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NEGATIVE-ENERGY QUANTA

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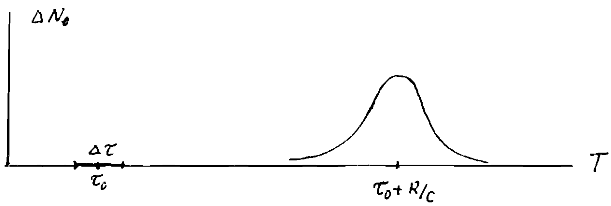
1. Introduction

In this paper the following effect is discussed. An atom $\mathcal{D}$ (detector) was initially ($t=0$) in the ground state 0, E_0 being its energy. Then in an interval (t_1, t_2) it is excited (e.g. by a laser beam) up to state 1 having an energy E_1. Another atom A is at a distance R from $\mathcal{D}$ and is effectively "turned on" in a time interval $\Delta\tau$, τ_0 being the interval center. As to the realization of this "turning on", see subsections 3.2 and 3.3 below. At a moment $t>t_2$ one measures the probability of finding $\mathcal{D}$ in the ground state 0. We are interested in the change of this probability induced by A at $0<\tau_0<t_1<t_2<t$. The exact definition of the induced probability (denoted by $\Delta N_0(t)$) is given in sections 2 and 3. $\Delta N_0(t)$ is here calculated by using the Heisenberg picture of the standard quantum electrodynamics. The result is represented in the Figure as the dependence of ΔN_0 upon the center T



of the interval (t_2, t), the interval being moved as a whole along the time axis. It is the relativistic retardation of the A action upon $\mathcal{D}$ that is the most important property of ΔN_0 for our purpose. Atom A does not absorb the radiation emitted by $\mathcal{D}$. It plays, instead, the active rôle: A creates initially the cause of the subsequent $\mathcal{D}'$ de-excitation.

The structure of the formulae for ΔN_0 (see, e.g. eq.(18) below) suggests the following interpretation of the effect. The excited atom $\mathcal{D}$ absorbs in the interval (t_2, t) something which has an energy x and has been emitted earlier by A. After this $\mathcal{D}$ turns out to be in the state 0. From the energy conservation equation $E_1 + x = E_0$ one has $x = E_0 - E_1 < 0$, i.e. that something has a negative energy.

Let us try to explain the effect using the notion of the photon. Atom $\mathcal{D}$ deexcites emitting in the interval (t_2 , t) a photon of energy $\Delta = E_1 - E_0$. But ΔN_0 is the probability induced by A , it vanishes if A is absent. Therefore in the effect the photon must interact with A . But A is "turned off" when $\mathcal{D}$ is emitting the photon (remind that $0<\tau_0<t_1<t_2<t$). The atom A can interact with this photon only if the photon moves backward in time. We cannot consider this possibility in the frame-work of the used standard QED, because the latter allows only retarded solutions.

One may try to interpret the effect without using the words "negative energy", e.g. as follows: A emits an electromagnetive field which later absorbs the $\mathcal{D}$ excitation. But let us try to interpret the field in terms of quanta. It cannot consist of photons, because $\mathcal{D}$ can only increase its energy absorbing the photon. So, we are brought again to the notion "quantum having the negative energy". In what follows I shall use the term "quen", the abbreviation from French "le quantum de l'énergie negative".

The paper is organized as follows. The problem formulation and its calculations are illustrated at first in sect.2 by a simplified model: the external current substitutes the atom A and QED without the Lorentz subsidiary condition is used. Section 3 is devoted to the discussion and elimination of the deficiencies of the simplified model. The section contains no calculations but only results. In the concluding section 4 I give main requirements which the appropriate experiment must meet, argue the existence of the quens of other (nonelectromagnetic) fields, discuss the existence of free quens and consider the role of the quen in the particle interpretation of quantized fields.

2. Negative energy transmission from external current to excited atom

2.1. Let us begin with the definition of the induced probability ΔN_0 . At a moment t one must measure the probability that $\mathcal{D}$ is in the state 0 . Stress that one must not detect photons or the state of atom A . The part of this inclusive probability which is due to the presence of atom A (of the external current in this section) will be defined as a difference of two quantities. In this section the first is

$$\sum_n |\langle d_0^+ \varphi_n , \mathcal{U}_J(t,0)\varphi_i\rangle|^2 . \qquad (1)$$

Here φ_i is the initial state vector: "$\mathcal{D}$ is in the state 1 , no photons" (for simplicity $\mathcal{D}$ is let to be now in an excited state at the moment $t=0$; the $\mathcal{D}$ preparation in state 1 during (t_1, t_2) is considered further in subsect 2,7); $\mathcal{U}_J(t,0)$ is the evolution operator when the external current is turned on at the moment $t=0$. The vectors φ_n constitute a complete set of states (including the vacuum), $d_0^+\varphi_n$ is the state which differs from φ_n by the presence of one more electron in the $\mathcal{D}$ state 0. Of course, some of $\langle d_0^+\varphi_n, \mathcal{U}_J\varphi_i\rangle$ vanish, e.g. if φ_i and $d_0^+\varphi_n$ differ by the electric charge. One can rewrite eq.(1) as

$$\sum_n \langle \mathcal{U}_J\varphi_i, d_0^+\varphi_n\rangle\langle d_0^+\varphi_n, \mathcal{U}_J\varphi_i\rangle = \sum_n \langle \mathcal{U}_J\varphi_i, d_0^+\varphi_n\rangle\langle \varphi_n, d_0\mathcal{U}_J\varphi_i\rangle =$$
$$= \langle \mathcal{U}_J(t,0)\varphi_i, d_0^+ d_0\, \mathcal{U}_J(t,0)\varphi_i\rangle = \langle \varphi_i, d_{J0}^+(t)\, d_{J0}(t)\, \varphi_i\rangle . \qquad (2)$$

The completeness $\sum_n |\varphi_n\rangle\langle\varphi_n| = 1$ is used; $d_{J0}(t)$ denotes the Heisenberg operator $\mathcal{U}_J^+(t,0)\, d_0\, \mathcal{U}_J(t,0)$.

The second quantity (subtrahend) differs from (1) only in one respect: the external current is absent. The evolution operator is denoted by $\mathcal{U}(t,0)$ in this case. So,

$$\Delta N_0(t) = \langle \mathcal{U}_J\varphi_i, d_0^+ d_0\, \mathcal{U}_J\varphi_i\rangle - \langle \mathcal{U}\varphi_i, d_0^+ d_0\, \mathcal{U}\varphi_i\rangle . \qquad (3)$$

This definition corresponds to the subtraction of the background which is used by experimentalists to obtain the part of the measured quantity which is due to the investigated cause. The subtrahend in eq.(3) is simply the probability of the $\mathcal{D}$ spontaneous transition from 1 to 0 . I designate (3) as $\Delta N_0(t)$ because $\langle \mathcal{U}\varphi_i, d_0^+ d_0\, \mathcal{U}\varphi_i\rangle$ is also the expectation value of the operator $N_0 = d_0^+ d_0$ of the number of electrons in state 0 .

2.2. One can calculate (3) using the known perturbation expansion of the interaction-picture evolution operator $T\exp[-i\int_0^t dt'\, H_{int}(t')]$. But a simpler way is to calculate (3) by finding the Heisenberg operators $d_{J0}(t) = \mathcal{U}_J^+ d_0\, \mathcal{U}_J$ and $d_0(t) = \mathcal{U}^+ d_0\, \mathcal{U}$ by means of a perturbation theory. The connection of $d_0(t)$ with the Heisenberg electron-positron field operator $\psi(\vec{x},t) = \mathcal{U}^+\psi(\vec{x},0)\,\mathcal{U}$ is given by the known expansion

$$\psi(\vec{x},t) = S_n\, u_n(\vec{x})\, d_n(t) + S_m\, v_m(\vec{x})\, b_m^+(t) . \qquad (4)$$

Here S_n means summation over a discrete part of the spectrum of the atom $\mathcal{D}$ electron and integration over its continuous part; $b_m^+(t)$ denotes the Heisenberg positron creation operator. Using the orthogonality of spinors u_n and v_m one gets from eq.(4)

$$d_o(t) = \int d^3x\ u_o^+(\vec{x})\ \psi(\vec{x}, t)\,. \tag{5}$$

The QED equations for Heisenberg operators ψ and A_μ are well known (e.g. see § 22 in /1/). Their integral forms are called the Källen-Yang-Feldman eqs. (e.g. see /2/). If the external current J_μ is present, the eqs. are

$$\psi_J(x) = \psi^f(x) - ie\int d^4y\ S^R(x,y)\ A_{J\mu}(y)\gamma_\mu\psi_J(y)$$

$$A_{J\mu}(x) = A_\mu^f(x) + \int d^4y\ \mathcal{D}^R(x-y)\left[ie\bar{\psi}_J(y)\gamma_\mu\psi_J(y) + J_\mu(y)\right]. \tag{6}$$

Here $x \equiv \{\vec{x}, x_o\}$, $x_o = t$; $y \equiv \{\vec{y}, y_o\}$. The integration over y_o runs from $y_o = 0$ to $y_o = x_o$, and therefore the operators $\psi_J(\vec{x}, 0)$ and $A_{J\mu}(\vec{x}, 0)$ coincide with the free operators $\psi^f(x)$ and $A_\mu^f(x)$, taken at the point $x_o = 0$ [1]. The latter can be written in terms of the Schrödinger creation-annihilation operators. To solve (6) means to find how $\psi_J(x)$ (and $d_{Jo}(t)$, see eq.(5)) is expressed in terms of the Schrödinger operators. The expression allows one to calculate (3) because one knows how the Schrödinger creation-annihilation operators act on the initial vector φ_i (e.g. see eq.(17) below).

Note that $\psi^f(x)$ satisfies the quasifree equation which contains potentials $U_\mu(\vec{x})$ binding the atom $\mathcal{D}$ electron

$$\left[\gamma_\mu\partial_\mu + m + i\gamma_\mu U_\mu(\vec{x})\right]\psi^f(\vec{x}, x_o) = 0\,. \tag{7}$$

The retarded function $S^R(x,y)$ satisfies a similar eq. which however, has $[-\delta^{(4)}(x-y)]$ in its r.h.s. The general solution of eq.(7) can be written as

$$\psi^f(\vec{x}, x_o) = S_n\ u_n(\vec{x})e^{-iE_n x_o} d_n + S_m\ v_m(\vec{x})e^{iE_m x_o} b_m^+\,, \tag{8}$$

where d_n is the Schrödinger electron annihilation operator.

[1] Equations (6) differ from the corresponding differential ones in three respects: 1) eqs. (6) contain initial conditions, 2) eqs. (6) are equivalent to the latter only when $x_o \geqslant 0$; 3) only retarded solutions are taken into account in eqs. (6) while the differential ones allow also advanced solutions.

If there is no external current, then one lets $J_\mu = 0$ in eqs.(6) and the corresponding operators will be denoted by $\psi(x)$ and $A_\mu(x)$.

To solve integral eqs. for $\psi_J, A_{J\mu}, \psi, A_\mu$ I insert in the eqs. the expansions $\psi_J(x) = \Sigma_\kappa e^\kappa \psi_J^{(\kappa)}(x)$ etc. and equate the terms with the same power of e . The external current is treated exactly: I do not consider J_μ being proportional to e .

One obtains in the zeroth approximation

$$\psi_J^{(o)}(x) = \psi^{(o)}(x) = \psi^f(x)\ ; \quad A_{J\mu}^{(o)} = A_\mu^f + \int \mathcal{D}^R(x-y)\ J_\mu(y)\ d^4y\,. \tag{9}$$

As to the following approximations I shall need only $\psi_J^{(1)}(x)$ and $\psi^{(1)}(x)$

$$\psi^{(1)}(x) = -i\int d^4y\ S^R(x,y)\ \gamma_\mu\ \psi^f(y) A_\mu^f(y), \tag{10}$$

$$\psi_J^{(1)}(x) = \psi^{(1)}(x) - i\int d^4y\ S^R(x,y)\,\gamma_\mu\,\psi^f(y)\int d^4z\ \mathcal{D}^R(y-z)\,J_\mu(z)\,. \tag{11}$$

Let us show that the first nonvanishing approximation for the quantity under calculation

$$\Delta N_o(t) = \left\langle \varphi_i\,,\ \left[d_{Jo}^+(t)\,d_{Jo}(t) - d_o^+(t)\,d_o(t)\right]\varphi_i\right\rangle \tag{12}$$

(see eqs.(2) and (3)) is of the order e^2 . Insert into eq. (12) the expansion $d_{Jo}(t) = d_{Jo}^{(o)}(t) + e\,d_{Jo}^{(1)}(t) + e^2 d_{Jo}^{(2)}(t) + \ldots$ and the analogous expansion for $d_o(t)$. Let $\varphi_i = d_i^+\Omega$, Ω being a no-particle state. Taking into account the eq. $d_{Jo}^{(o)}(t) = d_o^{(o)}(t) = d_o\exp(-iE_o t)$ see eqs.(5),(9),(8) and the eq. $d_o d_i^+\Omega = 0$, we ascertain that $d_{Jo}^{(2)}(t)$ and $d_o^{(2)}(t)$ do not contribute to $\Delta N_o(t)$ in the order e^2 :

$$\Delta N_o(t) = e^2\left\langle \varphi_i\,,\ \left[d_{Jo}^{(1)+}(t)\,d_{Jo}^{(1)}(t) - d_o^{(1)+}(t)\,d_o^{(1)}(t)\right]\varphi_i\right\rangle. \tag{13}$$

Using eqs.(5), (10) and (11) one gets

$$d_o^{(1)}(t) = e^{-iE_o t}\int d^3y\int_0^t dy_o\ \bar{u}_o(\vec{y})e^{iE_o y_o}\gamma_\mu\,\psi^f(y)\,A_\mu^f(y), \tag{14}$$

$$d_{Jo}^{(1)}(t) = d_o^{(1)}(t) + e^{-iE_o t}\int d^3y\int_0^t dy_o\ \bar{u}_o(y)\,\gamma_\mu\,\psi^f(y)\int d^4z\ \mathcal{D}^R(y-z)\,J_\mu(z)\,. \tag{15}$$

To obtain eqs. (14), (15), I substituted S for S^R, which is permissible if the upper limit $y_o = t$ of integration over y_o is explicitly written in eq. (10). Further I used representation /3/

$$-iS(x,y) = S_n\ u_n(x)\bar{u}_n(y) + S_m\ v_m(x)\ \bar{v}_m(y)$$

$$u_n(x) \equiv u_n(\vec{x})\exp(-iE_n x_o)\,, \qquad \bar{u} \equiv u^+\gamma_o \tag{16}$$

and orthonormality of the spinors u_n and v_m.

Now insert eqs. (14) and (15) into eq. (13); use further that $\langle \varphi_1 | A_\mu | \varphi_2 \rangle$ is zero if φ_1 and φ_2 are states with an equal number of photons[2], finally use the equation

$$\langle \varphi_i , \bar{\psi}^f(y) \psi^f(y') \varphi_i \rangle = \langle d_i^+ \Omega , \bar{\psi}^f(y) \psi^f(y') d_i^+ \Omega \rangle =$$
$$= \bar{u}_1(\vec{y}) e^{iE_1 y_0} u_1(\vec{y}') e^{-iE_1 y_0'} + S_m \bar{v}_m(\vec{y}) e^{-iE_m y_0} v_m(\vec{y}') e^{iE_m y_0'} . \tag{17}$$

As a result, one gets

$$\Delta N_0(t) = \Big| \sum_\mu \int d^3y \int_0^t dy_0\, e\, \bar{u}_0(\vec{y}) e^{iE_0 y_0} \gamma_\mu u_1(\vec{y}) e^{-iE_1 y_0} \int d^4 z\, \mathcal{D}^R(y-z) J_\mu(z) \Big|^2 +$$
$$+ S_m \Big| \sum_\mu \int d^3y \int_0^t dy_0\, e\, \bar{u}_0(\vec{y}) e^{iE_0 y_0} \gamma_\mu v_m(\vec{y}) e^{iE_m y_0} \int d^4 z\, \mathcal{D}^R(y-z) J_\mu(z) \Big|^2 . \tag{18}$$

I shall not interpret the term $S_m |\ \ |^2$ in the eq. because it turns out to be small and will be neglected (see below).

Let us analyse and evaluate the obtained expression (18) for $\Delta N_0(t)$.

2.3. I begin with the first term of the r.h.s. of eq.(18). It is the squared modulus of $\sum_\mu I_\mu$, where

$$I_\mu = e \int d^3y \int d^3z\, \bar{u}_0(\vec{y}) \gamma_\mu u_1(\vec{y}) \int_0^t dy_0 \int_0^{y_0} dz_0\, e^{i(E_0 - E_1) y_0} \mathcal{D}^R(y-z) J_\mu(z) . \tag{19}$$

The current $J_\mu(z) = J_\mu(\vec{z}, z_0)$ being a function of z_0 can be represented as the Fourier integral

$$J_\mu(\vec{z}, z_0) = \int_{-\infty}^{+\infty} d\omega\, \tilde{J}_\mu(\vec{z}, \omega) e^{-i\omega z_0} . \tag{20}$$

Use the known representation

$$\mathcal{D}^R(y-z) \equiv \mathcal{D}^R(\vec{y}-\vec{z}, y_0 - z_0) = \frac{1}{4\pi |\vec{y}-\vec{z}|} \delta\left(|\vec{y}-\vec{z}|/c - (y_0 - z_0) \right) \tag{21}$$

and calculate the integral $\int dy_0 \int dz_0 ...$ in eq.(19), using the notation $\tau \equiv |\vec{y}-\vec{z}|/c$ and $\Delta \equiv E_1 - E_0$[3].

2) Of course I must use a representation of A_μ in terms of photon creation-annihilation operators. It is important to take here into account the Lorentz subsidiary condition, which allows one to eliminate the nonphysical longitudinal and scalar photons and to consider φ_i as a state with zero number of transversal photons (see subsect 2.6).

3) The integral over z_0 is calculated by means of the substitution $z_0 = y_0 - \tau$ under the condition that ($y_0 - \tau$) is inside the integration interval ($0, y_0$). The condition is taken into account by using the function $\theta(y_0 - \tau)$ (see the middle of eq.(22)).

$$Y = \frac{1}{4\pi\tau} \int_0^t dy_0\, e^{i(E_0 - E_1) y_0} \int_0^{y_0} dz_0\, \delta(\tau - y_0 + z_0) e^{-i\omega z_0} =$$
$$= \frac{1}{4\pi\tau} e^{i\omega\tau} \int_0^t dy_0\, \theta(y_0 - \tau) e^{-iy_0(\omega + \Delta)} =$$
$$= e^{i\omega\tau} e^{-i(\omega+\Delta)(t+\tau)/2} \theta(t-\tau) \frac{t-\tau}{4\pi\tau} \frac{\sin(\omega+\Delta)(t-\tau)/2}{(\omega+\Delta)(t-\tau)/2} \tag{22}$$

I let in the following that $\mathcal{D}$ is localized in a region $V_\mathcal{D}$ near the origin and the current J_μ is localized in V_J, the distance R between $V_\mathcal{D}$ and V_J being much greater than $V_\mathcal{D}$, V_J dimensions. Therefore, $\tau \equiv |\vec{y}-\vec{z}|/c \cong R/c$. Let t be such a moment that $t - \tau \cong t - R/c > 0$ and $t - R/c >> 1/\Delta$ (of course also $R >> 1/\Delta$). Then $|Y|$ is maximal if $\omega = -\Delta$: one has $|Y| = (t-\tau)/4\pi\tau$ if $\omega + \Delta = 0$ and

$$|Y| \leq \frac{1}{2\pi\tau} \frac{1}{\omega+\Delta} << \frac{t-\tau}{4\pi\tau} \tag{23}$$

if $(\omega+\Delta)(t-\tau) >> 1$. So, it is the Fourier-components $\tilde{J}_\mu(\vec{z}, \omega)$ with $\omega \cong -\Delta$ that give a main contribution to I_μ, if $t - R/c >> 1/\Delta$. This reflect the approximate energy conservation in the considered effect.

Let us choose such a current in the following: $J_\mu(\vec{z}, z_0) = 0$ if z_0 is out of the interval (τ_1, τ_2), $0 < \tau_1 < \tau_2 < R/c$ and

$$J_\mu(\vec{z}, z_0) = K_\mu(\vec{z}, -\Delta) e^{iz_0\Delta} + K_\mu^*(\vec{z}, -\Delta) e^{-iz_0\Delta} , \quad z_0 \in (\tau_1, \tau_2) . \tag{24}$$

Note that J_μ must be real for the interaction $J_\mu A_\mu$ to be Hermitian. The support of the function $\tilde{J}_\mu(\vec{z}, \omega)$, the Fourier representation of (24), is in the vicinities of $\omega = -\Delta$ and $\omega = +\Delta$. The second term in eq. (24) gives a much less contribution to I_μ and $\Delta N_0(t)$ than the first one because it does not "conserve the energy" and gives in $|Y|$ the contribution $\sim 1/\tau\Delta$ if $t - \tau >> 1/\Delta$, cf. (23). $S_m |\ \ |^2$ contributes to $\Delta N_0(t)$ still less than the contribution just discussed. Indeed, one has in $S_m |\ \ |^2$ the integral $\int dy_0 \exp i(E_0 + E_m) y_0$ as compared to $\int dy_0 \exp i(E_1 - E_0) y_0$, see eq.(19). Here $E_0 \cong m_e$ and $E_m \geq m_e$, m_e being the electron mass. The result is that $|Y|$ is here of the order $1/\tau m_e$.

2.4. Remind that the current must be "turned off" when $\mathcal{D}$ is detected to be in the ground state (see the Introduction). Therefore one has to have $\tau_2 < R/c$. In this case the calculation of $\int dy_0 \int dz_0 \dots$ (see eq. (19)) when the current J_μ is given by eq. (24) results in

$$Y(z) \cong \frac{1}{4\pi z} e^{-i\Delta z} \, \theta\left(\min(t, \tau_2 + z) - (\tau_1 + z)\right) \int_{\tau_1 + z}^{\min(t, \tau_2 + z)} dy_0 =$$

$$= \frac{1}{4\pi z} e^{-iz\Delta} \cdot \begin{cases} 0 & t < \tau_1 + z \\ t - (\tau_1 + z) & \tau_1 + z < t < \tau_2 + z \\ (\tau_2 - \tau_1) & t > \tau_2 + z \end{cases} \tag{25}$$

(see footnote 3). The contribution of the second term of eq. (24) is neglected and this is justified if $t - (\tau_1 + z) \gg 1/\Delta$ and $(\tau_2 - \tau_1) \gg 1/\Delta$ (see above). Therefore, the inequality $t - (\tau_1 + z) > 0$ in the second line of the r.h.s. of eq. (25) must be understood in the sense $t - (\tau_1 + z) \gg 1/\Delta$.

Reming that z in eq. (25) means $|\vec z - \vec y|/c$ and that $\vec z \cong \vec R$ and $\vec y \cong 0$ (see the text after eq. (22)). More exactly, $\vec z = \vec R + \vec z'$, $|\vec z'| \le a_J$, and $|\vec y| \le a_{\mathcal D}$, where a_J is the dimension of V_J and $a_{\mathcal D}$ is the atom $\mathcal D$ dimension. Naturally one can use the fact that $a_J, a_{\mathcal D} \ll R$ (and therefore $z = |\vec R + \vec z' - \vec y|/c \cong R/c$) in order to evaluate I_μ. When $\mu = m = 1, 2, 3$ one can simply substitute $Y(R/c)$ for $Y(z)$. After this I_m divides into three factors: 1) $Y(R/c)$, 2) $e \int d^3y \, \bar u_0(\vec y) \gamma_m u_1(\vec y)$ and 3) $\int d^3z \, K_m(\vec z, -\Delta)$. One can show that

$$\int d^3y \, \bar u_0(\vec y) \gamma_m u_1(\vec y) = (E_0 - E_1) d_{01}^m \; ; \quad d_{01}^m = \int d^3x \, u_0^+(\vec x) \, x_m \, u_1(\vec x) \tag{26}$$

$$\int d^3z \, K_m(\vec z, -\Delta) = i\Delta \, d_J^m \; ; \quad d_J^m = \int d^3x \, x_m \, K_0(\vec x, -\Delta) \tag{27}$$

Here d_{01}^m is the dipole moment of the transition $1 \to 0$; analogously, d_J^m is the dipole moment of the external charge density[4]. So, we get in the described approximation (which can be called the dipole approximation)

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4) Current operator $j_\mu^{(0)} = ie : \bar\psi^+ \gamma_\mu \psi^+ :$ satisfies the eq. $\partial_\mu j_\mu^{(0)} = 0$. Hence the eq. $\int d^3x \, x_m \partial_0 j_0^{(0)} = -\int d^3x \, x_m \, \mathrm{div} \, \vec j^{(0)}$ follows. The matrix element of the l.h.s. of this eq. between $\langle d_0^+ \Omega|$ and $|d_1^+ \Omega\rangle$ is equal to $i(E_0 - E_1) e \, d_{01}^m$. The matrix element of the r.h.s. can be reduced to $ie \int d^3x \, \bar u_0(\vec x) \gamma_m u_1(\vec x)$ by integration by parts. Equation (27) can be proved analogously starting with $\partial_0 \int d^3x \, J_0 = -\int d^3x \, x_m \, \mathrm{div} \, \vec J$.

$$\sum_m I_m \cong -ie \, \Delta^2 \, (\vec d_{01} \cdot \vec d_J) \, Y(R/c). \tag{28}$$

In the case $\mu = 4$ the approximation $Y(z) \to Y(R/c)$ fails: it results in $I_4 = 0$ due to the orthogonality of the spinors u_0 and u_1 and due to $\int d^3z \, K_0(\vec z, z_0) = 0$ which is true for all z_0. The latter equation follows from $Q = \int d^3z \, J_0(\vec z, z_0) = 0$: the conserving total charge Q must be zero also in the interval (τ_1, τ_2). So, in this case $Y(z) = Y(|\vec R + \vec z' - \vec y|/c)$ has to be expanded into Taylor series in $z'_m - y_m$ powers. But $Y(z)$, see eq. (25), has discontinuous derivatives and this expansion needs a reservation. It is permissible to let $Y(|\vec R + \vec z' - \vec y|) = 0$ at all $|\vec z'| < a_J$, $|\vec y| < a_{\mathcal D}$ if $t < \tau_1 + R - a_J - a_{\mathcal D}$ see eq. (25). (here and in the following I let $c = 1$). Analogously, Y may be considered to be a smooth function, equal to $\exp(-ir\Delta)[t - \tau_1 - |\vec R + \vec z' - \vec y|]/4\pi z$ at all essential values of $\vec z'$ and $\vec y$ if $\tau_1 + R + a_J + a_{\mathcal D} < t < \tau_2 + R - a_J - a_{\mathcal D}$. At last, $Y = \exp(-iz\Delta)[\tau_2 - \tau_1]/4\pi z$ at all essential $\vec z'$ and $\vec y$ if $t > \tau_2 + R + a_J + a_{\mathcal D}$. If t belongs to above-mentioned intervals, one may expand Y in these intervals and to evaluate I_4 there. One cannot evaluate I_4 in such a manner only in microscopically small time intervals outside the above-mentioned ones.

For instance, at $\tau_1 + R + a_J + a_{\mathcal D} < t < \tau_2 + R - a_J - a_{\mathcal D}$ one has

$$\frac{t - \tau_1 - |\vec R + \vec z' - \vec y|}{|\vec R + \vec z' - \vec y|} e^{-i|\vec R + \vec z' - \vec y|\Delta} = \frac{t - \tau_1 - R}{R} e^{-iR\Delta} + \sum_m (z'_m - y_m) \frac{\partial}{\partial R_m} \left(\frac{t - \tau_1}{R} - 1 \right) e^{-iR\Delta} +$$

$$+ \frac{1}{2} \sum_{i,j} (z'_i - y_i)(z'_j - y_j) \left[-\Delta^2 \frac{R_i R_j}{R^2} \left(\frac{t - \tau_1}{R} - 1 \right) + \dots \right] e^{-iR\Delta}/R + \dots \tag{29}$$

In the square brackets of the r.h.s. I omit all terms which tend to zero more rapidly as $R \to \infty$ than the written one (it is the derivative of $\exp(-iR\Delta)$ which gives a main contribution). For the reasons discussed above after eq. (28) both the first and second term of expansion (29) do not contribute to I_4. Only $z'_i y_j$ from the third term contribute. So, one gets in the discussed time interval

$$I_4 \cong ie \, e^{-iR\Delta} \Delta^2 (\vec n \, \vec d_{01})(\vec n \, \vec d_J)(t - \tau_1 - R)/4\pi R \, , \quad \vec n \equiv \vec R/R. \tag{30}$$

Finally, one has

$$\sum_\mu I_\mu = \sum_m I_m + I_4 \cong -ie \, \Delta^2 \left[\vec d_{01} \vec d_J - (\vec n \vec d_{01})(\vec n \vec d_J) \right] Y(R), \tag{31}$$

where $Y(R)$ is given in eq. (25). The inequalities in this eq. mast be understood in the sense described above, see the text after eqs.(25) and (22).

2.5 So, we obtain that $\Delta N_0'(t) \cong |\Sigma_\mu I_\mu|^2$, where $\Sigma_\mu I_\mu$ is given by eq. (31).

$$\Delta N_0(t)=e^2\Delta^4\left|(\vec{d}_{01}^{\perp}\cdot\vec{d}_J^{\perp})\right|^2\frac{1}{(4\pi)^2R^2}\cdot\begin{cases}0 & t<\tau_1+R\\ [t-(\tau_1+R)]^2 & \tau_1+R<t<\tau_2+R\\ [\tau_2-\tau_1]^2 & t>\tau_2+R\end{cases}\qquad(32)$$

Here $c=1$, $\vec{d}^{\perp}$ is the $\vec{d}$ component which is perpendicular to $\vec{R}$: $(\vec{d}_{01}^{\perp}\,\vec{d}_J^{\perp})=(\vec{d}_{01}\,\vec{d}_J)-(\vec{n}\,\vec{d}_{01})(\vec{n}\,\vec{d}_J)$. The meaning of the inequalities in eq. (32) is described above.

Let us consider the induced probability $\Delta N_1(t)$ to find atom $\mathcal{D}$ in state 1 if initially $\mathcal{D}$ was in the ground state 0, the probability being induced by the same external current (24). Exactly the same numerical value (32) can be obtained for $\Delta N_1(t)$ [5]. The induced transition $0\to 1$ can be naturally explained as follows: the current emits a photon of energy $\sim\Delta$, this photon travels to $\mathcal{D}$ and is absorbed by $\mathcal{D}$. One can show that it is the positive-energy part $\mathcal{D}_+(y-z)$ of the function $\mathcal{D}(y-z)$ (that enters into an equation of type (19)) that gives a main contribution in this case. It is the negative-energy part $\mathcal{D}_-(y-z)$ of $\mathcal{D}(y-z)$ which gives a main contribution to the r.h.s. of eq. (19). So, one can explain the induced transition $1\to 0$ as follows: the current emits a quantum of negative energy $-\Delta$, this quantum travels to $\mathcal{D}$ and absorbs its excitation[6].

The induced probability $\Delta N_1(t)$ depends on dipole moment components perpendicular to $\vec{R}$ in the same manner as $\Delta N_0(t)$ does, see eq.(32). The fact can be explained by the transversality of the photon, transferred from J to $\mathcal{D}$. Indeed, the photon can be emitted and absorbed only by those $\vec{d}_J$, $\vec{d}_{01}$ components which are perpendicular to the direction $\vec{n}=\vec{R}/R$ of the photon propagation. Analogously, one can consider the presence of $\vec{d}_{01}^{\perp}$ and $\vec{d}_J^{\perp}$ in eq.(32) for $\Delta N_0(t)$ as an evidence for transversality of the electromagnetic quen.

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[5] Of course, the "background" probability is far less in this case as compared to the probability of spontaneous radiation which is a "background" in the effect considered in this paper.

[6] I define the energy of the quen as $-\Delta=E_0-E_1$. No other definition is available. As to the photon, it can be free (unlike the quen, see section 4 below) and its energy can be defined as an eigenvalue of the operator $\int d^3x\,(\vec{E}^2+\vec{H}^2)$.

2.6. The calculation of $\Delta N_0(t)$ presented in subsect 2.2 did not take into account the Lorentz subsidiary condition. The form " $\partial_\mu A_\mu(\vec{x},t)\varphi=0$ for all $\vec{x}, t$ " of the condition, e.g. see /4/, is equivalent to two equations at a fixed t, e.g. $t=0$ /4/:

$$\partial_\mu A_\mu(\vec{x},0)\varphi=0 \quad ; \quad [\operatorname{div}\vec{E}(\vec{x},0)-j_0(\vec{x},0)]\varphi=0 .$$

They are equations for permissible (physical) states φ. The initial vector $\varphi_i=d_1^+\Omega$ used in subsect 2.2 does not satisfy the second equation[7]. The cause is the noncommutativity of the operator ψ with $\operatorname{div}\vec{E}-j_0$ /4/. One has to describe electrons and positrons by another operator field which would commute with $\operatorname{div}\vec{E}-j_0$. The operator

$$\varphi(\vec{x},t)=\psi(\vec{x},t)\exp\left[\frac{ie}{4\pi}\int d^3y\ \operatorname{div}\vec{A}(\vec{y},t)/|\vec{x}-\vec{y}|\right]$$

is an example. Introducing φ instead of ψ, one gets the Coulomb gauge formulation of QED/4/ or its variants, e.g. see /5/. The calculation of $\Delta N_0(t)$ becomes more complicated in the formulations. But it can be shown that the resulting $\Delta N_0(t)$ differs insignificantly from that given by eq.(32). In particular, at $t>\tau_1+R$ the difference of the two results is $\sim(R\Delta)^2$ times less than the r.h.s. of eq.(32) and can be neglected if R is a macroscopic distance: $R\gg 1/\Delta$.

2.7. Consider a possible theoretical description of the process of preparation of the atom in an excited state. Let atom $\mathcal{D}$ be in the ground state at $t=0$. We turn on at a moment t_1 an external potential $V(\vec{x},t)$ which is additional to the static potential $U(\vec{x})$ binding the $\mathcal{D}$ electron, see eq.(7). After a moment t_2 $V(\vec{x},t)$ is switched off, becoming zero. Such a potential $V(\vec{x},t)$ can describe a laser beam. It is known that a laser of frequency Δ can quickly transfer the atom from the level E_0 to level $E_1=E_0+\Delta$ /6/. Suppose that $V(\vec{x},t)$ induces the transference in the interval (t_1, t_2) with probability 1. This drastically simplifies the calculations and result. At $t>t_2$ exactly the same expression follows for $\Delta N_0(t)$, see eq.(32), independently of the position of the interval (t_1, t_2) on the time axis until it is earlier than (τ_1, τ_2)[8]. If (t_1, t_2) is later

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[7] Even the no-particle vector Ω does not satisfy it because $j_0(\vec{x},0)$ contains terms of the type d^+b^+.

[8] Only the first nonvanishing approximation is implied throughout this paper. If spontaneous deexcitation of the atom $\mathcal{D}$ (radiation damping) is properly taken into account, then of course ΔN_0 does depend upon (t_1, t_2) position.

than (τ_1, τ_2) but earlier than the moment $\tau_1 + R$, one gets for $\Delta N_0(t)$ at $t > t_2$ approximately the same result as given by eq.(32), independently of the particular (t_1, t_2) position in the above-mentioned time region.

3. Quen in atom-atom interaction

The description of the electromagnetic field source by the external current has two deficiencies. First, the functions $J_\mu(\vec{x}, t)$ are prescribed at all $\vec{x}$ and t and describe the current which does not alter when radiating or absorbing the electromagnetic field while atoms change its state of course. Second, the hermiticity of $J_\mu A_\mu$ requires real J_μ . As a consequence, one has the equality $\tilde{J}_\mu(\vec{z}, -\omega) = \tilde{J}_\mu^*(\vec{z}, \omega)$ for J_μ Fourier representation, see eq.(20). This equality means that if J_μ can absorb the photon of energy ω , then it is sure also to emit it. This must be compared with the atom in the ground state which can only absorb photons.

Replacing J_μ by an atom A must give a better description of a real electromagnetic field source.

3.1. Consider the following simple variant of such a description. Let atom A be exactly the same as the atom $\mathcal{D}$ and let initial state be $a_0^+ d_0^+ \Omega$: "both atoms are in the ground state, no photons". Later $\mathcal{D}$ is excited from 0 to 1 in an interval (t_1, t_2), see subsect 2.7. The probability $\Delta N_0^A(t)$ (to find $\mathcal{D}$ in the state 0 at a moment t) induced by A is defined now as follows

$$\Delta N_0^A(t) = \langle a_0^+ d_0^+ \Omega \,|\, d_0^+(t)\, d_0(t) \,|\, a_0^+ d_0^+ \Omega \rangle - \langle d_0^+ \Omega \,|\, d_0^+(t)\, d_0(t) \,|\, d_0^+ \Omega \rangle \tag{33}$$

The "background" is now the probability of the spontaneous transition of the atom $\mathcal{D}$ from 1 to 0 when A is absent.

It turns out that the third order $e^3 d_0^{(3)}(t)$ of $d_0(t)$ is needed for the calculation of $\Delta N_0^A(t)$ in the first nonvanishing approximation which has the order e^4 . The calculation is much harder than in the preceding sect 2. But the result for $\Delta N_0^A(t)$ has the same important qualitative feature as $\Delta N_0(t)$ in sect.2: $\Delta N_0^A(t)$ is practically zero if $t < R/c$.

However, the presented variant has the following deficiency: the source A is working all the time after the moment $t = 0$ without being switched off. This circumstance permits the interpretation of the effect which does not use quen: the atom $\mathcal{D}$ emits a

photon in the interval (t_1, t) and then the photon is absorbed by A , cf. Introduction[9].

Before going to other variants in which A is "switched off" at a moment after $t = 0$, I ought to stress that atom A must be considered as being absent till the moment $t = 0$. Let us explain why. Proving the quen existence on the ground of the standard theory I allow inavoidably the following inconsistency: if quens exist, they should be taken into account from the beginning when setting and solving the problems considered above. In particular, there are no photons in the initial state φ_i , but are there the quens? This inconsistency is by-passed here as follows. The absence of photons at $t = 0$ can be secured if nothing has created them till $t = 0$. Analogously quens are absent at $t = 0$ if the external current vanishes at $t < 0$ or if atom A was "switched off" at $t < 0$.

Now I am going to discuss how one can realize the "turning on" and "switching off" of the atom A .

3.2. Let atom A is not the same as $\mathcal{D}$ so that $E_1^A - E_0^A \neq \Delta$, but $E_2^A - E_1^A = \Delta$. The atom A is initially in the state 0 , we begin to excite it at a moment $\tau_1' > 0$ (e.g., by a laser beam of the frequency $E_1^A - E_0^A$) so that at a moment τ_1 atom A turns out to be on a level E_1^A . At a moment $\tau_2 > \tau_1$ we begin to transfer A from 1 back to 0 , this process being accomplished at a moment τ_2' . In the interval (τ_1, τ_2), $\tau_2 - \tau_1 = \Delta\tau$, A can emit the quen of energy $-\Delta = E_1^A - E_2^A$ which can be absorbed by $\mathcal{D}$ later: $E_1^{\mathcal{D}} - E_0^{\mathcal{D}} = \Delta$.

A modification of this procedure is possible: being excited on the level E_1^A , atom A then spontaneously radiates going to E_0^A so that almost certainly it will be in the state 0 after the moment τ_2 being then unable to emit the quen $-\Delta$.

All other details are the same as in the preceding subsect. 3.1 except for the "background" definition. It is now the probability to find $\mathcal{D}$ at t in the state 0 under the condition that A is always in the state 0 , not suffering ang excitement.

The formulated problem is much harder to calculate than that of the preceeding subsection. It is the following variant of the effective "turning on" and "swiching off" of the atom A that has been calculated.

[9] However, this explanation leaves obscure why this mechanism had not acted before the moment R/c and resulted in the $\Delta N_0^A(t)$ non-vanishing for $t < R/c$.

3.3. Let A is the same atom as $\mathcal{D}$ but in distinction to subsect 3.1 it moves rectilinearly with a constant velocity $\vec{v}$ in passing $\mathcal{D}$. At a moment τ_c the distance $\vec{R}(t)$ between A and $\mathcal{D}$ is minimal, being equal to $\vec{R}$: $\vec{R}(t) = \vec{R} + \vec{v}(t-\tau_c)$.

So the "turning on" and "switching off" of the atom A is realized smoothly by changing $R(t)$.

As compared to subsect 3.1, it is harder to calculate the integral over y_c (of the type Y, cf. eq.(22)). The result is that the probability $\Delta N_0^A(t)$ at $t_2 > \tau_0$ depends practically only on two time differences: $t_2-\tau_0$ and $t-t_2$. Its dependence on the first argument ($t-t_2$ being fixed and $(t-t_2)\Delta << \sqrt{R\Delta}$) is qualitatively described by the curve of Fig.1. An estimate $R/c\,(v/c)^{-2}[(t-t_2)\Delta]^{-1}$ can be given for the width of the bump of the curve near the point $\tau_c + R/c$. The width is much less than R/c, if $t-t_2$ is a macroscopical time interval: $t-t_2 >> 1/\Delta$.

4. Conclusion

4.1. The existence of quen is a logical consequence of the effect of the retarded action of an unexcited source A on an excited detector $\mathcal{D}$. The mathematical cause of the effect is the presence of the function $\mathcal{D}^K$ in the formulae for the induced probability $\Delta N_c(t)$, e.g. see eq.(18). It is $\mathcal{D}^K$ (but not, e.g., the photon propagator $\mathcal{D}^c$) that describes the propagation of the electromagnetic field in the effect[10].

It can be shown that $\mathcal{D}^R$ appearance in its turn originates from two physical premises. The first is the inclusive character of ΔN_0 which means that measurements at t are performed with the atom $\mathcal{D}$ only in the région of its location. Nothing more is measured anywhere. The second is the subtraction of the "theoretical background". This secures the isolation of that part of the $\mathcal{D}$ deexcitation probability which is caused by the atom A. Therefore ΔN_c describes indeed the energy transmission from the atom A location to the $\mathcal{D}$ location.

This paper discusses the quen existence in the framework of QED. The corresponding experiment ought to reveal whether quens exist in Nature (in the sense as photons exist). Two necessary circumstances must be ensured in the experiment: the "turning on" of the source A (e.g. see subsect. 3.2 and 3.3) and confident detecting of the retardation of the atom A action on atom $\mathcal{D}$. It seems to be necessary for this purpose that ΔN_0 be comparable with the probability of the spontaneous $\mathcal{D}$ radiation (there must be sufficiently many atoms A around $\mathcal{D}$).

4.2. One can argue existence of the quens of other fields, e.g. as follows. A neutrino field is generated by an external spinor source η which is localized in a space region V_η and vanishes outside the interval $\Delta\tau$, τ_0 being its centre (in analogy with the external current in sect 2). The source changes the probability of the decay of a neutron which is in the region $V_\mathcal{D}$, the distance between $V_\mathcal{D}$ and V_η being R (the η action on the neutron can be described as $\eta + n \to p + e^-$ or $\nu + n \to p + e^-$). At a moment $t > \tau_0 + R$ one measures the number of pairs pe^- having a kinetic energy less than ($m_n - m_p - m_e$) (the pair total energy is less than m_n). One can show by calculations analogous to those used in /8/ that η induces a contribution to this number which mainly is due to quanta of the neutrino field having negative energy. Let us stress that these quanta are not antineutrinos because each of them has the same lepton number as the neutrino (or electron). In an analogous manner one can demonstrate the existence of the quens of other fields which mediate the interaction between observable particles.

4.3. Quens exist if besides their source A there is their absorber $\mathcal{D}$. Unlike the virtual particles quens exist during macroscopic time intervals of the order R/c. But what about the free quen, can it exist in the same sense as does the radiated photon?

Remind at first how the question is resolved in theories of the direct interaction of charges, using no electromagnetic field (Wheeler and Feynman/9/, Hoyle and Narlikar/10/, the review and other references see in /11/). In this theory an excited atom can emit a photon going "to infinity" only if there exists the Universe which is absorbing basically. The energy radiated by the atom is absorbed in the end by another atom of the Universe.

Using analogous arguments one can infer that a free quen would need the Universe which would be excited on the whole. As the contemporary Universe is not of this kind, the free quens do not exist now, cf./7/.

4.4. The known particle interpretations of a quantized field deal with positive-energy particles which can be free. Being not free, the quens ought not to be taken into account if one considers the "bare" or in-out fields.

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[10] M.Fierz has noted in /7/ that if $\mathcal{D}^c$ is replaced by another function, then there arise "Quanten negativer Energie". The term of Fierz is used in this paper.

The induced probability $\Delta N_c(t)$ considered here is defined by using the evolution operator $U(t,0)$ rather than the S-matrix. This elucidates why there arises the problem of describing the quen absence at $t=0$, see the end of subsect. 3.1.

The quen would be reasonable to use if one considers such a particle interpretation of the field which properly takes into account that real fields are interacting ones. In particular, one may take quens into account when considering the particle interpretation of the gluon field which cannot be free.

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Кванты с отрицательной энергией

Квантово-электродинамическим расчетом обнаружен эффект запаздывающего действия невозбужденного атома A на возбужденный атом D. Электромагнитное поле, которое сначала испустил A и которое D потом поглощает, не может состоять из фотонов, поскольку кванты этого поля должны иметь отрицательную энергию. Показано, что существуют отрицательно-энергетические кванты других полей, например фермионного. Обсуждаются соответствующий эксперимент и следствия существования таких квантов.

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Shirkov M.I. E4-86-705

Negative-energy quanta

The effect of the retarded action of an unexcited atom A on an excited atom D is revealed by means of QED calculation. The electromagnetic field emitted by A and later absorbed by D cannot consist of photons because quanta of the field must have a negative energy. It is argued that there exist also negative-energy quanta of other fields, e.g., the fermionic ones. The appropriate experiment and consequences of the existence of these quanta are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics,JINR.

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