объединенный
институт
ядерных
исследований
дубна

E4-87-35

V.B.Belyaev, V.E.Kuzmichev*, V.V.Peresypkin*,
M.L.Zepalova*

THE ROLE OF POLARIZABILITY IN THE $^{3}H(d,n)^{4}He$ REACTION

Submitted to "Ядерная физика"

* Institute Theoretical Physics Ac. Sc. UkrSSR, Kiev

1987

1. INTRODUCTION

Theoretical study of the exothermal reaction

$$d + {}^3H \rightarrow {}^4He + n + Q, \qquad (1)$$

where Q = 17, 589 MeV the triton incident energy E_t is of an order of 10 keV and lower, represents a special interest because of the absence of experimental data in that energy region. Till present, the cross section, astrophysical S -factor and yield of helium nuclei in reaction (1) in a low-energy region are measured for energies E_t from 12.5 to 375 keV /1/. A specific feature of the total cross section $\sigma(E_t)$ of reaction (1) at those energies is the existence of $3/2^+$ resonance with parameters $\sigma(E_t)$ = 5 b, $E_t \simeq 160$ keV, Γ = = 135 keV /2/. In a region of $E_t < 12.5$ keV, which is equivalent to the energy of relative motion $d\,{}^3H$ $E < 5$ keV, the cross section $\sigma(E)$ is extrapolated by the Gamov representation:

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-1.088\, E^{-1/2}\right), \qquad (2)$$

where E is the energy in c.m.s. (in MeV), S is a smooth function of the energy (astrophysical S -factor) having at E = 0 a finite value, $S(0)$ = 11.59 $\pm$ 0.06 MeV b /1/. Extrapolation into the energy region $E < 5$ keV by formula (2) is based on the assumption that at very low energies the interaction of deuterium and tritium nuclei is a pure Coulomb repulsive interaction, without any other peculiarities.

At the same time it is known[3] that an external electric field acting on a charged nuclear system whose center of mass does not coincide with the centre of charge distribution induces electric moments of different multipolarity in the nuclear system. Effective interaction at long distances between such masses at low energies should differ from the pure Coulomb interaction since it contains additional, to short-range and Coulomb, polarization attraction potentials decreasing at infinity in a power way. As a result of imposing the Coulomb and polarization potentials, the scattering of particles by nuclei should generally differ from the Rutherford scattering[4-6]. As is shown in[7], if the electric dipole polarizability of the deuteron in the reaction of radiative capture of protons by deuterons is taken into account, then the cross section of that reaction at energies of the relative pd-motion lower than 2 keV differs from the Coulomb one and has a narrow maximum. The astrophysical S-factor gets a strong energy dependence. As a result, in the energy region below 2 keV the yield of ^3He nuclei gets increasing. An analogous behaviour takes place for the cross section of reaction $p(p,d)e^+\nu$ at energies below 0.5 keV calculated with the account of electric dipole polarizability of the proton[8],[9]. It may be expected that the account of polarizability of the deuteron in an electric field of the ^3H nucleus will also lead to new peculiarities in the cross section of reaction (1) in the energy region $E < 5$ keV.

In this paper, we have studied the influence of electric dipole polarizability of the deuteron on the yield of nuclei in reaction (1). Since the deuteron polarizability $\alpha = 0.7 \pm 0.05$ fm^3 is by an order larger than the triton polarizability, we have not taken account of the latter.

2. THEORY

The transition amplitude for process (1) is of the form[11]

$$M(\vec{k}_f, \vec{k}_i) = \langle \varphi_{\vec{k}_f} \varphi_\alpha | V_f | \Psi^{(+)}_{\vec{k}_i} \rangle , \qquad (3)$$

where $\Psi^{(+)}_{\vec{k}_i}$ is the wave function of an initial state of the d^3H system with the momentum of relative motion of nuclei d and ^3H, $\vec{k}_i$, φ_α is the wave function of the ^4He nucleus, $\varphi_{\vec{k}_f}$ describes a free motion of the ^4He nucleus and neutron with a relative momentum $\vec{k}_f$ in a final state, V_f is the potential of interaction in an exit channel of reaction (1) which is a sum of nucleon-nucleon potentials characterizing the interaction of an incident neutron with nucleons of the ^4He nucleus and allowing for both central and tensor forces[12]. The wave function of α-particle will be represented by $|\varphi_\alpha\rangle = |\varphi_p\rangle|\varphi_t\rangle$, where $|\varphi_t\rangle$ is the wave function of ^3H nucleus, the overlap integral $\langle \varphi_\alpha | \varphi_t \rangle$ is proportional to the ^4He vertex function with asymptotics $\varphi_p(r) \sim \exp(-\varkappa r)/r$, $\varkappa^2 = 2\mu_{tp}(\theta + B)$, μ_{tp} is the reduced mass of $^3H + p$, B is the deuteron binding energy. Neglecting the effects of antisymmetrization for the function $\Psi^{(+)}_{\vec{k}_i}$, we arrive at the representation $\Psi^{(+)}_{\vec{k}_i} = \varphi_t \varphi_d \Psi_{\vec{k}_i}$, where φ_d is the deuteron wave function, $\Psi_{\vec{k}_i}$ is the wave function of relative motion of d^3H. Thus, the transition amplitude (3) assumes the form

$$M(\vec{k}_f, \vec{k}_i) = \langle \varphi_{\vec{k}_f} \varphi_p | \bar{V}_f | \varphi_d \Psi_{\vec{k}_i} \rangle , \qquad (4)$$

where

$$\bar{V}_f = \langle \varphi_t | V_f | \varphi_t \rangle . \qquad (5)$$

In the region (we are interested in) of energies of an order of several keV in the presence of Coulomb repulsion, a leading contribution to the matrix element (4) comes from the range of motion of d and ^3H nuclei outside of the action of nuclear forces between them. This allows us to calculate the amplitude M using the asymptotic representation of the wave function $\Psi_{\vec{k}_i}$:

$$\Psi_{k_i}(r) = \frac{e^{i\sigma_0}}{k_i r}\left\{F_0(k_i r) + k_i f(k_i)\left[G_0(k_i r) + iF_0(k_i r)\right]\right\}_{r>r_0} \quad (6)$$

where r_0 is the range of nuclear forces, σ_0 is the Coulomb d^3H -scattering phase, F_0 and G_0 are s -wave components of regular and irregular Coulomb functions, r is the distance between d and ^3H nuclei, f is the elastic scattering amplitude reckoned from the Coulomb amplitude and equal to

$$f(k_i) = -\frac{3M_{dt}}{k_i}\int_0^\infty r F_0(k_i r) V_i(r) \Psi_{k_i}(r)\,dr. \quad (7)$$

Here V_i is the interaction effective potential of d and ^3H nuclei, M_{dt} is their reduced mass. We choose V_i as a sum of the nuclear U_N and polarization U_p potentials:

$$2M_{dt}V_i = U_N + U_p. \quad (8)$$

where (fc.[5] and [6])

$$U_p = -\frac{\alpha}{2R}\,\frac{\Theta(r-r_0)}{r^4}. \quad (9)$$

Here α is the deuteron polarizability, $R=(2M_{dt}e^2)^{-1}$ is the Bohr radius, $\Theta(x)$ is a step function. In the limit of small k_i for amplitude (7) we have the representation

$$f \simeq -(a+ib)C_0^2 + \frac{4}{15}\alpha R^2 k_i^4, \quad k_i \to 0, \quad (10)$$

where $C_0^2 = \frac{\pi}{k_i R}\left[\exp\left(\frac{\pi}{k_i R}\right)-1\right]^{-1}$ is the Coulomb factor, a and b are constants which are close to the magnitudes of real and imaginary parts of the elastic d^3H scattering length because of the smallness of polarization interaction U_p. The astrophysical S -factor S_0

$$S_0(E) = \frac{M_{dn}}{8\pi^2}k_i k_f \int \left| e^{\frac{\pi}{2k_i R}} M\right|^2 d\Omega_{\vec{k}_f} \quad (11)$$

calculated by formulae (4),(6) and (10) is an exponentially growing function as $E \to 0$. The reason is that only terms linear in α/R^3 have been taken into account in the amplitude (10). Summation of the whole series of perturbation theory in polarization potential gives us the expression

$$S(E) = \frac{S_0(E)}{1+\frac{M_{dt}}{2\pi^2}k_i R\, S_0(E)}. \quad (12)$$

3. CALCULATION

The transition amplitude (4) depends on the magnitude of spin S in the initial state. For S = 1/2 and S -wave in the relative motion of d and ^3H the interaction in the final state $\bar{V}_f$ (5) also occurs in S -wave (ℓ = 0). In this case in a good approximation for $\bar{V}_f\cdot\varphi_d$ we have the representation[11]:

$$\bar{V}_f\,\varphi_d(r) \simeq V_{np}\varphi_d = -D\delta(r),$$

where

$$D = -\int d\vec{r}\, V_{np}\varphi_d(r)$$

is a constant which takes account of a finite-range nature of nuclear forces between nucleons. At S = 3/2 the interaction $\bar{V}_f$ takes place in d-wave (ℓ = 2) and may be parametrized by the expression

$$\bar{V}_f = -V\Theta\left(r_c - \tfrac{3}{4}r\right) + \frac{16}{3M_{dn}r^2}\Theta\left(\tfrac{3}{4}r - r_c\right),$$

where V = 1 MeV and r_c = 7 fm[13]. A potential of that sort describes the resonance in d- phase of elastic n^4He scattering.

In Fig.1 the astrophysical S -factor is drawn for reaction (1) as a function of the incident-triton energy E_t for $\ell = 0$ (curve1) and $\ell = 2$ (curve 2). There were used the following values of parameters:

$$a = 32.2\ fm^{[13]},\ b = -17.6\ fm^{[13]},\ R = 12.01\ fm \qquad (13)$$
$$\alpha = 0.7\ fm^{3\ [10]},\ \varkappa = 0.846\ fm^{-1},\ r_0 = 1.9\ fm.$$

As is seen from that Figure, the influence of the polarization potential U_p on the S -factor is significant in the energy region $E_t < 10$ keV and depends on the angular momentum ℓ . For $\ell = 0$ in the energy region $6.5 < E_t < 10$ keV the factor $S(E)$ has a small minimum due to interference of

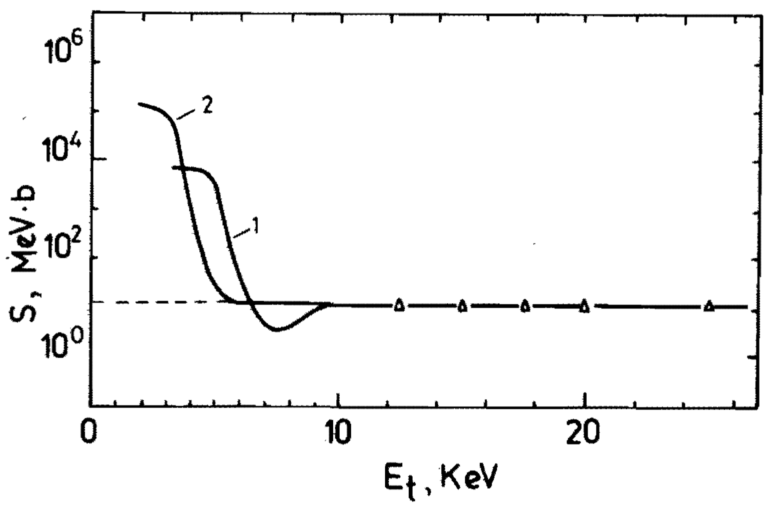

Fig.1. Astrophysical S-factor S of the reaction (1) as a function of energy E_t . The case $U_p \neq 0$ is the full line, $U_p = 0$ is the dashed line. The curves 1 and 2 correspond to relative angular momentum ^4He and n $\ell = 0$ and $\ell = 2$. The experimental data are shown by triangles [1].

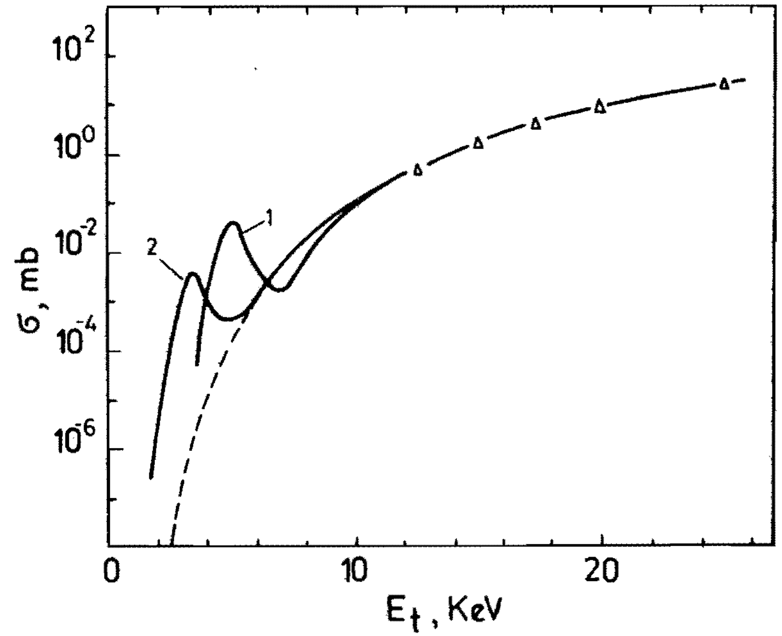

Fig.2. The cross section as a function of energy of incident nucleus ^3H. The notation is the same as in Fig.1.

nuclear and polarization parts of the amplitude (10). For $5 \leq E_t \leq 6.5$ keV the function $S(E) \simeq S_0(E)$ with decreasing energy grows exponentially, whereas for $E_t < 5$ keV in accordance with (13). For $\ell = 2$ the interference minimum is practically indistinguishable in the Figure, and the factor $S \simeq S_0$ grows exponentially for $3.2 \leq E_t \leq 6$ keV and like $E_t^{-1/2}$ for $E_t < 3.2$ keV. The dashed curve in Fig. 1 is the S -factor calculated without the deuteron polarizability ($U_p = 0$).

In Fig.2 the cross section of reaction (1) is plotted. When $E_t < 10$ keV, the cross sections calculated with (solid curves) and without (dashed curves) the effect of deuteron polarizability differ from each other. When $U_p \neq 0$, the cross section is characterized by a further muximum whose position and size depend on the angular momentum ℓ of the system of ^4He and neutron in the final state: $\sigma = 4.8\ 10^{-2}$ mb at E_t = 5 keV for $\ell = 0$ (curve 1) and $\sigma = 3.7\ 10^{-3}$ mb at E_t = 3.3 keV for $\ell = 2$ (curve 2). A small decrease of the cross section as compared to that for $U_p = 0$, at $7 < E_t < 10$ keV, and $\ell = 0$ is explained by interference of the nuclear and polarization parts of the elastic scattering amplitude (10).

The energy yield of reaction (1) is determined by the temperature dependence of the quantity $\langle \sigma v \rangle$, where v is the velocity of relative motion of colliding d and ^3H nuclei and averaging is made over the Maxwell distribution of velocities v in the d^3H system. In Fig.3 the dependence of $\langle \sigma v \rangle$ is shown on the temperature κT (κ is the Boltzmann constant) calculated with (solid curve) and without (dotted

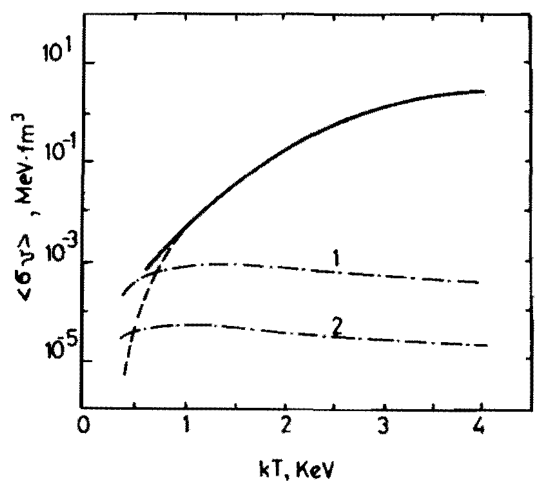

Fig.3. The temperature dependence reactivity $\langle \sigma v \rangle$ with (full line) and without (dashed line) the polarizability of the deuteron. The contributions of the deuteron polarizability to the value $\langle \sigma v \rangle$ shown by the dash-dotted lines 1 ($\ell = 0$) and 2 ($\ell = 2$).

curve) potential U_p . The contribution of the deuteron polarizability is shown by dash-dotted curves I (for ℓ = 0) and 2 (for ℓ = 2). When $\kappa T < 1$ keV (ℓ = 0) and $\kappa T < 0.6$ keV (ℓ = 2), the yield of reaction (1) increases; here the role of state with ℓ = 0 turns out to be more essential. The temperature dependence of the function $\langle \sigma v \rangle$ with ℓ = 0 in the range of $\kappa T < 1$ keV has the form

$$\langle \sigma v \rangle \sim (\kappa T)^{-3/2} \exp\left(-2/\kappa T\right) , \tag{14}$$

and differs from the dependence

$$\langle \sigma v \rangle_{\alpha=0} \sim (\kappa T)^{-2/3} \exp\left(-\frac{19,99}{(\kappa T)^{1/3}}\right) , \tag{15}$$

that takes no account of the polarization potential U_p /1/. When $\kappa T > 1$ keV, the deuteron polarizability has a negligible influence on the temperature dependence of $\langle \sigma v \rangle$ which is in this case determined by function (15). In Table the ratio

$$R(\kappa T) = \langle \sigma v \rangle / \langle \sigma v \rangle_{\alpha=0} \tag{16}$$

is given for ℓ = 0 in the temperature range $\kappa T \leq 1$ keV characterizing a relative increase of the energy yield of reaction (1) due to the effect of attraction polarization potential. The increase in the yield of reaction (1)for ℓ = 2 in the elastic n^4He channel is negligible.

Table

The dependence of the ratio $R(\kappa T)$ (16) on the temperature of the d^3H - system.

kT, KeV	0.4	0.5	0.6	0.7	0.8	0.9	1.0
R(kT)	41.8	12.7	4.8	2.5	0.66	1.12	1.0

4. CONCLUSION

Our theoretical analysis of the characteristics of nuclear fusion (1) at low energies with account of the deuteron polarizability in the triton electric field performed in the framework of the potential model (9) allows us to conclude that there exists a narrow maximum in the cross section of the reaction at energies $E_t < 10$ keV.

It is to be noted that the difference of cross sections calculated with an without the deuteron polarizability at the point of maximum exceeds the value of experimental error/11/ in the measured energy region. From this standpoint, experimental check of the theoretical predictions seems to be not so hopeless. However, the cross section at maximum is still very small, and for Louson-criterion to be fulfilled two orders of magnitude are not sufficient.

REFERENCES

1. Jarmie N., Brown R.E., Hardekopf R.A. Fujion-energy reaotion $^2H(t,\alpha)n$ from E_t= 12.5 to 117 keV. – Phys.Rev., 1984, v.C29, N 6, p.2031–2046.
2. Ajzenberg–Selove F. Energy levels of light nuolei A=5-10.– Nuol.Phys., 1984, v.A413, N 1, p.1–224.
3. Абелишвили Т.Л., Ситенко А.Г. Электрическая поляризация дейтрона при рассеянии в кулоновском поле. - УФЖ, I96I, т.VI, N I, с.3-II.
4. Квицинский А.А., Меркурьев С.П. Поляризационный потенциал и низкоэнергетические характеристики pd рассеяния. – Ядерная физика, 1985, т.4I, N 3, с.647-654.
5. Kuzmichev V.E., Zepalova M.L. Estimate of polarization effeot in the pd scattering. – Proc. of the X Europ.Symp. on the dynamics of few-body systems. Contributed papers. Balatonfured, Hungary, 1985, p.91–93.
6. Kuzmiohev V.E., Zepalova M.L. On the anomaly behaviour of the doublet pd phase shift oaused by polarization potential. – Phys.Lett., 1986, v.167B, N 3, p.268–270.
7. Belyaev V.B., Kartavtsev O.I., Kuzmiohev V.E. Influence of the deuteron polarizability on radiative pd–capture at astrophysical low energies. – Preprint JINR E4–86–66, Dubna, 1986, – 10 p.
8. Кузьмичев В.Е., Беляев В.Б. Роль поляризуемости протона в реакции $p+p\to d+e^{+}+\nu$ при астрофизических низких энергиях. - Препринт ИТФ-86-I22Р, Киев, 1986. - I3 с.
9. Belyaev V.B., Kuzmichev V.E. The role of proton polarizability in the reaction $pp\to de^{+}\nu$ at astrophysical low energies. – Proc. of the Intern.Conf. Few Body – IX, Japan, 1986.
10. Rodning N.L., Knutson L.D., Lynch W.G., Tsang M.B., Measurement of the electric polarizability of the deuteron.– Phys.Rev.Lett., 1982, v.49, N 13, p.909–912.
11. Ситенко А.Г. Теория ядерных реакций. Энергоатомиздат, М., 1983, 352 с.
12. Вильдермут К.,Тан Я. Единая теория ядра. Мир, М., 1980,502 с.
13. Беляев В.Б., Бренер С.Е., Гандыль Е.М., Зубарев А.Л., Иргазиев Б.Ф. О возможном влиянии сильного взаимодействия на свойства мезомолекулярных систем. - Препринт ОИЯИ, Р4-8I-72, Дубна, 198I. - 7 с.

Received by Publishing Department
on January 23, 1987.

SUBJECT CATEGORIES
OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Беляев В.Б. и др. E4-87-35

Учет поляризуемости дейтронов в реакции $^{3}H(d,n)^{4}He$

Исследовано влияние электрической дипольной поляризуемости дейтрона на сечение, астрофизический S-фактор и выход ядер гелия в реакции $^{3}H(d,n)^{4}He$ в области предельно низких энергий. Предсказано существование узкого максимума в сечении при энергиях налетающего тритона, меньших 10 кэВ, обусловленного действием поляризационного потенциала притяжения в $d^{3}H$-системе. Рост сечения $d^{3}H$-реакции приводит к увеличению выхода ядер ^{4}He при температурах, меньших 1 кэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Belyaev V.B. et al. E4-87-35

The Role of Polarizability in the $^{3}H(d,n)^{4}He$ Reaction

The influence is investigated of the deuteron electric dipole polarizability on the cross section, astrophysical S-factor, and the yield of helium nuclei in the $^{3}H(d,n)^{4}He$ reaction in the region of extremely low energies. Prediction is made of the existence of a narrow maximum in the cross section at energies of an incident triton lower that 10 KeV produced by the action of an attractive polarization potential in the $d^{3}H$ system. The growth of the cross section of $d^{3}H$ reaction increases the yield of ^{4}He nuclei at temperatures lower than 1 keV.

The investigation has beem performed at the Laboratory of Teoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987