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SEMICLASSICAL TREATMENT<br>OF A REALISTIC NUCLEAR AVOIDED LEVEL-CROSSING PROBLEM

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## 1. INTRODUCTION

Nucleonic molecular orbitals in nucleus-nucleus collisions have been widely used in theoretical descriptions both in quantal models/1/ and semiclassical dynamical treatments $/ 2 /$. An interesting problem in this field is the consideration of avoided level-crossings of adiabatic nucleon levels of equal symmetry. There are first experimental results allowing direct studies of a single avoided level crossing in the nuclear case ${ }^{3-5 /}$. Such pseudo-crossings between nucleonic molecular states generated in a two-center shell modell $/ 6,7 /$ or by linear combinations of nuclear orbitals/8/ are supposed to play an important role for large-amplitude nuclear motion. So, a resonance-1ike behaviour of the inelastic excitation function of the first $1 / 2+$ state of ${ }^{17} \mathrm{O}$ by ${ }^{13} \mathrm{C}$ ions $/ 3-5 /$ has been interpreted first by Abe and Park/9/ , later by Milek and Reif /10/ and by Park et al./11/ in various versions as an evidence for a transition mechanism between mainly two molecular states. Up to now to our knowledge there is no other type of explanation of the experimental data. Recently, Imanishi and von Oertzen/12/ considered the drastic change of angular distributions in the reaction ${ }^{13} \mathrm{C}\left({ }^{12} \mathrm{C},{ }^{12} \mathrm{C}\right){ }^{13} \mathrm{C}$ * ( $3.086 \mathrm{MeV}, 1 / 2^{+}$) in a very small range of the center of mass energy around 8 MeV as an evidence of physical importance of an avoided level crossing in their molecular basis.

Such studies of the influence of only one pseudo-crossing on physical quantities is of basic interest for all applications of the molecular picture in large-amplitude collective nuclear motion. Most of the quantitative considerations $/ 9,10 /$ and qualitative. estimates $/ 2,13 /$ of the nuclear twolevel problem make use of the Landau-Zener formula/ 14 / for the promotion probability of a particle, occupying initially the lower level, to the higher molecular orbit after passing the avoided level-crossing region. The analytical Landau-Zener Model (LZM) was derived a long time ago 14 /. under very restrictive conditions with the assumption of a uniform motion of the nuclei along classical trajectories.

The aim of the present work is a comparison of the simple LZM and more refined analytical models with numerical results for the same problem but with realistic single-particle levels, with realistic couplings between them and with realistic trajectories for the nuclear motion. In section 2 the problem is formulated and in section 3 numerical results are presented for the system ${ }^{17} \mathrm{O}+{ }^{13} \mathrm{C}$. These results are compared to the predictions of the LZM. The resulting discrepancies are explained in section 4 at least qualitatively in terms of more realistic analytical models. Finally, the physical consequences of a new view on the nuclear level-crossing problem are discussed in the conclusions.

## 2. FORMULATION OF THE PROBLEM

Starting from an adiabatic two-center molecular basis $|\phi\rangle^{\mathrm{a}}$ the time-dependent single particle wave function $|\psi(t)\rangle$ for the motion of a valence nucleon - for example the 1d5/2 neutron in ${ }^{17} 0^{-}$- can be expressed in the two-state problem as

$$
\begin{equation*}
|\psi(\mathrm{t})\rangle=\sum_{a=1}^{2} \mathrm{a}_{a \Omega}(\mathrm{t})\left|\phi_{a \Omega}(\mathrm{R}(\mathrm{t}))\right\rangle^{\mathrm{a}} \tag{2.1}
\end{equation*}
$$

Here, $\Omega$ denotes the projection quantum number of the total single, particle angular momentum on the symmetry axis of the system and the additional quantum number a distinguishes between two different levels with the same symmetry. Using the so-called "one trajectory approximation" $/ 17 /$, the time-dependent classical distance $R(t)$ between the two colliding nuclei does not depend on the motion of the valence nucleon. In this approximation the trajectory is fully determined by two inert cores which are accelerated by Coulomb and nuclear forces. From the time-dependent Schrödinger equation a set of two coupled linear equations in time for the expansion coefficients $(\partial / \partial t=R * \partial / \partial R)^{/ 15 /}$ can be derived
$\dot{a}_{1}=-i_{1}(R(t)) \cdot \dot{a}_{1}-\dot{R} T_{12}(R(t)) \cdot a_{2}$.
$\dot{a}_{2}=-\dot{i}_{2}(R(t)) \cdot a_{2}-\dot{R} T_{21}(R(t)) \cdot a_{1}$.

In eqs. (2.2) the commion index $\Omega$ has been suppressed. The arising coupling matrix element $T_{12} \stackrel{a}{=}\left\langle\dot{\phi}_{1}\right| \partial / \partial R\left|\phi_{2}\right\rangle^{\text {a }}=-T_{21}$
contains all non-adiabatic effects of the two-state problem which are related to the change of the structure in the adiabatic wave functions $\mid \phi>^{\mathrm{a}}$. In addition the time-derivative of the collective variable controlls non-adiabatic transitions, too. The molecular adiabatic term $\epsilon_{1}(R)$ denotes the lower level or ground state while $\epsilon_{2}(R)$ characterizes the higher or excited level. Initially ( $t \rightarrow-\infty$ ) the particle occupies the lower level, i.e. $a_{1}(t \rightarrow-\infty)=1$ and $a_{2}(t \rightarrow-\infty)=0$. Then, the physical question is twofold: what is the probability $P_{j}$ for making a jump after passing the pseudo-crossing at $R_{c}$ and what is the final occupation probability $\mathrm{P}_{12}=\left|\mathrm{a}_{2}(\mathrm{t} \rightarrow+\infty)\right|^{2}$ after passing it twice (see fig. 1)?


Fig. 1. The avoided level-crossing problem.
Because in molecular physics one has found for a large number of systems that $\mathrm{T}_{12}$ is a very narrow function (see, e.g., /16/ ), i.e. all non-adiabatic effects are localized in a very small region of the internuclear distance, it is some times convenient to introduce the so-called diabatic basis. The definition of such basis is arbitrary and serves for a minimalization of the coupling in the new basis. Following the physical reason for the strong coupling in the adiabatic basis, which is the change of the nodal structure of the wave functions in the pseudo-crossing region ${ }^{16 /}$, the diabatic
basis can be chosen in such a way that it continues the nodal structure of the adiabatic branches before and after the crossing region.

In the following the LZM will be discussed very briefiy. The first step is a transformation in a new basis $|\phi\rangle^{d}$ which is related to the adiabatic basis according to:

$$
\begin{equation*}
\left|\psi\left(\mathrm{t}^{i}\right)\right\rangle=\mathrm{b}_{1}\left|\phi_{1}\right\rangle^{\mathrm{d}}+\mathrm{b}_{2}\left|\phi_{2}\right\rangle^{\mathrm{d}} \tag{2.3}
\end{equation*}
$$

$\left|\phi_{1}\right\rangle^{a}=\cos \chi / 2 \cdot\left|\phi_{1}\right\rangle^{d}+\sin \chi / 2 \cdot\left|\phi_{2}\right\rangle^{d}$,
$\left|\phi_{2}\right\rangle^{a}=-\sin \chi / 2 \cdot\left|\phi_{1}\right\rangle^{d}+\cos \chi / 2 \cdot\left|\phi_{2}\right\rangle^{d}$.
The angle $\chi$ is determined by a constant interaction $\left|\mathrm{H}_{12}\right|$ and by the slope of the levels assuming straight lines for the energy levels $\epsilon$ in the diabatic basis which crosses each other at $R=R_{c}$.
$\tan \chi=2 \cdot\left|H_{12}\right| / \Delta F \cdot X$
with $\Delta F=\left|\frac{d_{\epsilon_{1}}}{d R}-\frac{d_{\epsilon_{2}}}{d R}\right|$ and $X=R-R_{c}$. Supposing a non accelerated motion of the nuclei with velocity $v_{c}$ the time scale is chosen in such a way that $X$ is equal to $v_{c}{ }^{*} t$. One gets $/ 17 /$ for the unknown coefficients
$\dot{b}_{1}=-i\left|H_{12}\right| \exp \left(-i \int_{0} \Delta F X d t\right) \cdot b_{2}$,
t
$\mathbf{b}_{2}=-\mathrm{i}\left|\mathrm{H}_{12}\right| \exp \left(\mathrm{i} \int \Delta \mathrm{FXdt}\right) \cdot \mathrm{b}_{1}$.

$$
0
$$

The system of equations (2.6) can be solved analytically in terms of parabolic cylindric functions. The next approximation of the LZM is the use of an infinite time-interval for the definition of the one-way jump probability $P_{j}$. While the initial conditions are given physically, the probability of jumping to the higher level after passing the crossing region once i's determined in the LZM in the limit of positive infinite times $P_{j}=\left|b_{2}(t \rightarrow+\infty)\right|^{2}$. For quasielastic or inelastic processes with a finite turning point of the trajectory this definition is not very suited. Finally, the LZM results in the .well-known formula:
$P_{j}{ }^{L Z}=\exp (-2 \pi \cdot G) \quad$ with $\quad G=\left|H_{12}\right|^{2} / \Delta F^{\prime} \cdot v_{c}$.
Disregarding the approximation of an infinite time interval, for the case of two subsequent passings through the :interaction region a two-way formula has been established by. the assumption of a sequence of independent jumps
$P_{12}=P_{j}^{L Z}\left(1-P_{j}^{L Z}\right)+\left(1-P_{j}^{L Z}\right) \cdot P_{j}^{L Z}, \quad P_{12}=2 \cdot P_{j}^{L Z}\left(1-P_{j}^{L Z}\right)$.
This procedure is of course a pure classical one and neglects the phases of the wave functions completly. Both formulae (2.7) and (2.8) have been used in a large extend in nuclear physics (see for example ${ }^{2,9,10,13 /}$ ) although the validity is rather questionable. Taking eqs. (2.7) and (2.8) seriouslÿ the consequences are drastically in heavy-ion collisions.: It was stated $/ 2 /$ that for real nuclear parameters $P_{j}^{L Z}$ it practically always 1 and $P_{12}$ is then equal to zero. Already at $\mathrm{T}_{\mathrm{c}}=\mu / 2^{*} \mathrm{v}_{\mathrm{c}}{ }^{2}=0.05 \mathrm{MeV}$ ( $\mu$ : reduced mass of the system) $\mathrm{P}_{\mathrm{j}}^{\mathrm{LZ}}$ is larger than $90 \%$. Consequently, in heavy-ion collisions a final non vanishing occupation probability could be achieved only under very restricted conditions when such small values of $\mathrm{T}_{\mathrm{c}}$ at the avoided level crossing can be organized/8/ .
3. NUMERTCAL RESULTS IN ${ }^{17} 0+{ }^{13} \mathrm{C}$

The diagram of adiabatic levels for the oxygen + carbon system exhibits a pronounced avoided level-crossing between two $\Omega=1 / 2$ states originating from the asymptotic $1 d 5 / 2$ and $2 \mathrm{~s} 1 / 2$ states in ${ }^{17} \mathrm{O}$. This feature has been observed in the two-center oscillator model $/ 7 /$ as well as in a double, well potential with fixed spherical Woods-Saxon potentials $/ 6,10$ ) used in our previous paper/18/ on non-adiabatic effects on the neutron emission in ${ }^{17} \mathrm{O}+{ }^{16} \mathrm{O}$, too. For the finite depth two-center potential figure 2 demonstrates the two branches with the same symmetry ( $\Omega=1 / 2$ ) in dependence of the distance between the nuclear cores. The corresponding coupling matrix element $T_{12}$ can be expressed mainly analytically (for a detailed discussion see ref. $18 /$ ) and shows a strong peak at the region of avoided level-crossing together with a long range tail.

The important role of the long range interaction (dashed line in fig. 2) on physical quantities as the excitation function of the ${ }^{17} \mathrm{O}^{*} 0,87 \mathrm{MeV}$ level has already been discussed
in ref. $/ 10$ / . Such a tail is not considered in the LZM at all. Because the long range neighbourhoud of the two levels in the case under consideration is surely a not very general feature in large amplitude nuclear motion, the tail in $\mathrm{T}_{12}$ was canceled in the present paper. The ramaining avoided level crossing problem with a sharp peak in $\mathrm{T}_{12}$ should reflect the problem displayed in section 2 quite well.


In order to solve the equations (2.2) for the unknown coefficients one has to fix the time dependence of R. This has been done by calculating the classical trajectory for each relative angular momentum $L$ (impact parameter) separately for a given bombarding energy $\mathrm{E}_{\mathrm{cm}}$, utilizing a modified version of the code TRAJEC $19 /$. The core-core interaction has been parametrized with a Woods-Saxon potential which was determined in ref. $/ 20 /$ by fitting elastic scattering data for ${ }^{12} \mathrm{C}+{ }^{16} \mathrm{O}$ at bombarding energies of $\mathrm{E}_{\mathrm{cm}}$ around $20 \mathrm{MeV}: \mathrm{v}_{\mathrm{o}}=-(6.5 \mathrm{MeV}+$ $\left.+0.4 * \mathrm{E}_{\mathrm{cm}}\right), \mathrm{R}_{\mathrm{o}}=6.55 \mathrm{fm}$ and $\mathrm{a}=0.45 \mathrm{fm}$.

With the underlying nuclear dynamics the crossing region around $\mathrm{R}_{\mathrm{c}}=6.7 \mathrm{fm}$ lies in the potential pocket of the effective core core potential. The barrier is located around 8 fm . When the collective energy $\mathrm{E}_{\mathrm{cm}}$ is just overcoming the barrier the relative radial kinetic energy $T_{c}$ at the crossing point $\mathrm{R}_{\mathrm{c}}$, is already clearly larger than 0.05 MeV which should lead
to a mainly diabatic single particle motion, i.e. $\mathrm{P}_{\mathrm{j}}^{\mathrm{LZ}} \approx 1$ and $\mathrm{P}_{12}^{\mathrm{Z}}=0$. But the numerical results contradict the predictions of the LZM completely, not only quantitativly but also qualitativly. Two statements can be made:
(i) Varying the bombarding energy for a fixed l-value (impact parameter) already at $\mathrm{T}_{\mathrm{c}} \approx 1 \mathrm{MeV}$ the one-way jump probability $\mathrm{P}_{\mathrm{j}}$ reaches a saturation value $\mathrm{P}_{\mathrm{j}}^{\mathrm{o}}$ which is constant in two or three numerical figures. up to very high energies ( $\mathrm{T}_{\mathrm{c}} \approx 30 \mathrm{MeV}$ ). The value for $P_{j}^{\infty}$ is much smaller than 1. For example for a partial wave $L=15 \mathrm{P}_{\mathrm{j}}^{\infty}$ amounts about 0.6 while the LZM already at $T_{c}=0.7 \mathrm{MeV}$ according to eq. (2.7) with $\left|\mathrm{H}_{12}\right|=$ $=0.1 \mathrm{MeV}, \Delta \mathrm{F}=2\left|\mathrm{H}_{12}\right| / \Delta \mathrm{R}, \Delta \mathrm{R}=\mathrm{fm}$ is expected to be $P_{j}{ }^{\text {Z }}=0.95$.
(ii) Instead of a monotonic decrease of the final occupation probability $\mathrm{P}_{12}$ with increasing energy $\mathrm{T}_{\mathrm{c}}$ the numerical results exhibit an increase in $\mathrm{P}_{12}$ with growing energy. Further in contradiction to the two-way formu1a (2.8) $P_{12}$ changes strongly while $P_{j}$ is a constant. In fugure 3 the final jump probability $\mathrm{P}_{12}$ is shown for a trajectory with $L=13$ (upper full curve). If the energy exce-

Fig. 3. Final occu- $P_{1}$ pation probability $\mathrm{P}_{12}$ of the excited state in dependence on the energy of the collective motion for a trajectory with relative angular momentum $L=13$. Upper full curve: numerical calculations, dashed curve: calculations with an added repulaive potential, lower full curve: predictions of the Lan-* dau-Zener model.

eds the potential barrier at $V_{B}=16.2 \mathrm{MeV}$ the avoided le-vel-crossing becomes active in populating the higher level. The lower full curve demonstrates the predictions of the LZM. It follows a monotonic decreasing function $\left(P_{j}^{L Z} \approx 1\right): P_{1}^{Z} 2(1-$ $\left.-\mathrm{P}_{\mathrm{j}}^{\mathrm{LZ}}\right) \sim\left(\mathrm{E}_{\mathrm{cm}}-\mathrm{V}_{\mathrm{B}}\right)^{-1 / 2}$. Both curves start at the potential barrier roughly at $P_{12} \approx 0.1$. Enlarging the energy the realistically calculated values for $\mathrm{P}_{12}$ grow up to about 0.5 highly above the barrier while the LZM gives numbers smaller than 0,005 at these energies. So, the difference in $P_{12}$ - and in related cross, sections - can amount two orders of magnitude!

It was found that the unexpected feature of $\mathrm{P}_{12}$ depends sensitively on the shape of the potential pocket. The dashed line in fig. 2 represents the same calculations but with an attractive potential between the nuclei added ( $\mathrm{X}=\mathrm{R}-6.5 \mathrm{fm}$ ):

$$
\delta \mathrm{V}=\left\{\begin{array}{ccc}
0 & \mathrm{R}>6.5 \mathrm{fm}  \tag{3.1}\\
\mathrm{X}^{2} \cdot & \mathrm{MeV} / \mathrm{fm}^{2} & \text { for } \\
\mathrm{R} \geq 6.5 \mathrm{fm}
\end{array}\right.
$$

This additional potential causes a small change of the turning points of the trajectories. For example at $\mathrm{E}_{\mathrm{cm}}=25 \mathrm{MeV}$ the turning point $\mathrm{R}_{\mathrm{t}}$ increases from 4.34 fm to 4.60 fm . Compared to these small modifications the resulting change in $P_{12}$ is quite large. Being aware of the fact that the coupling matrix is localized in the avoided level-crossing region one can conclude that the final occupation probability is determined by the whole range of the trajectory.

Stu甘ying the saturation property $\mathrm{P}_{\mathrm{j}}^{\infty}$ in a little more detail the coupling elenemt has been fit by a gaussian $\mathrm{T}_{12}=1.12$ $\left.\exp \left(\left(R-R_{c}\right) / \Delta R\right)^{2}\right) 1 / f m$. Leaving the strength of $T_{12}$ unchanged the width of the non-adiabatic region has been varied. With $\mathrm{R}_{\mathrm{c}}=6.7 \mathrm{fm}$ and $\Delta \mathrm{R}=0.34 \mathrm{fm}$ the numerically found coupling of fig. 2 is described quite well by such a gaussian. Repeating the calculations by solving again eqs. (2.2) a strong dependence of $P_{j}^{\infty}$ on $\Delta R$ can be observed. Figure 4 demonstrates that only in a small region around $\Delta R=0.6 \mathrm{fm}$ the diabatic property $P_{j}^{\infty}=1$ is confirmed. One has to state that a variation of $\Delta R$ keeping the strength of $T_{12}$ unchanged is somewhat artifical, because of course the strength can grow for small $\Delta \mathrm{R}$ and can fail for larger $\Delta \mathrm{R}$. Nevertheless the per$\therefore$ formed calculations show that $\mathrm{P}_{\mathrm{j}}^{\infty}$ depends sensitively on the parame'ters of the coupling element $\mathrm{T}_{12}$.

Finally, some consequences on physical cross sections are pointed out. Following refs. $/ 9-11 \%$, one can define an angle integrated cross section for the inelastic excitation of the first $1 / 2^{+}$state in 170 according to:


Fig. 4. Saturation jump probability $\mathrm{P}_{\mathrm{j}}^{\infty}$ depending on the width of the non-adiabatic coupling element chosen as a gaussian with fixed strength.
$\partial_{21}=\pi^{2} \pi \cdot \sum_{L}\left(2 L+1^{\prime}\right) P_{12}(L) / 3$.
The notation in (3.2) stresses the fact that the final occupation probability depends on the impact parameter. Figure 5 shows results extending the sum in (3.2) from $L=13$ to $L=$ $=18$. For energies smaller than 16.2 MeV all considered partial waves are scattered on the barrier. If the energy overcomes the $L$-dependent barrier the partial wave $L$ can reach the potential pocket and the avoided level-crossing mechanism starts to become active in populating the upper level. Due to the increasing final occupation probability - compare to figure 3 - the cross section enlarges slightly. When the next partial wave becomes active the cross section jumps because then a quite large additional term contributes in eq. (3.2).


Fig. 5. Excitation function of the ${ }^{17} \mathrm{O}, 1 / 2^{+}$level. Dashed curve: experimental results of ref. $/ 3 /$, full curve: cross section according to eq. (3.2) extending the sum over $L$ from 13 to 18, dashed lines: schematic illustration of the influence of absorpttion

A more refined calculation should consider also the absorption of the partial waves in the inner region. This could be done by introducting penetration factors of optical models or by inclusion of depletion of probability in eq. (2.2) as it was made in ref. ${ }^{11}$. Such a procedure would appreciably diminish $\mathrm{P}_{12}$ for energies well above the barrier. Schematic results of the inclusion of absorption are demonstrated by the dashed © lines of fig. 5. Comparing this resonance-like feature with those of refs. $/ 9-11 /$ it can be states that the large increasing final occupation probability for large energies of the system found in contradiction to the LZM - is capable to explain the irregular behaviour of the experimental data in the same ex-

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tend as the previous model considerations. We would like to stress again the fact that the LZM would give two orders smaller cross sections for the present model where the pseudocrossing is located inside the barrier.

## 4. EXTENDED ANALYTICAL MODELS

The extension and improvement of the LZM has a long history. For a review see for example ref. ${ }^{21 / \text {. In the following }}$ we would like to discuss only very briefly a small collection of models which can directly be related to. an overcome of the approximations of the LZM and which make the numerical results of the previous section more transparent.
4.1. Quantum Treatment of Sudden Jump Sequences

Already at $1932^{/ 22 /}$ after publication of the LZM Stücke1berg proposed a more realistic formula for the two-way jump probability $\mathbf{P}_{12}$. Not adding the probabilities but the amplitudes of the two possible transition sequences between the basis wave functions one can easily formulate a more general two-way formula ${ }^{17 /}$.

$$
\begin{equation*}
P_{12}=4 \cdot P_{j}\left(1-P_{j}\right) \sin ^{2} \phi . \tag{4.1}
\end{equation*}
$$

The Stücke1berg phase $\phi$ describes a quantal interference phenomenon and can be devided in two pieces $\phi_{0}$ and $\phi_{1}$. The phase $\phi_{1}$ is related to a change in the wave functions during the sudden transition process in the crossing region. This phase cannot be derived by any general constraint (unitarity, symmetry, etc. $)^{(23 /}$. For the LZM it was found ${ }^{/ 24 /}$ that $\phi_{1}$ goes to zero for small velocities and to $\pi / 4$ for $\mathrm{v}_{\mathrm{c}} \rightarrow \infty$. A1so for more realistic models it is restricted between 0 and $\pi / 4 / 24 /$. The other phase $\phi_{0}$ can be understood as an adiabatic phase lag between the two possible transition sequences. In the one trajectory approximation $/ 17 / \phi_{0}$ is given by

$$
\phi_{\mathrm{o}}=\sqrt{2 \mu} \int_{R_{\mathrm{t}}}^{\mathrm{R}_{\mathrm{c}}}\left[\left(\mathrm{E}_{\mathrm{cm}}-\mathrm{V}_{\mathrm{L}}(\mathrm{R})-\epsilon_{1}(\mathrm{R})\right)^{1 / 2}-\left(\mathrm{E}_{\mathrm{cm}}-\mathrm{V}_{\mathrm{L}}(\mathrm{R})-\epsilon_{2}(\mathrm{R})\right)^{1 / 2}\right] \mathrm{dR}(4.2)
$$

Approximating roughly the potential pocket as an oscillator and the energy levels as straight lines with an equal slope $\mathrm{F}=\left|\mathrm{a}_{\epsilon} / \mathrm{dR}\right|$ the phase lag $\phi_{0}$ can be expanded in powers of $\delta=$ $=1-R_{t} / R_{c}$. A straightforward calculation leads' to
$\phi_{o}=\sqrt{2 \mu} \frac{\mathrm{~F}}{4}\left(\mathrm{E}_{\mathrm{cm}}-\mathrm{V}_{\mathrm{B}}+\left|\epsilon_{\mathrm{o}}\right|\right)^{-1 / 2} \cdot\left(\mathrm{R}_{\mathrm{c}}-\mathrm{R}_{\mathrm{t}}\right)^{2}+0\left(\delta^{4}\right)$,
with $\epsilon_{o}=\left(\epsilon_{1}\left(R_{c}\right)+\epsilon_{2}\left(R_{c}\right)\right) / 2$. Using expression (4.3) with realistic parameters for the system ${ }^{17} \mathrm{O}+{ }^{13} \mathrm{C}$ the sensivity of the numerical results on the position of the turning point and also the correct tendencies making the potential more attractive (eq. (3.1)) can be well reproduced. So, we conclude that formula (4-1) is much more valid than eq. (2.8) for a selected level-crossing and the observed behaviour of $\mathrm{P}_{12}$ can mainly be explained as an interference effect. It should be in general not only one maximum in $\mathrm{P}_{12}$ at low radial kinetic energies observable but also a second or even more péaks could exist.

### 4.2. Extension of the Uniform Motion Approximation

If the turning point is located near the crossing point the condition $\mathrm{v}_{\mathrm{c}}=$ const is clearly unreasonable. But unfortunately closed simple analytical expressions could not be achieved in the past $/ 17 /$ even for a uniform accelerated motion $R-R_{t} \sim t^{2}$. Under certain conditions, when a perturbation method in the coupling element $\left|\mathrm{H}_{12}\right|$ in the diabatic basis is valid, the final occupation probability can be expressed by Airy-functions depending on two dimensionless parameters $\omega$ and $\beta$. The parameter $\omega$ is proportional to the radial kinetic energy at the crossing point and $\beta$ to $\left|\mathrm{H}_{12}\right|^{3 / 2}$. For large $\omega$ the analytical result can be approximated (eq. (19.17a) of ref. $17 /$ ) by
$\mathrm{P}_{12}=\pi \cdot \beta \cdot \omega^{1 / 2} \sin ^{2}\left(2 / 3 \cdot \beta \cdot \omega^{3 / 2}+\pi / 4\right)$.
Consequently, also the extension of the uniform motion approximation leads to an oscillating behaviour for $\mathrm{P}_{12}$. A similar result was given by Bates $/ 259$ who described after a first peak in $P_{12}$ in the high energy region a second peak as an effect of the neighbourhoud between crossing point and turning point.

### 4.3. More Refined Shapes of the Adiabatic Terms

- The , so-called exponential model $/ 1^{7 /}$ was derived for the one way jump probability for two-state problems where the straight line approximation of the interacting terms is out of any relevance. All other shortcomings of the LZM are in' corporated, too. It can be formulated in a chosen diabatic y
as well as in the well defined adiabatic basis. Preferring the adiabatic basis the difference between the two levels is defined by $/ 17 /$
$\epsilon_{2}(R)-\epsilon_{1}(R)=\Delta \epsilon(1-2 \cos \theta \exp (-x)+\exp (-2 x))^{1 / 2}$
with $\mathrm{x}=\alpha \cdot\left(\mathrm{R}-\mathrm{R}_{\mathrm{p}}\right)$.
The parameter $\Delta \epsilon$ describes the constant difference for $R \rightarrow \infty$; $\alpha$ is a decay constant for the level distance and $R_{p}$ is related to the distance $R_{c}$ of maximal approach by $R_{p}=R_{c}+$ $+1 / \alpha \ln (\cos (\theta))$. Introducing a coupling element
$\mathrm{T}_{12}=\alpha / 4 \cdot \sin \theta /\left(\cos \mathrm{x}-\cos \theta^{\prime}\right)$
the model can be solved analytically. The angle $\theta$ is related to the width of the coupling $\Delta R$ according to $\sin \theta / 2 \simeq \Delta R^{*} \cdot \alpha$ (eq. (25.8) of $/ 17 /$ ). The one way jump probability is fully given by two parameters $\xi=\Delta_{\epsilon} / a \cdot \mathrm{v}_{\mathrm{c}}$ and $\gamma=\sin ^{2}(\theta / 2 \mathrm{i})$
$P_{j}=\exp (-\pi \cdot \xi \cdot \gamma) \sin (\pi \xi(1-\gamma)) / \sin (\pi \xi)$.
Considering the limit $\mathrm{v}_{\mathrm{c}} \rightarrow \infty$, i".e. $\xi \rightarrow 0$ the exponential model predicts a saturation value of
$P_{j}^{\infty}=1-\sin ^{2} \theta / 2$.
For example, the not very unreasonable parameter combination $\Delta R=0.5 \mathrm{fm}$ and $1 / a=1 \mathrm{fm}$ gives $P_{j}^{\infty}=0.75$. Only for very narrow couplings when the finite extension of the coupling is negligible and $\theta$ goes to zero the LZM limit or diabatic limit could be achieved.


### 4.4. Finite Time Interval

The limit $t \rightarrow+\infty$ for the definition of the one way jump probability is in most of the interesting cases not physically justified. Either a turning point exists, or a further interaction at finite times could occur., A recent work ${ }^{26}$ has been devoted to the level-crossing problem at finite times with realistic nuclear parameters but leaves all other approximations like straight lines, single particle terms and a constant velocity of the collective variable unchanged. In such a model $P_{j}$ at a certain distance far from the crossing region depends on two parameters as in the exponential model.

It was found that for a given value of the collective variable the jump probability $P_{j}$ exhibits a saturation feature.
So, at a distance of 1 fm from the crossing point with the parameters $\left|\mathrm{H}_{12}\right|=0.25 \mathrm{MeV}$ and $\Delta F=1 \mathrm{MeV} / \mathrm{fm}$ the one way jump probability cannot exceed $P_{j}^{\infty}=0.8$ (compare with the table 1 of ref. $/ 26 /$ ).

## 5. CONCLUSIONS

As was shown in the previous section every improvement of the oversimplified LZM leads to analytical expressions which in principal confirm the presented numerical results. It does not mean that the LZM is not suited for any application. But if only one avoided level-crossing is considered, quantal interference between transitions at the incident and excident stage of the reaction requires a more general expression for the two way formula. The observed saturation property for the one-way jump probability is related to the finite size of the coupling region in the nuclear case. As a conlcusion, in nuclear physics the picture of pure adiabatic motion interrupted by sudden "sure" hoppings addapted from molecular physics is in general not valid. Any avoided level-crossing problem should be examined carefully.

The avoided level-crossing problem in heavy-ion collisions should be viewed in a new manner. It is not in every case true that for high radial collective energies at the crossing point such molecular transition mechanism is ineffective in populating the final excited state of a reaction. So, for quasi-elastic collisions a two-level excitation mechanism could be observable even in situations when the crossing region is located inside the barrier. The present interpretation of experimental data in ${ }^{12} \mathrm{C}+{ }^{13} \mathrm{C}$ as an evidence for the nuclear Landau-Zener effect $/ 12 /$ could serve as an example. According to fig. 30 of ref. ${ }^{/ 28 /}$ the responsible avoided $1 e-$ vel crossing is located behind the barrier of the effective - adiabatic potential. For bombarding energies slightly above the barrier the radial kinetic energy $T_{c}$ amounts already 10 MeV ! In a simple diabatic picture it cannot be understood that under these conditions the avoided level crossing could effect any physical quantities in the final stage of the reaction.

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## Милек 6., Райф P.

E4-87-475 Полуклассическое описание реалистической пдерной задачи квазипересечения мешду деушн термами

Дли реакции тяшелах нонов полуклассическал ядернап задача каазилересечения мешду пэуми тернаии исспедуется рамках двухцентровой оболочечно

 Резупьтатм расчета демонстрируются ала вероитности прымка и вероптности перекода для реакиии ${ }^{18} \mathrm{C}\left({ }^{17} \mathrm{O}^{1}{ }^{17} \mathrm{O}^{*}\right)^{18} \mathrm{C}$, $1 \mathrm{~d} 8 / 2 \rightarrow 2 \mathrm{~s} 1 / 2(\Omega=1 / 2)$. Резупьтаты не соответствуот предсказаниям формупы Ландау - Змнера; подтвермдают некоторые своАства расмиренных анапитических подходов к задаче квазиперет сечения мешду двуми уровнлми.

Работа виполнена в Лаборатории теоретической физики омяи.

Сообиения О

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Mllek B., Reif R.
Semiclassical Treatment of a Reallistic Nuclear
Avolded Level-Crossing Problem
For a heavy-ion collision a nuclear two-state problem referring to an avolded level crossing of adlabatic single-particle states of the same symmetry is treated semiclassically within a two-center shell model with finlte-depth mean potentlals. Using a realistic trajectory for the relative mot Ion and a realistic time-dependent coupling strength, numerical results for the one-way and two-way Jump probabilities are presented for the tran-
 In agreement with the predictions of the Landau-Zener formula, but confirm some propertles of extended analytical treatments of the avolded levelcrossing problem.

The Investigation has been performed at the Laboratory of Theoretical Physics, JiNR.


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