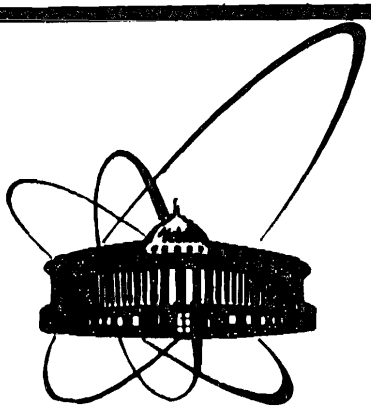

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DEUTERON POLARIZABILITY AND S-WAVE π^+d-SCATTERING AT ENERGIES BELOW 1 keV

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The investigation of scattering of a slow charged particle by a weak-bounded charged complex is of a special interest. The matter is that the nonpointlike pair charge distribution and the change of this distribution in the Coulomb field of the projectile cause an additional effective interaction decreasing as r^{-p} when the distance r between the projectile and complex c.m. increases. The power of decreasing p and form of this interaction essentially depend on the geometry of the complex and on spectrum of its excited states[1]. In a low-energy limit ($E = \hbar^2 k^2 / 2\mu \to 0$) elastic scattering of a particle by a charged complex with the same sign of charge is generally determined by a long-range behavior of the effective potential V_{eff}. Therefore, the peculiarities of such scattering may be explored in the framework of the effectively two-body (particle + complex) Shrödinger equation. A detailed analysis of phase-shift asymptotics as $E \to 0$ for charged particle scattering by a superposition of the Coulomb and polarization $V_p(r) \sim r^{-4}$, $r \to \infty$, potentials was first performed in paper[2]. An elegant method of the calculation of the leading phase-shift asymptotics for a slow particle scattering by the Coulomb field containing the power corrections $r^{-\beta}$ with an arbitrary power $\beta > 1$, has recently been built in paper[3]. In works[4], for a pd -system it was first demonstrated that the polarization interaction must be taken into account in the problems of ultralow-energy (~1 keV) nuclear physics. In these papers it was shown that the S -wave phase shift for pd -scattering at $E \lesssim 10$ keV is defined by the polarization instead of the nuclear interaction. As a result, the usual definition of the pd -scattering length as a limit as $E \to 0$ of the nuclear-Coulomb effective range function becomes meaningless. Modification of the effective range theory in the case of nuclear-Coulomb field including the polarization interaction has been considered in a recent paper[6]. A rich bibliography on the theory of long-range potential scattering is presented in reviews[7].

In the present work, the influence of the polarization interaction on the S -wave $\pi^+ d$ -scattering at energies below 1 keV is studied within the framework of the two-body ($\pi^+ d$ [illegible] contents of the work is as foll[illegible]

range of the polarization potential, giving a main contribution to the $\pi^+ d$ S -wave phase shift additional to the Coulomb one, estimation of the energy intervals when this phase-shift is formed by the pion-nuclear short-range potential or the polarization one, and estimation of a lower limit of distances r when the $\pi^+ + d$ -wave function may be approximated with a required accuracy by its leading long-range asymptotical term.

So, we shall explore the influence of the deuteron polarizability on the S -wave $\pi^+ d$ -scattering within the framework of effectively-two-body Shrödinger equation. In the $\pi^+ d$ centre-of-mass system it has the following form

$$\{ \partial_r^2 + k^2 - V_{eff}(r) \} u(r,k) = 0. \qquad (1)$$

The effective potential is the sum $V_{eff} = V_c + U$ of two terms. One term is the repulsive Coulomb potential $V_c(r) = n/r$, where $n = 2\mu (e/\hbar)^2 = 0.0096$ fm and the other is the potential $U = V_S + V_P$. The pion-nuclear potential V_S decreases exponentially with increasing distance and has a finite range of action r_S which is of an order of the deuteron size, ~ 4 fm. The potential V_P additional to V_S decreases as $O(r^{-4})$ with increasing distance and is due to the electric polarizability of the deuteron. For the polarization potential we limit ourselves to the representation

$$V_P(r) = -\frac{\alpha n}{2r^4} \theta(r - r_P), \qquad (2)$$

where $\alpha = 0.7$ fm^3[8] is the constant of electric polarizability of the deuteron. We assume that the parameter r_P satisfies the relations $r_S \leq r_P \sim R$, where $R = 1/n = 104$ fm is the Bohr radius of the $\pi^+ d$ -system. Further, varying the parameter r_P in wide limits, we show that our results weakly depend on a particular value of r_P. The asymptotic

$$u(r,k) \underset{r \to \infty}{\to} \sin(\theta_c(\rho,\eta) + \delta_c(k) + \delta(k)) \qquad (3)$$

of the regular solution of equation (1) contains the Coulomb phase shift $\delta_c(k)$ and phase shift $\delta(k)$ due to the additional potential U. Here we denote

$$\rho = kr, \quad \eta = n/2k, \quad \theta_c(\rho,\eta) = \rho - \eta \ln 2\rho.$$

In the framework of the variable phase approach the phase shift $\delta(\kappa)$ is calculated as a limit for $\rho \to \infty$ of the phase function $\delta(\rho,\kappa)$. The latter is a solution of the following problem

$$\partial_\rho \delta(\rho,\kappa) = -\kappa^{-2} U(\rho/\kappa)\{F_0(\rho,\eta)\cos\delta(\rho,\kappa) + G_0(\rho,\eta)\sin\delta(\rho,\kappa)\}^2, \quad (4)$$
$$\delta(0,\kappa) = 0,$$

where F_0 and G_0 are, respectively, regular and irregular Coulomb functions. By the symbols $\delta_s(\rho,\kappa)$ and $\delta_\rho(\rho,\kappa,r_\rho)$ we denote, respectively solutions of equation (4) at $\alpha = 0$, $U = V_s$ and at $V_s \equiv 0$, $U = V_\rho$. Let us study the phase function $\delta_\rho(\rho,\kappa,r_\rho)$, i.e. solutions of equation (4) with the boundary condition $\delta_\rho(\rho_\rho,\kappa,r_\rho)=0$, $\rho_\rho = \kappa r_\rho$ and potential $U = V_\rho$. For a hight accuracy calculation of the Coulomb functions we have used a method, described in paper /10/. Setting $r_\rho = \gamma R$ and choosing the value of parameter $\gamma = 0.1, 1.0, 10$ we have calculated the phase function $\delta_\rho(\rho,\kappa,r_\rho)$ for $E = 0.01$, 0.1, 1 keV. The fourth significant digits of these functions do not change with increasing ρ in the region $\rho \geq 15\rho_c$, where $\rho_c = \kappa r_c$ and $r_c = n/\kappa^2 = e^2/E$ is the Coulomb classical turning point corresponding to a given energy E. The relative accuracy $\sim 10^{-4}$ is quite enough for our aims, therfore, everywhere we use the values of phase functions $\delta_\rho(15\rho_c,\kappa,r_\rho)$ as the values of the corresponding phase shifts $\delta_\rho(\kappa,r_\rho)$. Table 1 demonstrates a weak dependence of these phase shifts on the parameter r_ρ at sufficiently low energies. The graphs of the function

$$B(\rho,\kappa,r_\rho) = \delta_\rho(\rho,\kappa,r_\rho)/\delta_\rho(\kappa,r_\rho) \quad (5)$$

are plotted in Fig. 1. The values of the variable $\rho = \kappa r$ in units $\rho_c = \kappa r_c$, where $r_c = e^2/E$ is the Coulomb classical turning point, corresponding to a given energy, are given along the abscissa. The solid lines are graphs of the function (5) at $E = 1$ keV. The numbers near these lines are the values of the parameter r_ρ in units of R. If the energy is decreased, then the dependence of the phase function and function (5) on parameter r_ρ becomes more weak. So, if $E = 0.1$ keV, then the functions (5) corresponding to three different values of parameter $r_\rho = \gamma R$, $\gamma = 0.1$, 1, 10 are equal to each other in the region $\rho > 0.7\rho_c$ with relative accuracy $\sim 10^{-4}$. The dashed line is their common graph. As follows from Fig. 1, the role of region $\rho \leq 0.8\rho_c$ (i.e. $r \leq 0.8 r_c$) in forming of the phase shift $\delta(\kappa,r_\rho)$ is small. If $E \leq 1$ keV, and $r_\rho \geq 0.1R$ these phase shifts are formed generally on the intervals $(0.8\rho_c, 2\rho_c)$,

Table 1. The phase shifts $\delta_\rho(\kappa,r_\rho)$ (radian) as function of the energy E (keV) and parameter r_ρ (fm)

E \ r_ρ	0.1R	R	10R
0.01	2.0141 10^{-15}	2.0141 10^{-15}	2.0141 10^{-15}
0.1	7.5129 10^{-13}	7.5129 10^{-13}	7.5129 10^{-13}
0.2	4.3480 10^{-12}	4.3479 10^{-12}	4.3470 10^{-13}
0.4	2.6058 10^{-11}	2.5948 10^{-11}	2.5774 10^{-11}
0.6	7.9769 10^{-11}	7.6569 10^{-11}	7.1999 10^{-11}
0.8	1.9417 10^{-10}	1.7003 10^{-10}	1.3951 10^{-10}
1.0	4.1678 10^{-10}	3.2091 10^{-10}	2.1325 10^{-10}

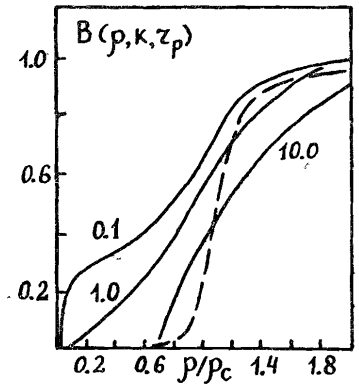


Fig.1. Function (5) at E = 1 keV (solid lines), E = 0.1 keV (dashed line). Numbers near solid lines are the values of parameter r_ρ in units of R.

i.e. in the interval $(0.8\,r_c\ ,\ 2\,r_c\ )$. Both the boundaries of this region depend on the energy and shift with decreasing energy to larger distances. If $E \leq 1$ keV and $r_\rho \in [0.1R, 10R]$, then the region $\rho \geq 2\rho_c$ is asymptotical for the phase function $\delta_\rho(\rho, K, r_\rho)$. Here it is necessary to point out that the asymptotical representation

$$u_\rho(r,K) = F_0(\rho,\eta)\cos\delta_\rho(K,r_\rho) + G_0(\rho,\eta)\sin\delta_\rho(K,r_\rho) \qquad (6)$$

for the regular solution of equation (1) at $U = V_\rho$ and a given energy E is valid only in the region $r \geq 2r_c = 2e^2/E$. As calculations show, the asymptotical representation $u_\rho(r,K) = \sin(\theta_c + \delta_c + \delta_\rho)$ is valid with relative accuracy ~ 0.01 in the region $r \geq 10\,r_c$. The calculation of phase shifts δ_ρ by solving the Shrödinger equation is a very complicated problem. Really, if $E = 1$ keV, then phase shifts δ_c and δ_ρ are equal respectively to ~ 1 and $\sim 10^{-10}$, the value of the Coulomb turning point is $r_c = 1440$ fm. Thus, the calculation of the phase shift δ_ρ at this energy requires solution of equation (1) with the relative accuracy $\sim 10^{-10}$ in the interval $0 \leq r \leq 10\,r_c = 14,400$ fm and subsequent extraction of phase shift δ_ρ from a rapidly oscillating asymptotics. The calculation of the wave-function in the interval $(0,\ 10\,r_c\ .)$ becomes with decreasing energy a more complicated problem. Numerical solution of phase equation (4) for $E \in [0.1,\ 1]$ keV has no difficulties. Moreover, if $E \leq 1$ keV and $r_\rho \geq 0.1R$, then a first iteration of this equation, i.e. the Born approximation

$$\delta_\rho^B(\rho,K,r_\rho) = -K^{-2}\int_{\rho_\rho}^{\rho} d\rho'\, V_\rho(\rho'/K)\, F_0^2(\rho',\eta) \qquad (7)$$

reproduces the corresponding phase function with a relative accuracy not worse than $\sim 10^{-3}$. Therefore the behavior of functions (5) with increasing ρ is explained so as the dependence of integrals (7) on their upper limit. The asymptotical representation $F_0(\rho,\eta) = C_0(\eta)\cdot\rho$ is valid in the region $\rho \ll 1$. The factor $C_0(\eta) = \{2\pi\eta/(\exp(2\pi\eta)-1)\}^{1/2}$ rapidly decreases as $K \to 0$. Therefore the contribution of this region to the integral (7) is small, and the value of the Born phase shift $\delta_\rho^B(15\rho_c, K, r_\rho)$ slightly depends on the value of parameter at sufficient low energies when the inequality $Kr_\rho \ll 1$ is valid. A main contribution to integral (7) comes from the region $(0.8\rho_c, 2\rho_c)$, where the function F_0 has a first local maximum, of an order of $O(\eta^{1/6})$. It is known[11], that the representation of Coulomb funtions in this region does not contain the factor $C_0(\eta)$. Therefore, the leading term of the Born-phase-shift asymptotics[2]

$$\delta_\rho^B(\infty,K,r_\rho) \underset{K\to 0}{\to} \frac{4}{15}\,\alpha K^5 R^2 \qquad (8)$$

also does not contain the function $C_0(\eta)$. The phase shift $\delta_S(K)$ owing to a rapid decrease of the potential V_S as $r \to \infty$, is formed in a finite distance range $r \leq r_S$ independently of the energy. If the energy is sufficiently low, i.e. if $\rho_S = Kr_S \ll 1$, then from the asymptotics $F_0 \sim C_0\cdot\rho$, $G_0 \sim C_0^{-1}$ in the region $\rho \leq \rho_S \ll 1$ there follows the representation

$$\delta_S(K) = -a_S K C_0^2(\eta), \qquad (9)$$

where a_S is the pion-nuclear scattering length. In the framework of the variable phase approach and our assumptions $r_S \leq r_\rho \sim R$ on the values of parameter r_ρ, the calculation of the phase shift $\delta(K)$ in the general case $U = V_S + V_\rho$; $V_S, V_\rho \neq 0$ is possible without knowing a concrete shape of the potential V_S. Really, the phase function $\delta(\rho,K)$ at point $\rho = \rho_S$ is equal to the phase shift (9), which we calculate using the experimental value $a_S = 0.079$ fm[12]. The inequality $|V_S(r)| \ll |V_\rho(r)|$ is valid for $r \geq r_\rho$. Therefore, for $\rho \geq \rho_\rho$ we substitute $U = V_\rho$ into equation (4) and further solve this equation with the boundary condition $\delta(\rho_\rho, K, r_\rho) = \delta_S(K)$. Thus, we have calculated the phase shift $\delta(K, r_\rho) = \delta(15\rho_c, K, r_\rho)$. The value of phase shift (9), at point ρ_ρ, i.e. the boundary value of the phase function $\delta(\rho, K, r_\rho)$, is sufficiently small, $\sim 10^{-8}$ at energies below one keV. Therefore if $E \leq 1$ keV and $r_\rho \geq 0.1R$, then the phase shifts $\delta(K, r_\rho)$ are approximated with a relative accuracy not worse than $\sim 10^{-3}$ by the corresponding sums $\delta_S(K) + \delta_\rho^B(K, r_\rho)$, where $\delta_\rho^B(K, r_\rho)$ is the value of integral (7) at point $\rho = 15\rho_c$. Both the phase shifts $\delta(K, r_\rho)$ and $\delta_\rho(K, r_\rho)$ slightly depend on the parameter r_ρ. Obviously, the same properties are characteristic of the functions of phase shifts $\delta(K, r_\rho)$, for instance,

$$A(E,\alpha) = -K^{-1}(E,\alpha) = -\{K C_0^2(\eta)\,\mathrm{ctg}\,\delta(K,r_\rho) + nh(\eta)\}^{-1} \qquad (10)$$

and the S-wave cross section of $\pi^+ d$-scattering, connected with the amplitude $f(K) = (2iK)^{-1}\{\exp(2i(\delta_c+\delta)) - 1\}$ by the equality $\sigma(E) = 4\pi\cdot|f(K)|^2$.

Following works[4], we consider the behavior of function (10) as $E \to 0$. The solid line in Fig. 2 is a common graph of three functions (10) calculated at $r_\rho = \gamma R$, $\gamma = 0.1,\ 1,\ 10$. The dash-

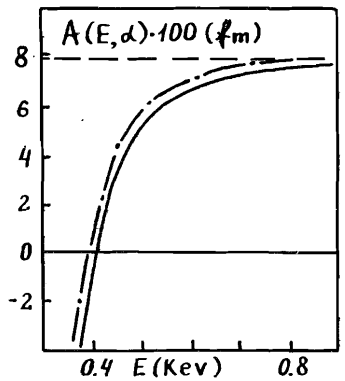
dotted line is a graph of function (10), calculated in our previous work[13] by the WKB-method. If the polarization potential is switched off ($\alpha = 0$, $\delta = \delta_S$), then $A(E,0)$ is the negative of the inverse effective-range Coulomb function. This function slightly depends on the energy in the considered energy region and has at point $E = 0$ a finite limit equal to α_S. The graph of the function $A(E,0)$ is plotted in Fig. 2 by the dashed line. If $\alpha \neq 0$, then the graph of the function $A(E,\alpha)$ (10)at 0.8 keV deviates from its horizontal asymptote $A(E,0) \approx \alpha_S$. It means that the upper boundary of the energy interval, where polarization effects occur in the S-wave π^+d-scattering, is equal to $\approx$ 0.8 keV. With further decreasing energy the phase shift δ_P decreases more slowly than the phase shift δ_S of opposite sign. Therefore, the phase shift δ and function A vanish at $E \approx 0.4$ keV. At the elastic scattering threshold, the conditions $\delta \approx \delta_P \gg \delta_S$ are valid owing to different asymptotics (8) and (9). The function $A(E,\alpha)$ approaches negative infinity as $E \to 0$. Let us study the influence of the polarization potential on the π^+d S-wave cross section. The latter may be represented by the sum

$$\sigma(E) = \sigma_c(E) + \frac{8\pi}{k^2} \sin\delta_c \cdot \sin\delta \cdot \cos(\delta_c + \delta) + \sigma_{sp}(E)$$

of three terms. The first term is the Coulomb cross section. When $E \to 0$ it, oscillating, tends to infinity as 1/E. The second, also oscillating, term has a threshold behavior $O(k^3)$. The third term, i.e. a smooth part of the total cross section, or the nuclear-Coulomb-polarization cross section $\sigma_{sp}(E) = (4\pi/k^2)\sin^2\delta(k,r_P)$is plotted in Fig.3 by the solid line. If $E \leq 0.35$ keV, then $\delta_P \gg \delta_S$ $\delta \approx \delta_P$. Therefore scattering on the polarization potential gives a dominant contribution to the cross section σ_{sp}. The cross section of the polarization potential scattering $\sigma_P = (4\pi/k^2)\sin^2\delta_P$is plotted in Fig.3. by the dash-dotted line. In the interval 0.35 keV$< E <$ 0.8 keV the values of phase shifts δ_P and δ_S are comparable, and both the potentials V_S and V_P give a comparable contribution to the scattering. If $E \geq 0.8$ keV, then the cross section σ_{sp} is equal to the cross section $\sigma_S = (4\pi/k^2)\sin^2\delta_S$, plotted in Fig.3 by the dashed line. The relations $\delta \approx \delta_S \gg \delta_P$ take place in this energy region, and therefore the influence of deutron polarizability on π^+d-scattering is negligible.

Studying the phase shifts $\delta(k,r_P)$ we have established that they slightly depend on a single free parameter r_P used, by us. The phase shift δ was normalized to its experimental value (9) in the

Fig. 2. Function (10), calculated at $\alpha = 0.7$ fm^3 in the frameworks of variable phase approach (solid line) and of WKB-method (dash-dotten line). The graph of function (10) at $\alpha = 0$ plotted by dashed line.



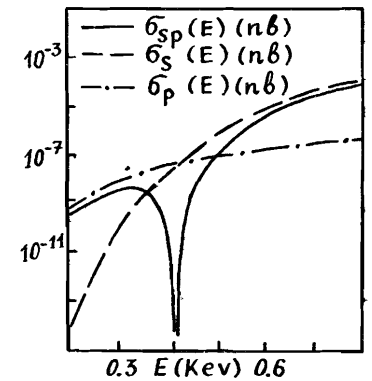


Fig. 3. Nonoscillating part of S-wave π^+d cross section. Solid line - $\sigma_{sp}(E)$, dash-dotted line $\sigma_P(E)$, $\sigma_S(E)$- dashed line.

energy region $E \geq 0.8$ keV, when one may neglect the polarization interaction. Therefore we believe that the above determined energy intervals, where a dominant contribution to the σ_{sp} cross section comes from the polarization potential or short-range potential, are quite exact. The most interesting result is the existence of a deep and sharp minimum of the cross section $\sigma_{sp}(E)$ at $E \approx 0.4$ keV; this minimum is due to the interference of scattering on potentials V_S and V_P. The same phenomenon in atomic physics is known as the Ramsauer effect[15].

In conclusion we point out the existence of such a minimum of the σ_{sp} cross section is evidently a common property of scattering of an arbitrary positive-charged particle (for example, the proton) on the H^2-nucleus if of course the phase shifts δ_S and δ_P at low energies opposite in sign. The experimental investigation of the cross section σ_{sp} at sufficiently low energies should allow to directly define a value of constant α, i.e. electric polarizability of deuteron.

REFERENCES

I. Ландау Л.Д., Лифшиц Е.М. Квантовая механика. М.: Наука, I974, с. 392.
2. Berger R.O., Spruch L. Phys.Rev. 1965, 138, p. B1106.
3. Квицинский А.А. ТМФ, I985, 65, с. 226.
4. Квицинский А.А. Письма в ЖЭТФ, I982, 36, с. 375
Kvitsinsky A.A., Merkyriev S.P. In: Few-Body Problems in Physics. Proc. of IX European Conf. Tbilisi, 1984,p.38; 1985,41, c.647.
5. Браун Дж. Е., Джексон А.Д. Нуклон-нуклонные взаимодействия. М.: Атомиздат, I979, с. 54.
6. Benze Gy. et al. Preprint LA-UR-86-3790, Los Alamos, 1986.
7. Kuperin Yu.A., Kvitsinsky A.A.,Merkuriev S.P.Preprint FT-FP-5-85, Barselona, 1985. Квицинский А.А. и др. ЭЧАЯ, I986, I7, с. 267.
8. Rodning N.L. et al. Phys.Rev.Lett., 1982, 49, p. 909.
9. Бабиков В.В. Метод фазовых функций в квантовой механике. М.: Наука, I976.
IO. Strecok A.J., Gregory J.A. Math.Comp. 1972, 26, p. 955
II. Абрамовиц М., Стиган И. Справочник по специальным функциям. М.: Наука, I979, с. 354.
12. Bovet E. et al. Phys.Lett. 1985; 153B, p. 231.
I3. Пупышев В.В., Соловцова О.П. ОИЯИ, Сообщение Р4-86-346, Дубна, I986.
I4. Фреман Н., Фреман П.У. ВКБ-приближение М.: Мир, I967.
I5. Друкарев Г.Ф. Столкновение электронов с атомами и молекулами М.: Наука, I978, с. 62.

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Поляризуемость дейтрона и S-волновое π^+d-рассеяние при энергиях ниже 1 кэВ

Методом фазовых функций исследуется влияние поляризуемости дейтрона на S-волновое π^+d-рассеяние в пределе низких энергий. Показано, что неосциллирующая часть S-волнового сечения π^+d-рассеяния имеет глубокий резкий минимум в области энергий ~ 0,4 кэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Pupyshev V.V., Solovtsova O.P. E4-87-467

Deuteron Polarizability and S-Wave π^+d-Scattering at Energies Below 1 keV

The influence of deuteron polarizability on the S-wave π^+d-scattering in a low-energy limit is explored in the framework of the variable phase method. It is shown that the nonoscillating part of the S-wave cross section of π^+d-scattering has a deep and sharp minimum in the energy region ~ 0.4 keV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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