

$$P_\alpha = |\langle M_{s.p}^{(\alpha)} \rangle|^2 F_\alpha \tag{47}$$

and

$$\sigma_2 = |\langle M_{s.p}^{(2)} \rangle|^2 F_2 . \tag{48}$$

Here $F_{\alpha(2)}$ are the corresponding superfluid enhancement factors defined by

$$F_\alpha = P_p^2 P_n^2 \tag{49}$$

and

$$F_2 = P_{p(n)}^2 \tag{50}$$

when $P_{p(n)}$ are given by eq.(17).

In the schematic model with equidistant twice degenerated single particle energy levels presented in section III these quantities have the following simple form:

$$F_{\alpha(2)}(t) = R_{\alpha(2)}(t)\ F_{\alpha(2)}(0) . \tag{51}$$

The ratios $R_{\alpha(2)}$ have the form

$$R_\alpha(t) = x^2(t)\ S_{-1}^4(t, x(t)) \{x^2(0)\ S_{-1}^4(0, x(0))\}^{-1} \tag{52}$$

$$R_2(t) = x(t)\ S_{-1}^2(t, x(t)) \{x(0) S_{-1}^2(0, x(0))\}^{-1} \tag{53}$$

and $F_{\alpha(2)}^{(0)}$ are calculated, for instance, in table 2 of ref./7/.

A simple inspection of eqs.(52,53) shows that the superfluid enhancement factors $F_{\alpha(2)}(t)$ are monotonic decreasing to zero functions of temperature.

However, the inclusion of α-type correlations can produce a region of nuclei with their ground states lying in the α-type superfluid phase (figs. 3,7).

For such nuclei the $F_{\alpha(2)}$ factors are still large for temperatures **larger than the critical one for the "pair" phase. This result may explain the large yield of α-particle emission in some heavy ion reactions.**

V. CONCLUSIONS

The finite-temperature pairing plus α-type BCS equations have been derived by minimising the grand potential (Gibbs free energy). These equations have the same form as for zero temperature/7,8/. The difference between the $T \neq 0$ and $T = 0$ cases is the presence of non-zero quasiparticle occupations when $T \neq 0$. The finite temperature pairing plus α-type BCS equations were solved within a schematic model with equidistant twice degenerated single-particle energy levels. One demonstrates that a first and second order "phase transitions" from a superfluid state to a normal state is caused by the increase of the temperature.

For α-type superfluid nuclei the enhancement factors for favoured α-clusterization processes (α-decay and α-transfer reactions) and two-nucleon transfer reactions may be still large for larger temperatures than the critical one for the pairing phase.

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Брандуш И. и др. E4-87-48

Корреляция типа α-частичных в атомных ядрах при конечной температуре

Включением парных и α-частичных взаимодействий в гамильтониан получены уравнения для щелей и химических потенциалов типа БКШ при конечных температурах (КТБКШ) при минимизации большого термодинамического потенциала (свободной энергии Гиббса). Эти уравнения имеют подобную форму как при конечных температурах, так и при нулевой температуре. Единственная разница между $T \neq 0$ и $T = 0$ моделями - это присутствие числа квазичастиц при $T \neq 0$. КТБКШ уравнения были решены в рамках одночастичной модели с 42 протонными и 42 нейтронными дважды вырожденными уровнями. Показано, что: 1) "фазовые переходы I-го и II-го рода из сверхтекучих состояний в нормальное состояние определены путем возрастания температуры; 2) сверхтекучие изомеры и/или их полосы элементарных возбуждений еще существуют при разумных больших температурах и 3) сверхтекучие факторы усиления процессов α-кластеризации и реакций двухнуклонной передачи для некоторых ядер могут иметь большие величины при температурах выше критической парной температуры.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Alpha-Type Correlations in Atomic Nuclei at Finite Temperature

By including the pairing and α-type interactions in the Hamiltonian the finite-temperature BCS (FTBCS) gap and Fermi energies equations have been derived by minimizing the grand thermodynamic potential (the Gibbs free energy). These equations have the same form both for the finite temperature and for the zero temperature. The only difference between the $T \neq 0$ and $T = 0$ cases is the presence of nonzero quasiparticle occupations when $T \neq 0$. The FTBCS equations for the schematic single-particle model with 42 proton and 42 neutron twice degenerated levels have been solved demonstrating that: 1) "first and second order phase transitions" from superfluid states to a normal state are caused by raising the temperature; 2) the superfluid isomers and/or their bands of elementary excitations may survive even at large temperatures and 3) the superfluid enhancement factors for α-clusterization processes and for two-nucleon transfer reactions for some nuclei may have large values for temperatures larger than the pairing critical one.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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