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# DWIA IN THE MOMENTUM SPACE FOR ($\gamma$, $\pi^{\circ}$) REACTION NEAR THRESHOLD

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## 1. Introduction

Recently, much attention has been paid to the threshold region of neutral pion photoproduction ( $E_\gamma^{LAB} \sim 136$-$200$ MeV) off nuclei. New experimental data /1,2/ make it possible to establish the mechanisms of this reaction /3,4/;on the other hand, it becomes possible to determine more precisely the $\pi^0$ -photoproduction amplitude on a nucleon.

At present, the traditional dispersion method seems not to be the only way to describe the threshold region of the $\pi^0$-photoproduction process. The effective chiral Lagrangian method, which is being developed now rather intensively, gives a possibility of determining the invariant photoproduction amplitude on a single nucleon using the partially-conserved axial current (PCAC) hypothesis as well. The differential cross-section of the $\pi^0$-photoproduction off nuclei is very sensitive to the choice of the elementary amplitude /2/. Therefore, comparing the calculated cross-section with the experimental data, one can discriminate between various versions of the amplitude.

However, for a successful solving of this problem one needs a reliable method for the description of the pion-nuclear interaction in the final state. The role of this interaction has been analysed in ref./5/ in the framework of DWIA in the coordinate space in the threshold region, where the interaction of the outgoing pion with nuclei is shown to be the reason for a 50-100% increase of the total cross section of the $^{12}C(\gamma,\pi^0)$ reaction. The results of ref./5/ are in good agreement with the experimental data; however, the 50-100% effect of the final state interaction can hardly be understood proceeding from the present concept of the pion-nuclear interaction. For example, one cannot correlate the result of ref./5/ with the well-known fact that the pion-nuclear interaction in the final state decreases the total cross-section at $E_\gamma >$ 200 MeV /6,7/.

A consistent use of DWIA for the coherent $\pi^0$-photoproduction in the resonance region gives a good description of the experimental data /7-9/.

The present calculations are aimed mainly at estimating the role of the strong pion-nuclear interaction in the $(\gamma,\pi^0)$ processes. We use the DWIA in the momentum space representation which allows us to avoid some approximations used in ref./5/ and to take into account the photoproduction amplitude terms depending nonlinearly on $\pi^0$ momentum and nucleon Fermi-motion. We use a unitarised version of the BL-amplitude /10/ as in ref./5/.

The elementary amplitude and PWIA approach are discussed in Sections 2 and 3, respectively. The fourth section is devoted to the DWIA calculations in the momentum space. The last section contains the results of the calculations on $^4$He, $^{12}$C and $^{40}$Ca.

## II. $\pi^0$-photoproduction amplitude on a free nucleon

The only component of the full elementary $\pi^0$-photoproduction amplitude, which is to be used in the coherent neutral pion photoproduction off the nuclei with spin $J^\pi = 0^+$ and isospin $T=0$, is the spin-independent one. According to Blomqvist and Laget /10/, this component can be represented as /5/

$$f_{\pi\gamma}(E_\pi;\vec{q},\vec{K}\lambda) = \frac{m}{4\pi\omega}\left(t_B^{(\lambda)} + t_\Delta^{(\lambda)} + t_\omega^{(\lambda)}\right), \tag{1}$$



where

$$t_B^{(\lambda)} = \frac{eg}{4m}(\mu_p-\mu_n)\left[\frac{1}{E_a(p_a^0-E_a)} + \frac{1}{E_b(p_b^0-E_b)}\right]\left[\vec{q} - \frac{E_\pi}{2m}(2\vec{p}_f+q)\right][\vec{K}\times\vec{\epsilon}_\lambda], \tag{2a}$$

$$t_\Delta^{(\lambda)} = \frac{4}{9}\frac{G_1 G_3}{\omega^2-M^2+iM\Gamma}\left(\vec{q} - \frac{E_\pi}{M}\vec{p}_a\right)\left[\left(\vec{K} - \frac{M-m}{m}\vec{p}_i\right)\times\vec{\epsilon}_\lambda\right], \tag{2b}$$

$$t_\omega^{(\lambda)} = \frac{1}{m_\pi}\frac{g_{\omega 1}\, g_{\gamma\pi\omega}}{(q-K)^2-m_\omega^2}(\vec{q}-\vec{K})[\vec{K}\times\vec{\epsilon}_\lambda]. \tag{2c}$$

Here $\vec{\epsilon}_\lambda$ is the photon polarization vector; $K=(E_\gamma,\vec{K})$ $q=(E_\pi,\vec{q})$, $p_i=(p_i^0,\vec{p}_i)$; $p_f=(p_f^0,\vec{p}_f)$ are photon, pion, initial and final nucleon four-momenta, respectively; $p_a = q+p_f$; $p_b = p_f - K$; $E_a = (\vec{p}_a^{\,2}+m^2)^{1/2}$;
$E_b = (\vec{p}_b^{\,2}+m^2)^{1/2}$; $\omega^2 = p_a^2$;
$m$, $m_\pi$, $m_\omega$ and $M$ are the masses of nucleon, pion, $\omega$-meson and $\Delta$-isobar, respectively. Their values and constants $G_1, G_3, g_{\omega 1}, g_{\gamma\pi\omega}$ and $\Delta$-isobar width are taken from ref./5/.

The complicated dependence of the amplitude (1) on the pion momentum arises a certain difficulty in considering correctly the nonlocality effects of the photoproduction operators within the traditional DWIA in the coordinate space (where $\vec{q}\to -i\vec{\nabla}$). Usually, this difficulty is avoided by linearisation of the amplitude (1) with the following approximation:

$$E_a = E_b \approx m;\quad p_a^0 \approx m+E_\pi;\quad p_b^0 \approx m-E_\gamma;$$
$$\omega^2 \approx m^2+m_\pi^2+2mE_\pi;\quad q\cdot K \approx E_\pi\cdot E_\gamma. \tag{3}$$

Using this approximation and neglecting terms of the $(m_\pi/m)^2$ order the authors of ref./5/ derived the following expression for the single-nucleon amplitude:

$$f'_{\pi\gamma} = \frac{m}{4\pi\omega}(t'_B + t'_\Delta + t'_\omega)\,\vec{q}\cdot[\vec{K}\times\vec{\varepsilon}_\lambda]\,, \tag{4}$$

where

$$t'_B = \frac{eg}{8m^2}(\mu_p - \mu_n)\left(\frac{1}{E_\gamma} - \frac{1}{E_\pi}\right), \tag{5a}$$

$$t'_\Delta = \frac{4}{9}G_1G_3[m^2 + m_\pi^2 + 2mE_\pi - M^2 + iM\Gamma]^{-1}, \tag{5b}$$

$$t'_\omega = \frac{1}{m_\pi}g_{\omega 1}g_{\gamma\pi\omega}[m_\pi^2 - m_\omega^2 - 2E_\pi E_\gamma]^{-1}. \tag{5c}$$

According to /11/, the approximations (3) are reliable in the threshold region for the charged pion photoproduction. The dominant contribution in the case of charged pion photoproduction is that of the "seagull" diagram that corresponds to the $S$-wave $E_{0+}$ multipole. However, this diagram does not contribute to the neutral pion photoproduction and that is why the approximations (3) seem to be incorrect in that case.

The absence of the "seagull" diagram in the ( $\gamma, \pi^0$ )-reaction can be the reason for an enhancement of the nucleon Fermi-motion effect. The analysis of this effect in the coherent $\pi^0$-photoproduction is easier because of the identity of the initial and final nuclear states. Indeed, one can represent the momenta of nucleons in nuclei as follows:

$$\vec{p}_i = -\frac{\vec{K}}{A} - \frac{A-1}{2A}(\vec{K}-\vec{q}) + \vec{V} \equiv \vec{p}_i^{\,eff} + \vec{V}\,, \tag{6a}$$

$$\vec{p}_f = -\frac{\vec{q}}{A} + \frac{A-1}{2A}(\vec{K}-\vec{q}) + \vec{V} \equiv \vec{p}_f^{\,eff} + \vec{V}. \tag{6b}$$

It can be shown /12,13/ that after averaging over the momentum distribution of nucleons in nuclei, the contribution of the linear over $\vec{V}$ terms equals zero. The quadratic over $\vec{V}$ terms are of the $(m_\pi/m)^2 \approx 0.02$ order; therefore, one can neglect these terms by putting $V=0$ in



exp.(6) (factorization approximation). Simplifying the calculations, this approximation allows one to coordinate the pion-nucleon kinematics with the pion-nuclear one. The last is quite important in the threshold region /14/.

## III. Plane wave part of the $\pi^0$-photoproduction amplitude off the nuclei.

When the final pion-nuclear interaction is not taken into account one can represent the coherent $\pi^0$-photoproduction amplitude as follows /15/:

$$V_{\pi\gamma}(\vec{q},\vec{K}\lambda) = W_A(q,K)\langle 0|\sum_{j=1}^{A} e^{i(\vec{K}-\vec{q})\vec{r}_j} f_{\pi\gamma}(E_\pi;\vec{q},\vec{K}\lambda)|0\rangle, \tag{7}$$

where the kinematic factor is

$$W_A(q,K) = \left[\frac{E_A(K)\cdot E_A(q)}{\mathcal{E}(K)\cdot\mathcal{E}(q)}\right]^{1/2}\frac{\omega}{m}\,, \tag{8}$$

$E_A(K) = (m_A^2 + \vec{K}^2)^{1/2}$ ; $E_A(q) = (m_A^2 + \vec{q}^2)^{1/2}$ are nuclear energies in the initial and final states, respectively; $m_A$ is the nuclear mass in the ground state; $\mathcal{E}(K) = K + E_A$ and $\mathcal{E}(q) = E_\pi(q) + E_A(q)$ are total energies of photon-nuclear and pion-nuclear systems.

Neglecting the effect of nucleon motion in nuclei or taking it into account by "factorization approximation", one obtains the single-nucleon amplitude as follows:

$$f_{\pi\gamma}(E_\pi;\vec{q},\vec{K}\lambda) = \frac{1}{\sqrt{2}}f_0(E_\pi;\vec{q},\vec{K})\,\mathrm{Sin}\,\theta_\pi\, e^{i\lambda\varphi_\pi}. \tag{9}$$

Then it is possible to represent the nuclear amplitude in the following form:

$$V_{\pi\gamma}(\vec{q},\vec{k}\lambda)=\frac{A}{\sqrt{2}}W_A(q,k)f_0(E_\pi;\vec{q},\vec{k})\sin\theta_\pi e^{i\lambda\varphi_\pi}F_0(Q), \tag{10}$$

where $\theta_\pi$ and $\varphi_\pi$ are the polar and azimuthal angles of an outgoing pion; $\vec{Q}=\vec{k}-\vec{q}$ is the transferred momentum.

The nuclear formfactor $F_0(Q)$ is a Fourier transform of the nuclear density $\rho(r)$ normalized to 1. Here $r$ is the nucleon coordinate in the nuclear c.m. frame. This formfactor is expressed in terms of the experimental charge formfactor as follows:

$$F_{exp}^{ch}(Q)=F_0(Q)\cdot f_p^{ch}(Q), \tag{11a}$$

where $f_p^{ch}(Q)$ is the proton charge formfactor. In the shell model calculations, the appropriate nuclear density is obtained as a function of different coordinate $r'$. The transition to the nuclear c.m. frame $r'\to r$ leads to the following correction in $F_0(Q)$:

$$F_0(Q)=F(Q)\cdot F_{c.m.}(Q),\; F_{c.m.}(Q)=exp(R^2Q^2/6A). \tag{11b}$$

For determination of $F(Q)$, we use here the phenomenological fit of nuclear electromagnetic formfactors by taking the symmetrized Fermi density /16,17/, which permits one to produce a unified description of the charge formfactors in a wide region of transferred momenta for light and heavy nuclei.

In terms of this model

$$F(Q)=-\frac{3\pi b[\cos(Qc)-\pi b\sin(Qc)\,\mathrm{cth}(\pi bQ)/c]}{Qc^2\,\mathrm{sh}(\pi bQ)[1+\pi^2b^2/c^2]}. \tag{12}$$

The numerical values of $b$, $c$ and $R$ for the $^{12}C$ and $^{40}Ca$, which have been determined in /17/ from the



elastic electron scattering with the help of expressions (11) and (12), are reproduced in Table 1.

Taking into account (10), one easily gets the following expression for the differential coherent $(\gamma,\pi^0)$ cross-section in the PWIA (in the $\pi A$ -center of mass frame):

$$\frac{d\sigma}{d\Omega}(PWIA)=\frac{q}{2k}A^2W_A^2(q,k)|f_0(E_\pi;\vec{q},\vec{k})|^2\sin^2\theta_\pi F^2(Q)F_{c.m.}^2(Q). \tag{13}$$

To complete the discussion of the PWIA-approach, it is necessary to say that we have reproduced the results of /5/ by using the linearized BL-amplitude (4,5) when $F_{c.m.}(Q)=1$. If $F_{c.m.}(Q)\neq 1$, one obtains a 10%-increase of the cross-section.

Table 1. Symmetrized Fermi-density parameters for $^4He$, $^{12}C$ and $^{40}Ca$ (in fm)

| | b | c | R |
|---|---|---|---|
| $^4He$ | 0.406 | 1.231 | 1.806 |
| $^{12}C$ | 0.478 | 2.220 | 2.462 |
| $^{40}Ca$ | 0.537 | 3.573 | 3.287 |

## IV. DWIA in the momentum space

As it has been mentioned in the introduction, the most complete and consistent way to allow for the strong $\pi A$ interaction in the final state is to use the DWIA in the momentum space, as it has been done in /15,18/. In this case, one can use the full BL-amplitude (1,2) without the approximations (3).

In the present paper we use the DWIA in the momentum space /15/, according to which the amplitude of the coherent $\pi^0$ -meson photoproduction off nuclei is

$$F_{\pi\gamma}(\vec{q},\vec{k}\lambda)=V_{\pi\gamma}(\vec{q},\vec{k}\lambda)-\frac{a}{(2\pi)^2}\int\frac{d\vec{q}'}{m(q')}\frac{F_{\pi\pi}(\vec{q},\vec{q}')V_{\pi\gamma}(\vec{q}',\vec{k}\lambda)}{\mathcal{E}(q)-\mathcal{E}(q')+io}, \quad (14)$$

where $m(q')=E_\pi(q')E_A(q')/\mathcal{E}(q')$ is the reduced $\pi A$ -system mass, $V_{\pi\gamma}(\vec{q},\vec{k}\lambda)$ is the plane wave part of the amplitude which has been discussed in III.

The strong $\pi A$ interaction in the final state is introduced by the second term of (14). The coefficient $a=(A-1)/A$ removes double counting of pion rescattering on one and the same nucleon because such effects are already included in $V_{\pi\gamma}$. This coefficient appears naturally in the Multiple Scattering Theory KMT /19/ in which the $\pi A$ interaction is described with the help of

$$F'_{\pi\pi}(\vec{q},\vec{q}')=a\,F_{\pi\pi}(\vec{q},\vec{q}')$$

which satisfies the following integral equation:

$$F'_{\pi\pi}(\vec{q},\vec{q}')=U_{opt}(\vec{q},\vec{q}')-\frac{1}{(2\pi)^2}\int\frac{d\vec{q}''}{m(q'')}\frac{U_{opt}(\vec{q}',\vec{q}'')F'_{\pi\pi}(\vec{q}'',\vec{q})}{\mathcal{E}(q)-\mathcal{E}(q'')+io}. \quad (15)$$

The details of the optical potential construction are discussed in /20,21/. Let us only mention that there are two terms in $U_{opt}$

$$U_{opt}(\vec{q},\vec{q}')=U_1(\vec{q},\vec{q}')+U_2(\vec{q},\vec{q}'). \quad (16)$$

The first term $U_1(\vec{q},\vec{q}')$ in the first-order optical potential constructed with the help of the free t-matrix of $\pi N$-scattering

$$U_1(\vec{q}',\vec{q})=-(A-1)\sqrt{m(q')m(q)}\langle\vec{q}_f|t(z_0)|\vec{q}_i\rangle F_0(\vec{q}-\vec{q}')/2\pi, \quad (17)$$

where $\vec{q}_i(\vec{q}_f)$ is the initial (final) pion momentum in the $\pi N$ c.m. system; $z_0$ is the reaction energy; $F_0(Q)$ is the nuclear formfactor.



The second term in (16) is introduced phenomenologically and is intended to imitate the true pion absorption and the second order optical potential effects. According to /20,21/, one has

$$U_2(\vec{q}',\vec{q})=(A-1)^2\frac{\sqrt{m(q')m(q)}}{m_\pi}(B_0+C_0\vec{q}'\cdot\vec{q})\frac{g_{\pi N}(q')g_{\pi N}(q)}{g^2_{\pi N}(q_0)}G_0(\vec{q}'-\vec{q}), \quad (18)$$

where $G_0(Q)=4\pi\int \sin(Qr)\,\rho^2(r)\,r\,dr/Q$ is the Fourier transform of the squared density $\rho^2(r)$ of the nucleon distribution in a nucleus $q_0$ is the asymptotic (on shell) value of pion momentum and $g_{\pi N}(q)$ is the pion-nucleon formfactor which controls the off-shell behaviour of the amplitude. We use the following expression for $g_{\pi N}(q)$ :

$$g_{\pi N}(q)=(1+\alpha q^2)^{-2}, \quad \alpha=0.224\ fm^2, \quad (19)$$

which follows from the separable $\pi N$ -interaction model /22/.

Complex values $B_0, C_0$ are taken from the pion elastic scattering data /20/. In the threshold region they are comparable with mesoatomic results /23/. The prameters $B_0$ and $C_0$ are universal for nuclei with $A=4\div 40$. Their energy dependence is shown in Fig.1.

It is convenient to perform a partial wave decomposition of the pion photoproduction amplitude $F_{\pi\gamma}$. The full analysis of this expansion is given in /15/. As far as the spin-zero nuclear states are concerned, one obtains

$$F_{\pi\gamma}(\vec{q},\vec{k}\lambda)=\sum_L Y_{L\lambda}(\Omega_{\vec{q}})\mathcal{F}^L_{\pi\gamma}(q,k\lambda). \quad (20)$$

Just the same expansion is to be used for the plane wave

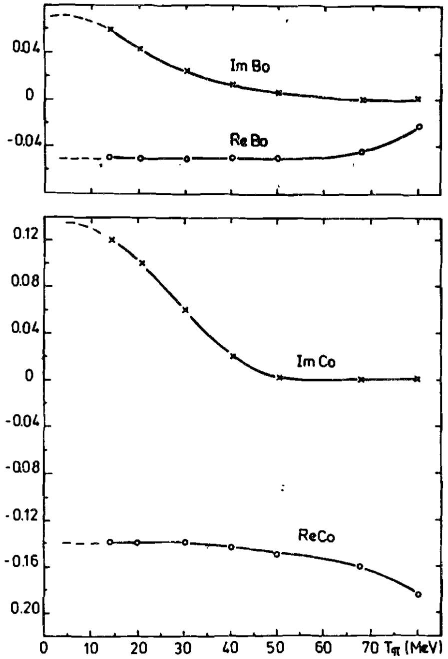



Fig. 1. Energy dependence of the parameters $B_0$ and $C_0$ in $U_2$ (see exp.(18) extracted from the elastic scattering data /20/; $B_0$ is given in $[m_\pi^{-4}]$ and $C_0$ in $[m_\pi^{-6}]$ .

amplitude $V_{\pi\gamma}(\vec{q},\vec{k}\lambda)$ (the corresponding partial amplitude is $V^L_{\pi\gamma}(q,k\lambda)$ ). The partial amplitudes $\mathcal{F}^L_{\pi\gamma}(q,k\lambda)$ satisfy the following equation:

$$\mathcal{F}^L_{\pi\gamma}(q,k\lambda)=V^L_{\pi\gamma}(q,k\lambda)-\frac{a}{\pi}\int\frac{q'^2dq'}{\mathfrak{M}(q')}\,\frac{\mathcal{F}^L_{\pi\pi}(q,q')V^L_{\pi\gamma}(q',k\lambda)}{\mathcal{E}(q)-\mathcal{E}(q')+i0}, \tag{21}$$

where $\mathcal{F}^L_{\pi\pi}(q,q')$ is the partial elastic pion scattering amplitude. The on-shell behaviour of $\mathcal{F}^L_{\pi\pi}(q,q')$ is determined by the phase shift $\delta_L(q)$

$$\mathcal{F}^L_{\pi\pi}(q,q)=\frac{1}{2iq}\left(\exp(2i\delta_L)-1\right). \tag{22}$$

The differential and total cross-sections can be expressed in terms of $\mathcal{F}^L_{\pi\gamma}$ with the help of



$$\frac{d\sigma}{d\Omega}=\frac{q}{2k}\sum_{L\lambda}\left|Y_{L\lambda}(\Omega_{\vec{q}})\mathcal{F}^L_{\pi\gamma}(q,k\lambda)\right|^2, \tag{23a}$$

$$\sigma=\frac{q}{2k}\sum_{L\lambda}\left|\mathcal{F}^L_{\pi\gamma}(q,k\lambda)\right|^2. \tag{23b}$$

In calculations one also needs the off-shell values of the elementary amplitude $f_{\pi\gamma}$ in the region where

$$E'_\pi(q')=\sqrt{m_\pi^2+\vec{q}'^2}\neq E^0_\pi=\sqrt{m_\pi^2+\vec{q}_0^{\,2}}. \tag{24}$$

We use the following parametrization;

$$f_{\pi\gamma}(E'_\pi;\vec{q}',\vec{k}\lambda)=f_{\pi\gamma}(E^0_\pi;\vec{q}',\vec{k}\lambda)\,\frac{g_{\pi N}(q')}{g_{\pi N}(q)}, \tag{25}$$

taken in a form analogous to that of the off-shell elastic pion scattering amplitude /13/.

## V. Results and discussion

Let us begin with the investigation of the approximations (3). For this purpose, we consider the BL-amplitude in following versions:

A - reduced BL-amplitude /5/ obtained with the help of approximations (3) (see exp.(4) and (5)).
B - full BL-amplitude (1-2) when $\vec{p}_i=0$ and $\vec{p}_f=\vec{k}-\vec{q}$ ;
C - full BL-amplitude (1-2) when $\vec{p}_i=\vec{p}_i^{\,eff}$, $\vec{p}_f=\vec{p}_f^{\,eff}$, where $\vec{p}_i^{\,eff}$ and $\vec{p}_f^{\,eff}$ are determined in (6).

All the versions are considered when $g_{\pi N}(q)=1$ .

In Table 2 the components $t'_B$, $t'_\Delta$ and $t'_\omega$ (see (9)) are calculated in (A-C) versions when $E^{LAB}_\gamma=160$ MeV and $\theta_\pi=60^\circ$ for the $^{12}C(\gamma,\pi^0)$ reaction. One can see

Table 2. Numerical values for different components of the BL-amplitude (in $fm^3$); $E_\gamma^{LAB}$ = 160 MeV, $\theta_\pi = 60°$.

| | $t'_B \times 10^3$ | Re $t'_\Delta$ | Im $t'_\Delta$ | $t'_\omega$ |
|---|---|---|---|---|
| A | 0.661 | -0.249 | -0.110 | -0.112 |
| B | 4.98 | -0.252 | -0.113 | -0.114 |
| C | 5.92 | -0.277 | -0.123 | -0.114 |

that the main differences of all the versions are those of the Born $t'_B$ and isobar $t'_\Delta$ terms. The relation between the PWIA total cross-sections $\sigma$ calculated in /5/ and in (A-C) versions are as follows:

$$\sigma_{Boffi} : \sigma_A : \sigma_B : \sigma_C = 1 : 1.13 : 1.26 : 1.32 , \qquad (26)$$

where the difference between $\sigma_{Boffi}$ and $\sigma_A$ is caused by neglecting in /5/ the center-of-mass factor $F_{c.m.}(Q)$ (see(13)). From expression (26) it is clear that our PWIA result in C-version is 30% larger than that of Boffi /5/ and more than a half of this increase is explained by using the full BL-amplitude.

The energy dependence of the cross-section of the coherent $^{12}C(\gamma,\pi^0)$ -reaction is described in Table 3, where both the PWIA and DWIA results (in the case $g_{\pi N} = 1$ ) are presented. The strong $\pi$ A-interaction in the final state does not change the relationship (26) drastically. But our DWIA result in A version differs from the result of /5/. For example, $\sigma_{Boffi}(DWIA)/\sigma_A(DWIA) \approx 0.80$ at $E_\gamma^{LAB}$ = 180 MeV. It is to be emphasized that the effect of pion wave distortion in our calculations is of 25% (compare with 70% effect in ref./5/).

It is convenient to introduce a factor of distortion

$$\eta = \sigma(DWIA)/\sigma(PWIA) \qquad (27)$$



Table 3. Energy dependence of the cross-section (in $\mu b$ ) of the $^{12}C(\gamma,\pi^0)$ coherent photoproduction calculated with different versions of the BL-amplitude /10/. ( $g_{\pi N}(q) = 1$ ).

| $E_\gamma^{LAB}$ (MeV) | A | | B | | C | |
|---|---|---|---|---|---|---|
| | PWIA | DWIA | PWIA | DWIA | PWIA | DWIA |
| 136 | 0.079 | 0.100 | 0.086 | 0.110 | 0.091 | 0.116 |
| 137 | 0.998 | 1.275 | 1.088 | 1.402 | 1.156 | 1.482 |
| 138 | 2.434 | 3.134 | 2.655 | 3.449 | 2.819 | 3.643 |
| 139 | 4.231 | 5.488 | 4.620 | 6.044 | 4.903 | 6.381 |
| 140 | 6.320 | 8.250 | 6.907 | 9.093 | 7.325 | 9.597 |
| 142 | 11.21 | 14.83 | 12.28 | 16.38 | 13.00 | 17.26 |
| 145 | 19.98 | 26.83 | 21.93 | 29.71 | 23.19 | 31.27 |
| 150 | 37.43 | 50.47 | 41.26 | 56.13 | 43.52 | 58.95 |
| 155 | 57.48 | 77.06 | 63.63 | 86.09 | 66.98 | 90.26 |
| 160 | 79.42 | 105.6 | 88.28 | 118.6 | 92.75 | 124.1 |
| 165 | 102.8 | 134.8 | 114.7 | 152.1 | 120.3 | 158.9 |
| 170 | 127.2 | 163.6 | 142.5 | 185.3 | 149.3 | 193.5 |
| 175 | 152.5 | 191.7 | 171.6 | 218.0 | 178.5 | 227.5 |
| 180 | 178.5 | 219.1 | 201.6 | 249.7 | 210.7 | 260.8 |
| 185 | 205.1 | 245.6 | 232.7 | 280.4 | 242.8 | 293.1 |
| 190 | 232.4 | 270.0 | 264.7 | 308.3 | 276.0 | 323.0 |

for further analysis of the pion-nuclear interaction in the final state.

The energy dependence of this factor in version C for photon energy $E_\gamma^{LAB}$=136-290 MeV is plotted in Fig.2 (dashed curve). The results of other authors for this factor are depicted in

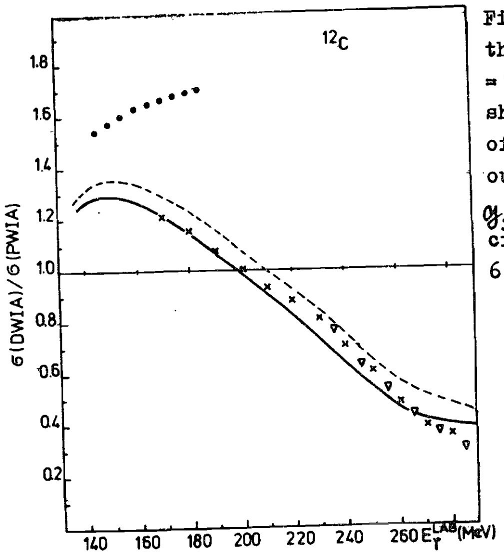



Fig. 2. Energy dependence of the distortion factor $\eta = \sigma(DWIA)/\sigma(PWIA)$. Solid (dashed) line is the results of this calculation with (without) the off-shell form-factor $g_{\pi N}(q)$. Dots, triangles and crosses are from papers /5, 6,7/ respectively.

Fig.1 as well. It is obvious that the results of /5/ are hardly in agreement with the fact that $\eta<1$ at $E_\gamma^{LAB}>200$ MeV.

The solid curve corresponds to the version C results with $g_{\pi N}(q)$ satisfying expression (19). One can see that the inclusion of the off-shell effects decreases the factor of distortion $\eta$. The corresponding values of cross-sections and the results of their comparison with the experimental data are given in Table 4 and Fig.3,4 respectively. The results of calculations for $^4He$ and $^{40}Ca$, which are under experimental investigation, are in Table 4 as well.

It seems to be premature to do some specific conclusion while comparing the theoretical results with the experimental data. On the one hand there is a discrepancy between the data of different groups. On the other hand, there is uncertainty in choosing the elementary amplitude. To illustrate this, we



Table 4. Energy dependence of the cross-section (in $\mu b$) of the $\pi^0$– coherent photoproduction, calculated in version C with $g_{\pi N}(q)$ from eq.(19).

| $E_\gamma^{LAB}$ (MeV) | $^4He$ | | $^{12}C$ | | $^{40}Ca$ | |
|---|---|---|---|---|---|---|
| | PWIA | DWIA | PWIA | DWIA | PWIA | DWIA |
| 136 | | | 0.091 | 0.111 | 2.448 | 3.055 |
| 137 | | | 1.156 | 1.421 | 8.129 | 10.26 |
| 138 | 0.068 | 0.078 | 2.819 | 3.494 | 15.47 | 19.67 |
| 139 | 0.299 | 0.347 | 4.093 | 6.118 | 23.97 | 30.60 |
| 140 | 0.625 | 0.729 | 7.325 | 9.196 | 33.36 | 43.12 |
| 142 | 1.497 | 1.756 | 13.00 | 16.53 | 54.08 | 68.74 |
| 145 | 3.258 | 3.872 | 23.19 | 29.88 | 88.36 | 108.5 |
| 150 | 7.235 | 8.723 | 43.52 | 56.15 | 149.9 | 171.3 |
| 155 | 12.41 | 15.17 | 66.98 | 85.69 | 213.8 | 231.5 |
| 160 | 18.74 | 23.23 | 92.75 | 117.4 | 278.8 | 305.2 |
| 165 | 26.24 | 32.89 | 120.3 | 149.7 | 344.6 | 386.2 |
| 170 | 34.93 | 44.08 | 149.3 | 181.6 | 411.6 | 470.9 |
| 175 | 44.85 | 56.78 | 179.5 | 212.7 | 480.4 | 555.3 |
| 180 | 56.05 | 70.85 | 210.7 | 242.7 | 551.5 | 635.0 |
| 185 | 68.60 | 86.26 | 242.8 | 271.6 | 625.4 | 708.6 |
| 190 | 82.58 | 102.5 | 276.0 | 297.7 | 702.6 | 767.9 |

give in Fig.4 the PWIA-results of ref./2/ *) with two different versions of the elementary amplitude. The third reason is the absence of a relevant evaluation of noncoherent process contri-

*) Unfortunately, the erroneous results have been shown in ref. /2/. In Fig.4. we used the correct results which have been given to us by the authors of ref./2/.

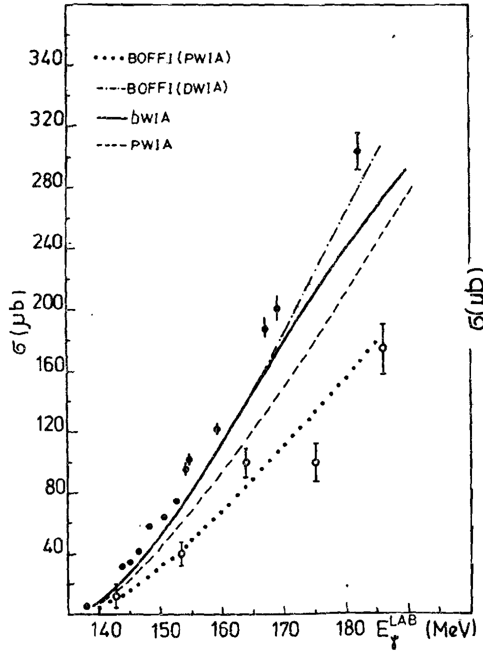



Fig. 3. Total cross sections for the $^{12}C(\gamma,\pi_0)$ reaction. Solid and dashed lines are the results of this calculation in the DWIA and PWIA with $g_{\pi N}(q)$ according to eq. (19); dash-dotted and dotted lines are the DWIA and PWIA results of ref./5/. The experimental data are from ref./1/ and ref./24/ .

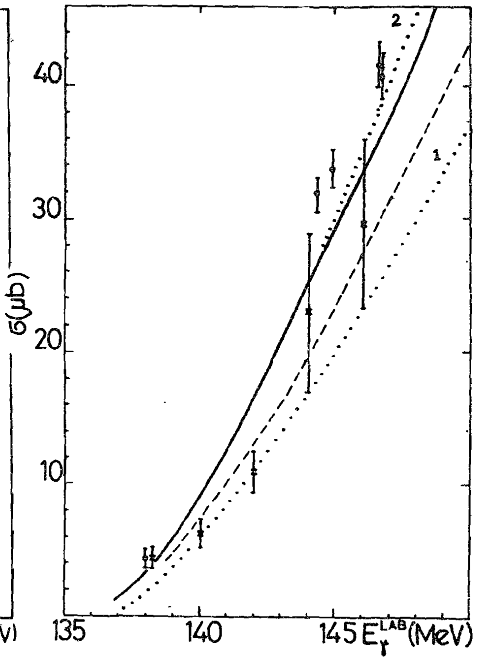
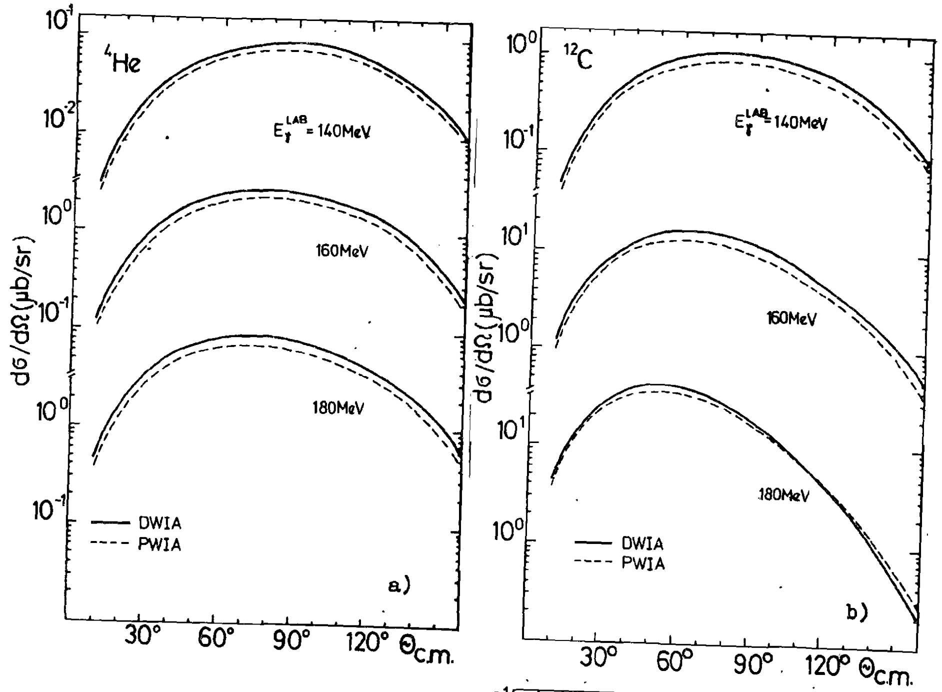



Fig. 4. Total cross-section for the $^{12}C(\gamma,\pi^0)$ reaction. Dashed and solid lines present the PWIA and DWIA results of this paper, respectively, with C-version of the BL-amplitude and off-shell formfactor $g_{\pi N}$ . Dotted lines 1 and 2 are the PWIA-result of ref./2/ with the amplitudes from ref./25/ and /26/. Experimental data from ref./1/ and ref./2/

butions to the $\pi^0$-photoproduction which are included in the experimental data.

Let us compare the DWIA- and PWIA-differential cross-sections in Fig.5. Two facts are to be pointed out. The first is a strong sensitivity of the cross-section in the region of the second maximum to the pion wave distortion. The second is the





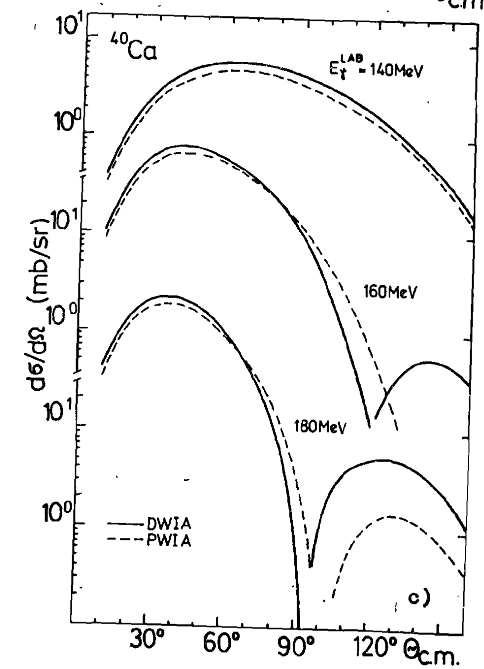




Fig. 5. Differential cross-sections for the neutral pion photoproduction off a - $^4$He, b - $^{12}$C and c - $^{40}$Ca in C version of the BL amplitude /10/ and with the off-shell formfactor $g_{\pi N}$ . Solid lines are the DWIA calculations; dashed lines are the PWIA calculations.

impossibility to describe the DWIA-differential cross-section by introducing the factor of distortion $\eta$ when the photon energy increases. When the photon energies are small, the main contribution is that of the $p$-wave partial amplitude. In this case one may approximately write

$$\frac{d\sigma}{d\Omega}(DWIA) \approx \eta \frac{d\sigma}{d\Omega}(PWIA) \cdot \qquad (28)$$

VI. Summary

The results of the present calculations make it possible to estimate the distorted pion wave effect as to be less than 25% in the threshold region of neutral pion photoproduction. It is in contradiction with the results of ref./5/.

We have achieved good agreement with the experimental data /1/ by using the full elementary amplitude including the pion-momentum nonlinear terms and the terms depending on nucleon momenta. The transferred momentum is not small $Q \sim m_\pi$; therefore, one cannot neglect the center-of-mass motion factor $F_{c.m.}(Q) = exp(Q^2R^2/6A)$ while using the shell model wave function for the nuclear ground state. All these effects give a 30%-increase of the PWIA cross-section of the $^{12}C(\gamma, \pi^0)$ -reaction and open an opportunity to describe the experimental data when distorted wave effects are small.

Since the distorted wave effects in the threshold region are small, the investigation of the elementary amplitude with the help of the nuclear data is simplified. The reliable scheme for this analysis is to use the PWIA and to take into account the pion-nuclear interaction in the final state by introducing the factor of distortion (see exp.(28)).



It is necessary to keep in mind the fact that reliability of such an approximation depends on photon energy and a concreate nucleus. For example, it is not correct for the $\pi^0$- photoproduction of $^{12}C$ with $E_\gamma^{LAB} \geqslant 170$ MeV (see fig.5 (b)).

The authors are grateful to Prof. J.M.Laget, who attracted their attention to the problem and Prof. H.Ströher, B.Schoch and N.De Botton for the discussion of the experimental situation. We would like also to note the fruitfulness of the remarks given by Glavanakov I.V. and Radutsky G.M.

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DWIA-подход в импульсном представлении для ($\gamma, \pi^0$) -реакции возле порога

Рассчитаны дифференциальные сечения и полные сечения процесса когерентного фоторождения нейтральных пионов на ядрах $^4$He, $^{12}$C и $^{40}$Ca с использованием импульсного приближения и метода искаженных волн в импульсном представлении. Показано, что эффекты искажения пионной волны в околопороговой области энергий фотона не превышают 25%. Хорошее согласие с экспериментом достигнуто благодаря использованию полной элементарной амплитуды, учитывающей как нелинейные по импульсу пиона члены, так и члены, зависящие от импульса пиона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Chumbalov A.A., Eramzhyan R.A., Kamalov S.S. E4-86-162

DWIA in the Momentum Space for ($\gamma, \pi^0$) Reaction near Threshold

The differential and total cross-sections of neutral pion photoproduction off $^4$He, $^{12}$C and $^{40}$Ca nuclei are calculated in the framework of the DWIA in the momentum space. It is shown that the inclusion of pion wave distortion gives a less than 25% increase of the total cross sections. A good agreement with experimental data is achieved by using the full elementary amplitude that includes the pion-momentum nonlinear terms and the terms depending on nucleon momenta.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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