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INDUCED TOROID STRUCTURES AND TOROID POLARIZABILITIES

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There is now interest in toroid moments [1,2] which are being studied in a variety of domains [1-4]. In refs.5 a new type of polarizability, hereafter called "toroid" (different from the usual electric and magnetic ones) has been introduced (and some effects related to it considered) in connection with the toroid multipole moments induced in a system by a external magnetic field $\vec{H}^{exp}$ of nonvanishing $\nabla \times \vec{H}^{ext}$ (or alternatively, by an external conduction or displacement current). While the intrinsic toroid moments of elementary quantum systems are ruled-out either by parity conservation or by invariance under time reversal, the corresponding induced ones (as emphasized in refs.5) are not forbidden by these discrete symmetries and the toroid polarizability is just measuring their size. In the multipole decomposition of the Hamiltonian (time dependent, in general) describing the system's interaction with the external electromagnetic field, alongside with the usual electric and magnetic dipole (and higher multipole) pieces, there are also contributions expressing the interaction of the toroid moments with the external fields [1,2]; the latter begin the toroid dipole piece [1,2]

$$H_{(\text{Toroid-dipole})}(t) = -\vec{T}(t)\cdot[\nabla\times\vec{H}^{ext}]_{\vec{x}=0,t} = -\vec{T}(t)\,[\frac{4\pi}{c}\vec{J}^{ext} + \frac{1}{c}\frac{d\vec{D}^{ext}}{dt}]_{\vec{x}=0,t}, \qquad (1)$$

where $\vec{J}^{ext}$ and $(4\pi)^{-1}\,d\vec{D}^{ext}/dt$ are the external conduction and diplacement currents, while [2]

$$\vec{T}(t) = \frac{1}{10c}\int\{\vec{x}[\vec{x}\cdot\vec{j}(\vec{x},t)] - 2\vec{x}^2\vec{j}(\vec{x},t)\}d^3x \qquad (2)$$

is the toroid dipole moment ($\vec{j}(\vec{x},t)$ denotes the system's current density). According to the well-known nonstationary perturbation rules, the response of a quantum system to the particular interaction from Eq.(1) is described by the following dynamic (i.e., frequency (ω) dependent) toroid dipole polarizability [5]

$$\gamma_{ij}(\omega) = i\int e^{i\omega t}\,\theta(t)\,dt\,\langle 0|[T_i(t),\,T_j(0)]|0\rangle = \sum_n\,[\frac{\langle 0|T_i|n\rangle\langle n|T_j|0\rangle}{E_n - E_0 - \omega - i\epsilon} + \frac{\langle 0|T_j|n\rangle\langle n|T_i|0\rangle}{E_n - E_0 + \omega + i\epsilon}]. \qquad (3)$$

The ground state ($|0\rangle$) contribution, as usually, is to be taken-off from Eqs.(3); E_0, E_n denote the energies of the ground and excited states. The toroid dipole moment induced in the system (irrespective of whether or not it does possess a nonvanishing intrinsic one) has the Fourier components

$$T_i^{\text{induced}}(\omega) = \sum_j \gamma_{ij}(\omega)\,[\nabla \times \vec{H}^{\text{ext}}(\omega)]_j \,. \qquad (4)$$

As is shown in refs.5, unlike the (static) electric and magnetic dipole polarizabilities $\alpha_{\ell=1}(\omega=0)$, $\beta_{\ell=1}(\omega=0)$ (the subscript ℓ indicates the multipole (2^{ℓ}-pole) order) which establish the angular structure of the amplitude for elastic scattering of low energy photons on the considered system (Compton scattering) in the second (photon) energy order, the (static) toroid dipole polarizability $\gamma_{\ell=1}(\omega=0)$ enters only begining with the fourth energy order, together with the usual (static) electric and magnetic quadrupole polarizabilities $\alpha_{\ell=2}(\omega=0)$, $\beta_{\ell=2}(\omega=0)$ and some derivatives of the usual (dynamic) dipole polarizabilities, like $\alpha'_{\ell=1}(\omega=0) = (d\alpha_{\ell=1}(\omega)/d\omega^2)|_{\omega=0}$ [7].

The purpose of this Note is to look into the relative importance of the induced toroid moments (measured by $\gamma_\ell(\omega)$) as against the (induced) usual electric and magnetic ones (measured by the multipole polarizabilities $\alpha_\ell(\omega)$, $\beta_\ell(\omega)$, first for atoms and then for hadrons, in order to try getting some guesses on what might happen at even smaller distances (or larger characteristic excitation energies), at the sub-hadronic level. As an example for the atomic physics case we shall consider a (nonrelativistic spinless) hydrogen-like atom in its ground state (we are then able to compute $\gamma^{(H)}_{\ell=1}(\omega)$ exactly and use for comparison the available (also exact) results for $\alpha^{(H)}_\ell(\omega)$ derived in ref.8). As a typical example for hadrons we shall consider the (charged) pion and use some numerical estimates given in refs.5 for $\gamma^{(\pi)}_{\ell=1}$ (in conjunction with previous ones for $\alpha^{(\pi)}_\ell$ found in refs.9) to perform an analogous comparison. Looking then at what the situation is in the two cases, at the length scale of 10^{-8} cm on one side and 10^{-13} cm on the other, we shall put forward a speculative idea about what might happen, say, at yet five orders of magnitude lower, down to 10^{-18} cm., i.e., at such distances which are expected to be explored, for instance, by the HERA electron-proton collider at DESY.

To calculate the toroid dipole polarizability of (ground state) H-like atoms, one starts with the definition Eq.(3) in which the one particle operator for the toroid dipole moment, by Eq.(2), is [2]

$$T = \frac{e}{10mc} \sum_k (-2\vec{x}^{\,2}\delta_{ik} P_k + x_i x_k P_k), \qquad (5)$$

where $P_k = -i\hbar\,\partial/\partial x_k$ (e and m are the charge and mass of the electron). The (exact) calculation is nonstandard, long and tedious and will be presented in detail elsewhere [10]. It is essentially based on the use of the integral representation for

the nonrelativistic Coulomb Green's function [11] in the form obtained by Schwinger [11] and on the fact that a certain "basic" momentum space integral (which is at the root of many exact calculations in studies concerning the interaction of nonrelativistic H-like atoms with the radiation) can be taken exactly [12]. Next, we shall give only the result. Due to the spherical symmetry of the ground s-state, one has $\gamma^{(H)}_{ij}(\omega) = \delta_{ij}\,\gamma^{(H)}_{\ell=1}(\omega)$. We have obtained the formula

$$\gamma^{(H)}_{\ell=1}(\omega) = \frac{\alpha^2}{20}\,\frac{a_0^5}{Z^4} \sum_{i=1,2} \frac{r_i^2}{(r_i+1)^4}\cdot\frac{1}{2-r_i}\cdot\frac{1}{3-r_i} \times$$

$$\times \Big[\frac{8r_i^2(r_i^2+1)^2}{(r_i+1)^2(4-r_i)}\,F(1,-1-r_i,5-r_i;\zeta_i) - \qquad (6)$$

$$-15r_i^4 + 7r_i^3 + 53r_i^2 + 57r_i + 18\Big],$$

where $F(a,b,c;z)$ is the usual Gauss hypergeometric function with the series expansion

$$F(a,b,c;z) = 1 + \frac{a\cdot b}{c}\,\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\,\frac{z^2}{2!} + \ldots$$

and

$$r_1 = (1-\omega_0)^{-1/2}, \quad r_2 = (1+\omega_0)^{-1/2}, \quad \zeta_i = \left(\frac{r_i-1}{r_i+1}\right)^2,$$

$$\omega_0 = \frac{\hbar\omega}{|E_0|} = \frac{2\hbar\omega}{\alpha^2 Z^2 mc^2} = \frac{2}{\alpha Z}\,\frac{a_0}{Z}\,\frac{\omega}{c}, \qquad (7)$$

$$a_0 = \frac{\hbar}{\alpha mc} \simeq 0{,}53\times 10^{-8}\ \text{cm}, \quad \alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137\ldots}\ldots .$$

$\gamma^{(H)}_{\ell=1}(\omega)$ as given by Eq.(6) is an even analytic function of ω having (in the complex ω-plane) the right singularities at the right place: simple poles at $\omega = (E_n - E_0)/\hbar$, $n = 2,3,4,\ldots$ ($E_n = E_0/n^2$ represents the discrete spectrum of the H-like atom) and a branch cut along the real ω-axis above the ionization threshold $\omega > |E_0|$. For the static (i.e. $\omega = 0$) toroid dipole polarizability of a (nonrelativistic, spinless) H-like atom in its ground state we find then from Eq.(6) the very simple result

$$\gamma^{(H)}_{\ell=1}(\omega=0) = \frac{23}{60}\,\alpha^2\,\frac{a_0^5}{Z^4} \simeq Z^{-4}\times 0.86\times 10^{-46}\ \text{cm}^5. \qquad (8)$$

This is the toroid analog of the well-known static electric dipole polarizability

$$\alpha^{(H)}_{\ell=1}(\omega=0) = \frac{9}{2}\,\frac{a_0^3}{Z^4} \qquad (9)$$

found in 1926 by Epstein and by Waller[13].

Now, we have to assess the relative importance of the toroid effects with respect to the usual, electric ones. To that aim, $\gamma^{(H)}_{\ell=1}(\omega=0)$ is to be compared with $\alpha'^{(H)}_{\ell=1}(\omega=0)$ and $\alpha^{(H)}_{\ell=2}(\omega=0)$. Using Eq.(8) above and the results of ref.8 (rewritten in the conventions used in this paper), we get the (exact) formulas:

$$\frac{\gamma^{(H)}_{\ell=1}(\omega=0)}{\alpha'^{(H)}_{\ell=1}(\omega=0)} = \frac{23(\alpha Z)^4}{1595}; \quad \frac{\gamma^{(H)}_{\ell=1}(\omega=0)}{\alpha^{(H)}_{\ell=2}(\omega=0)} = \frac{23(\alpha Z)^2}{900}. \qquad (10)$$

Thus one sees that for H-like atoms (and this holds also in atomic physics in general) the effects of the induced toroid moments appear very small indeed with respect to those of the corresponding usual electric ones. Perhaps it is also for this reason that the induced toroid moments have not been so far analyzed in atomic physics. In a larger perspective, what is important to us, is that they are there, whatever small. Moreover, as is seen from Eq.(10), the toroid effects are increasing with αZ and this allows for possible applications even in atomic physics problems (for instance, in what concerns the neutral component of plasma), but this subject is outside the scope of this paper.

Next we turn to see what a similar comparative analysis will say when instead of a H-like atom one is considering a typical hadron, the (charged) pion, for example. In refs.5 an order of magnitude estimate of the static toroid dipole polarizability of $\pi^{\pm}$, $\gamma^{(\pi)}_{\ell=1}(\omega=0)$, has been obtained by evaluating the contribution of the A_1 (1270 MeV) resonance (in terms of the experimentally known radiative width $\Gamma(A_1 \to \pi\gamma) \doteq 0.6$ MeV; see also refs.9 and the literature cited therein with the result $\gamma^{(\pi)}_{\ell=1}(\omega=0) \simeq 1.2\cdot 10^{-5}$ fm^5 Under the same approximations in refs.5 it has been found that $\alpha'^{(\pi)}_{\ell=1}(\omega=0) \simeq 0.8\cdot 10^{-5}$ fm^5. From the results obtained in the first of refs.9 it is known that the static electric and magnetic quadrupole polarizabilities of $\pi^{\pm}$ are expected to be of the order $\alpha^{(\pi)}_{\ell=2}(\omega=0) \sim \beta^{(\pi)}_{\ell=2}(\omega=0) \sim 10^{-5}$ fm^5. So, the picture which then emerges for (charged) pions looks in sharp contrast with the corresponding one for H-like atoms:

$$\frac{\gamma^{(\pi)}_{\ell=1}(\omega=0)}{\alpha'^{(\pi)}_{\ell=1}(\omega=0)} \sim \frac{\gamma^{(\pi)}_{\ell=1}(\omega=0)}{\alpha^{(\pi)}_{\ell=2}(\omega=0)} \sim 1. \qquad (11)$$

Comparing Eqs.(10) and Eqs.(11), one sees that unlike the case of atoms, for hadrons not only the toroid polarizabilities and effects related to them can no longer be neglected, but, on the contrary, they are expected to be of the same order of magnitude as the usual electric (and magnetic /9/) ones (of one order of multipolarity higher, of course, since it is with them that the comparison has to be made).

Eqs.(10) and (11) are here to stay and they must be taken seriously. In a context in which there is nowadays such an intense activity in supersymmetric, string, superstring theories, we can not refrain from putting forward the following speculative idea: Eqs.(10), (11) seem to tell us that the more "elementary" the object is (or otherwise, the higher are the characteristic excitation energies of the system), the better might it respond to an external current $(\nabla \times \vec{H}^{ext})$ rather than to the external fields $\vec{E}^{ext}$, $\vec{H}^{ext}$ themselves; towards yet smaller distances (at 10^{-18} cm., say) the role of the (induced) toroid moments might increase further and become as predominant over the usual (induced) electric and magnetic ones as the latter were dominating over the toroid moments in atomic physics. Such a "linear" extrapolation from things reliably known in atomic and hadronic physics, down to the next substructure, would come in line with at least two features of the topical theories mentioned above:

Firstly, string-like objects are likely to provide good candidates for systems having large toroid polarizabilities but small electric and magnetic ones. Indeed, while all types of polarizabilities are more or less extensive quantities (i.e., more or less proportional with the volume of the body), for the toroid ones (if the material properties are properly choosen) one may expect comparatively large values in the case of (closed) filiform structures (strings) on account of large numbers of turns of winding. We recall (see refs.2,5) that for a classical toroidal current the toroid dipole moment (calculated by means of Eq.(2)) is

$$\vec{T}_{Torus} = \vec{n}\,\frac{NIV_T}{4\pi c} \qquad (12)$$

(I - the current intensity, N - the number of turns of winding (N - even), V_T - the volume of the toroid, $\vec{n}$ - unit vector along the toroid axis). When we deal with induced toroid moments, some kind of external current flowing through the system induces in it closed toroidal secondary currents; this is, in a sense, a usual transformator effect and the toroid polarizability measures, in fact, its intensity. In a quantum field theoretical picture, while the usual electric and magnetic polarizabilities of a quantum object express the ability of the cloud of virtual particles surrounding it to get deformed in electric and magnetic fields, the toroid polarizabilities represent analogously a measure of the cloud's deformations which are topologically nontrivial and nonstationary.

Secondly, if Majorana particles (currently occuring in grand unified and supersymmetric theories as well as in connection with neutrinos) are really to play a role both as the next "ultimate" constituents of matter and as terribly massive gauge fields, then it could not be only a mere coincidence that the only possible electromagnetic structure they might possess is just represented by toroid moments and distributions; for them, all other usual electric and magnetic multipole moments and distributions are forbidden [4]. Of course, we are speaking now about intrinsic toroid moments, but bound systems of such Majorana fermions with nonvanishing intrinsic toroidal electromagnetic structure would rather have large toroid polarizabilities, in much the same way as a usual macroscopic piece of matter composed of polar molecules would have all chances to possess a large electric polarizability. Anyway, the happening that Majorana particles single them out in a clear-cut manner by choosing to possess only toroid electromagnetic structures (or no electromagnetic structure at all), might have other consequences, too, if supersymmetry has something to do with facts; the boson-fermion symmetry is very special, indeed, and sometimes leads to strange surprises(see for instance, ref.14).

We end with the remark that a certain experimental information on the (static, dipole) toroid polarizability of a system may be obtained by determining the fourth order frequency part of the low energy Compton scattering amplitude; for direct extraction one has to go, however, to charge scattering, or virtual Compton scattering, or other processes in which a current is flowed through the system [15]. Details will be presented elsewhere [16].

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6. To understand intuitively what means inducing a toroid dipole moment, we note that when a conduction or displacement current is flowed through a system, some of the constituent charges may well begin, for instance, to "move" (speaking in a classical language) on "eight-like" (closed) orbits. While inducing a usual magnetic dipole moment means, say, inducing a closed circular current, inducing a toroid dipole would mean inducing an "eight-like" closed current, or a (coplanar) pair of circular currents (equal, but circulating opposite to each other), or a coaxial collection of such pairs of currents (a toroidal current); these latter current configurations, topologically nontrivial, have no resultant magnetic moment but still represent a certain kind of "dipole" characteristic, a toroid dipole.
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Костеску А., Радеску Е.Е. E4-86-700
Наведенные тороидные структуры и тороидные поляризуемости

Динамическая тороидная дипольная поляризуемость $\gamma(\omega)$ /нерелятивистского, безспинного/ водородоподобного атома в основном состоянии вычислена аналитически в терминах двух гауссовых гипергеометрических функций. Статический результат имеет простую форму $\gamma(\omega=0) = (23/60)\alpha^2 Z^{-4} a_0^5$ /α – константа тонкой структуры, Z – зарядовое число ядра, a_0 – Боровский радиус/. Сравнивая данные вычисления для водородоподобных атомов с предыдущими оценками для пионов, находим, что роль наведенных тороидных моментов /относительно к обычным электрическим/ растет значительно в физике адронов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1986

Costescu A., Radescu E.E. E4-86-700
Induced Toroid Structures and Toroid Polarizabilities

The frequency-dependent toroid dipole polarizability $\gamma(\omega)$ of a (nonrelativistic, spinless) hydrogen-like atom in its ground state is calculated analytically in terms of two Gauss hypergeometric functions. The static result reads simply $\gamma(\omega=0) = (23/60)\alpha^2 Z^{-4} a_0^5$ (α - fine structure constant, Z - nucleus charge number, a_0 - Bohr radius). Comparing the present evaluations for H-like atoms with previous ones for pions, one sees that the role of the induced toroid moments (as against that of the usual electric ones) increases considerably when passing from atomic to particle physics. It might become dramatic at the sub-hadronic level.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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