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**SPINOR SYMMETRY
IN ODD Xe AND Ba NUCLEI**

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After the intriguing extension of the IBA-model to a mixed system of bosons and fermions^{/1,2/} a further promising development was initiated^{/3/} in the realm of the comparatively new applications of the graded Lie algebras theory to physical problems. The possibility of classifying multiplets of even-even and odd-even nuclei using the latter theory has already been shown for the Os-Ir-Pt-Au region (see, for instance, Refs.3,4). The even-even nuclei of Xe and Ba display features similar to those of Pt-nuclei^{/5/} and their description in terms of the IBA-model (O(6) - limit) is in good agreement with the experiment. So, it is reasonable to look for supersymmetry multiplets in the odd-even systems of the Xe-Ba-region, and as an initial step we shall present here three examples of the simplest "Spinor symmetry"^{/6/} in ^{131,133}Xe and ¹³⁵Ba. In the present application, a simplifying assumption is made that the quasiparticle states are built on a $j = 3/2$ level though the other three single particle orbits ($j = 1/2, 5/2, 7/2$) are not negligible, as follows also from our previous study of these nuclei^{/7/}. However, for a first test of the theory for the Xe-Ba region, one can admit the minor importance of those orbitals.

As is discussed in detail in^{/6/}, in the case the algebras of $SO^{(B)}(6)$, $SO^{(B)}(5)$, $SO^{(B)}(3)$, and $SO^{(B)}(2)$ are isomorphic to those of $SU^{(F)}(4)$, $Sp^{(F)}(4)$, $SU^{(F)}(2)$, and $SO^{(F)}(2)$, respectively, a spinor symmetry for the mixed system of bosons and fermions is possible, and the symmetry group of the Hamiltonian H is that of the direct product

$$U^{(R)}(6) \otimes U^{(F)}(4) \supset SO^{(B)}(6) \otimes SU^{(F)}(4) \quad (1)$$

$$\supset Spin(6) \supset Spin(5) \supset Spin(3) \supset Spin(2).$$

In terms of the eigenvalues of the quadratic Casimir operators of spinor groups Spin(6), Spin(5) and Spin(3), the Spin(6) invariant Hamiltonian has the form

$$H = -A C_6 + B C_5 + C C_3 \quad (2)$$

which contains the main terms necessary for calculating the excited state energies. The complete eigenvalue expression obtained in^{/6/} by using any combination of the linear and quadratic Casimir operators in the chain (1) reads

$$E(N, M, \Sigma, (\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), \nu_{\Delta}, J, M_J) =$$

$$= E_0 + E_1 N + E_2 N^2 + E_3 M + E_4 M^2 + E_5 MN -$$

$$\begin{aligned}
& - \frac{A}{4} 1 \Sigma (\Sigma + 4) - \frac{A}{4} 2 [\sigma_1 (\sigma_1 + 4) + \sigma_2 (\sigma_2 + 2) + \sigma_3^2] \\
& + \frac{B}{6} [\tau_1 (\tau_1 + 3) + \tau_2 (\tau_2 + 1)] + c J(J + 1).
\end{aligned}
\tag{3}$$

The compact analytic solution of the eigenvalue problem found in^{6/} provides the expressions for the state energies as well as for the transition probabilities, in which only the quantum numbers, which label the nuclear states, enter. These quantum numbers characterize the representations of various groups appearing in (1) and are discussed in detail in^{2,6/}. A typical classification of an odd-A nucleus in the discussed theoretical scheme is shown in Fig.1, where the excited states of ¹³⁵Ba are grouped in several multiplets with $(\tau_1, \tau_2) = (1/2, 1/2); (3/2, 1/2); (5/2, 1/2); \dots$ closed in broken lines, and the Spin(6) quantum numbers $(\sigma_1, \sigma_2, \sigma_3)$ are written at the top of the figure. Four parameters are needed to calculate the level energies: $A_1/4$, $A_2/4$, $B/6$, and C . They are obtained by using the experimentally known energies of the $3/2^+$ states found in the $(n, n' \gamma)$ reaction study of ¹³⁵Ba^{9/} and of 0^+ - states in ¹³⁴Ba^{10/}. These levels we suppose to belong to different σ_1 - multiplets ($\sigma_1 = N, N-2$ for $M = 0$ and $\sigma_1 = N+1/2, \dots$ for $M = 1$). Using only these four parameters, one can describe the energy-spectra of all six nuclei: the odd ones (^{131,133}Xe, ¹³⁵Ba) and the neighbour even-even ones (^{130,132}Xe, ¹³⁴Ba), with mean deviation $\Delta E = |E^{exp} - E^{calc}|/E^{exp}$ being within the range of (8-16)% and (15-20)%, respectively. A more refined treatment of the even-even nuclei can be found in Ref.5, where a cubic term is added in the Hamiltonian in order to obtain a precise description of the experimental spectra including the observed staggering of the states in the γ -bands. However, we try here to obtain a simultaneous description of both odd and even nuclei, so we have chosen the parameters presented in Table 1 (a), that give reasonable results besides for the even-even nuclei, for ¹³³Xe and ¹³⁵Ba, also, being slightly irrelevant for ¹³¹Xe. A little change of the parameter $B/6$ (Table 1, set (b)) gives already better results for ¹³¹Xe, too. All this seems to be a consequence of a strong cut-off of a single particle basis. We note also that the bad lacking of the experimental data (the reaction study, for exemple) makes the test very difficult, and compeled us for some states with $E > 1$ MeV to assume their characteristics I^π inbetween few experimentally determined possibilities (see Refs. 8,9). Such a problem one meets with the states 1225 and 1238 KeV (¹³⁵Ba), which could be classified by two different ways. The first one is present in Table 1 and the second is shown in Fig.1a, where these states have characteristics $(\sigma_1, \tau_1, J) = (7/2, 3/2, 5/2)$ and $(7/2, 5/2, 3/2)$, respectively. It is clear that in such cases besides the level spins, more information about the electromagnetic transitions would be decisive and necessary.

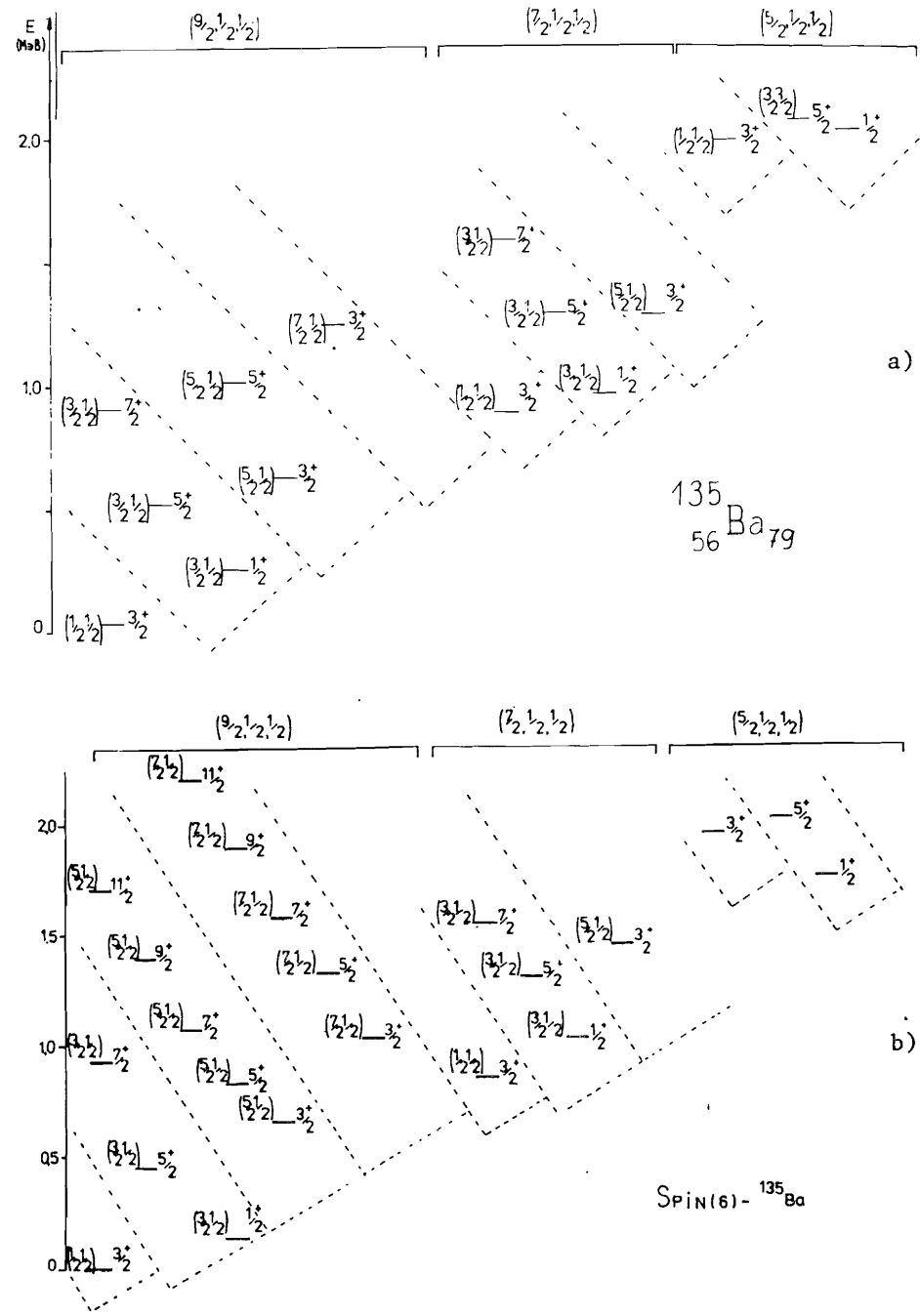


Fig.1.

Table 1

Comparison of the calculated and experimental energies in odd-Xe and Ba-nuclei and the deviation $\Delta E = (E^{exp} - E^{calc})/E^{exp}$, (%)

τ_i	J_i^{π}	^{135}Ba			^{133}Xe		^{131}Xe		
		E_{calc}^a (keV)	E^{exp} (keV)	ΔE (%)	E^{exp} (keV)	ΔE (%)	E_{calc}^b (keV)	E^{exp} (keV)	ΔE (%)
($\sigma_i = N+1/2; \Sigma = N$)									
1/2	$3/2^+$	0	0	-	0	-	0	0	-
3/2	$1/2^+$	210	220	4,5	262	19	60	80	25
	$5/2^+$	450	480	6	529	15	300	364	17
	$7/2^+$	660	874	24	875	24	510	636	19
5/2	$3/2^+$	720	587	22	680	6	360	404	11
	$5/2^+$	870	979	11	1052	17	510	722	29
	$7/2^+$	1080	-	-	1236	13	720	973	26
	$9/2^+$	1350	-	-	1386	2,5	990	971	2
7/2	$11/2^+$	1620	-	-	-	-	1260	1397	10
	$3/2^+$	1260	1213	4	911	38	630	699	14
	$5/2^+$	1410	1225	15	1298	8	780	994	21
	$7/2^+$	1620	-	-	1590	2	990	-	-
	$9/2^+$	1890	-	-	-	-	1260	1456	13
	$11/2^+$	2160	-	-	-	-	1530	1641	6,7
	$13/2^+$	2610	-	-	1743	50	1980	1584	25
	$15/2^+$						2430	2296	6
($\sigma_i = N-1/2; \Sigma = N$)									
1/2	$3/2^+$	720	854	16					
3/2	$1/2^+$	930	910	2					
	$5/2^+$	1170	1238	5,5					
	$7/2^+$	1380	1557	11					
5/2	$3/2^+$	1440	1871	17					
($\sigma_i = N+1/2; \Sigma = N-2$)									
1/2	$3/2^+$	1920	1940	1					
3/2	$1/2^+$	2130	1970	8					
	$5/2^+$	2370	2075	14					

In the calculations two sets of parameters are used:

- a) $1/4A_1 = 30$ keV; $1/4A_2 = 60$ keV; $1/6B = 60$ keV and $C = 30$ keV;
 b) $1/4A_1 = 30$ keV; $1/4A_2 = 60$ keV; $1/6B = 30$ keV and $C = 30$ keV.

Table 2.

Summary of calculated and measured values of $B(E2)$

$J_i^{\pi} \rightarrow J_f^{\pi}$	^{131}Xe		^{135}Ba		$B(E2)$ (theory) (e^2b^2)		
	E_f (keV)	$B(E2)^{exp}$ (e^2b^2)	E_f (keV)	$B(E2)^{exp}$ (e^2b^2)	Spin(6) pres.work	^{131}Xe IBFM 11	^{135}Ba PVM 12
$1/2_1^+ \rightarrow 3/2_1^+$	80	0.0039(5)	220	0.0192(14)	0.1	0.009	0.019
$5/2_1^+ \rightarrow 3/2_1^+$	364	0.10(1)	480	0.116(5)	0.1*	0.080	0.110
$7/2_1^+ \rightarrow 3/2_1^+$	637	0.081(6)	874	0.081(4)	0.1	0.081	0.091
$5/2_1^+ \rightarrow 1/2_1^+$	284	0.030(5)			0.013	0.028	
$7/2_1^+ \rightarrow 5/2_1^+$	272	0.005($^{+0}_{-4}$)			0.019	0.003	0.067
$3/2_2^+ \rightarrow 3/2_1^+$	405	0.057(4)	588	0.069(6)	forb.	0.065	0.065
$1/2_2^+ \rightarrow 3/2_1^+$	565	0.048(4)	909	0.044(6)	forb.	0.070	0.019
$3/2_3^+ \rightarrow 3/2_1^+$	699	0.027(2)	855	0.029(4)	forb.	0.022	
$5/2_2^+ \rightarrow 1/2_1^+$	643	0.068(9)			0.057	0.071	
$5/2_2^+ \rightarrow 3/2_1^+$	722	0.013(1)			forb.	0.019	
$7/2_2^+ \rightarrow 3/2_1^+$	974	0.0050(5)			forb.	0.0002	

Additionally to the level energies the E2-transition probabilities in ^{131}Xe and ^{135}Ba are calculated and compared both with measured values and the calculated ones in the IBFM^{12/} and PVC^{13/} models, Table 2. Using a one-body E2-transition operator of the form

$$T(E2) = \tilde{q}_2^2 G^{(2)} \quad (4)$$

where $G^{(2)}$ is a generator of the group Spin(6), in^{6/} all the expressions for B(E2)- values are obtained, which depend only on the quantum numbers N and τ_1 . The parameter \tilde{q}_2^2 , which appears in the expressions referred is the same for the two odd nuclei and is obtained by normalizing all B(E2)- values by this of $5/2_1^+ \rightarrow 3/2_1^+$ transition in ^{131}Xe , marked by asterisk in Table 2. The calculated B(E2)-values are in agreement with the measured ones for the transitions with $\Delta\tau_1 = 1$. The main disagreement appears for the experimentally observed transitions with $\Delta\tau_1 > 1$, which are forbidden in the model and for B(M1), which are known as very sensitive to admixtures of the single particle levels, which in the present calculations are reduced to only d3/2.

However, the present attempt to apply "Spinor" symmetry to the odd nuclei of Xe and Ba is unexpectedly promising. We believe that further extension of the single particle basis, corroborated with more experimental data, should enable one to check every detail of this new theoretical scheme and to obtain a quantitative description of the discussed nuclei.

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Михайлова М.М.

E4-86-717

Спинорная симметрия в нечетных ядрах Хе и Ва

Проверяется возможность интерпретировать ядра $^{131,133}\text{Xe}$ и ^{135}Ba , классифицируя их возбужденные состояния с помощью квантовых чисел, характерных для "спинорной" симметрии в смешанных системах бозонов и фермионов. Рассчитанные энергии уровней и вероятности E2-переходов находятся в качественном согласии с экспериментом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1986

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E4-86-717

Spinor Symmetry in Odd Xe and Ba Nuclei

The possibility of interpreting the nuclei of $^{131,133}\text{Xe}$ and ^{135}Ba by classifying their excited states using the quantum numbers, characteristic for the "spinor" symmetry of a mixed system of bosons and fermions, is tested. The level-energies and E2-transition probabilities are calculated in qualitative agreement with the experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1986