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SQUEEZING IN A MIXTURE OF TWO MODES INTERACTING WITH STRONGLY DRIVEN TWO-LEVEL ATOMS

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A squeezed state of the radiation field[1-3] which may have potential application in the optical communication system[4] and gravitational radiation detector[5] has become the subject of extensive theoretical[6-28] and experimental studies[29].

A number of nonlinear optical systems susceptible to producing a squeezed state has been analysed theoretically. These include the degenerate parametric oscillator[6-10], fourwave mixing[11-17], resonance fluorescence[18-24], optical bistability[25,26] two-photon processes[27,28] and others.

In this work we present the squeezed-state generation by the mixture of the two-signal modes interacting with strongly driven two-level atoms. The signal modes are assumed to be located near two sidebands of resonance fluorescence. The collective effects, cavity damping and the effects of atomic and field reservoirs are accounted for. A large squeezing has been obtained for suitable values of the parameters of the system (cavity damping constants, frequency detuning of resonance, number of atoms, etc.). For the case of a large number of atoms the factor of squeezing can be reached to a limiting value.

The N two-level atoms concentrated in a region small compared to the wavelength of all the relevent radiation modes interact with an intense driving mode $\vec{E}$ at frequency ω and with two signal modes $\vec{E}_1$, $\vec{E}_2$ at frequencies ω_1. , ω_2 (Fig.1). The external field $\vec{E}$ is assumed so intense that it can be treated classically.

The coherence part of the hamiltonian in the rotating wave approximation and interaction picture is

$$\begin{aligned} H_{coh} = {} & \frac{\Delta_0}{2}(J_{22}-J_{11}) + G\,(J_{21}+J_{12}) \\ & + g_1(a_1^+ J_{12} + J_{21} a) + g_2(a_2^+ J_{12} + J_{21} a_2) \\ & + \Delta_1 a_1^+ a_1 + \Delta_2 a_2^+ a_2 , \end{aligned} \tag{1}$$

where $\Delta_0 = \varepsilon_2 - \varepsilon_1 - \omega$ (system with $\hbar = 1$):

$\Delta_{1,2} = \omega_{1,2} - \omega$; $G = -\vec{d}_{21}\vec{E}$; $g_{1,2} = -\vec{d}_{21}\cdot\vec{E}_{1,2}/|\vec{E}_{1,2}|$

here $\vec{d}$ is the electric dipole operator for the system; a_1^+ , a_1 and a_2^+ , a_2 are the creation and annihilation operators of the signal modes $\vec{E}_1$ and $\vec{E}_2$ respectively. The operators

$$J_{ij} = \sum_{k=1}^{N} |i\rangle_k\, {}_k\langle j| \qquad (i, j = 1,2)$$

are the collective angular momenta of the atoms. They satisfy the commutation relation

$$[J_{ij}, J_{i'j'}] = J_{ij'}\delta_{ji'} - J_{i'j}\delta_{ij'}. \qquad (2)$$

Considering the operator a_1 , a_1^+ and a_2 , a_2^+ in hamiltonian (1) as c -numbers and using the Markovian approximation, one finds the master equation for atomic system as follows[30]:

$$\frac{\partial\rho}{\partial t} = -i[H_{coh}, \rho] + \frac{\partial\rho}{\partial t}\Big|_A \equiv L\rho, \qquad (3)$$

where the dissipative term for the atoms is

$$\frac{\partial\rho}{\partial t}\Big|_A = -\gamma_{21}(J_{21}J_{12}\rho - 2J_{12}\rho J_{21} + \rho J_{21}J_{12}).$$

The term $2\gamma_{21}$ is the transition rate caused by the atomic reservoir from level $|2\rangle$ to $|1\rangle$. Following refs.[31,32] we introduce the Schwinger representation for angular momentum

$$J_{ij} = c_i^+ c_j \qquad (i, j = 1,2),$$

where c_i obey the boson commutation relation

$$[c_i, c_j^+] = \delta_{ij}.$$

Further, we shall consider only the case of intense driving field or large detuning Δ_0 so that

$$\Omega = (\Delta_0^2 + 4G^2)^{1/2} \gg N\gamma_{21};\ g_{1,2}|\vec{E}_{1,2}|. \qquad (4)$$

After performing the canonical transformation

$$C_1 = \cos\varphi\, Q_1 + \sin\varphi\, Q_2$$
$$C_2 = -\sin\varphi\, Q_1 + \cos\varphi\, Q_2, \qquad (5)$$

where

$$\mathrm{tg}\, 2\varphi = \frac{2G}{\Delta_0},$$

one can find that the Liouville operator L appearing in eq.(3) splits into two components L_0 and L_1 . The component L_0 is slowly varying in time whereas L_1 contains rapidly oscillating terms with frequencies Ω and 2Ω. For the case of intense driving field or large detuning Δ_0 so that the condition (4) is satisfied, it is reasonable to make the secular approximation, i.e., to retain only the slowly varying part[32,33]. A correction to the results obtained in this fashion will be of an order of $(\gamma_{21}N/\Omega)^2$ or $(g_{1,2}|\vec{E}_{1,2}|/\Omega)^2$.

Making the secular approximation, one can find a stationary solution of the master equation

$$\tilde{\rho} = U\rho U^+ = Z^{-1}\sum_{N_1=0}^{N} x^{N_1}|N_1\rangle\langle N_1|, \qquad (6)$$

where U is a unitary operator representing the canonical transformation (5)

$$x = \mathrm{ctg}^4\varphi \quad ; \quad Z = \frac{x^{N+1}-1}{x-1},$$

$|N_1\rangle$ is an eigenstate of the operators R_{11} , $\hat{N} = R_{11} + R_{22}$ here $R_{ij} = Q_i^+ Q_j$ $(i, j = 1,2)$. The operators Q_i satisfy the boson commutation relation

$$[Q_i, Q_j^+] = \delta_{ij}$$

so that

$$[R_{ij}, R_{i'j'}] = R_{ij'}\delta_{i'j} - R_{i'j}\delta_{ij'}. \qquad (7)$$

By using solution (6) one can calculate the statistical moments $\langle R^n\rangle_s$ where $\langle B\rangle_s$ indicates the expectation value of an operator B in steady-state (4). In particular, we find

$$\langle R_{11}\rangle_s = \frac{Nx^{N+2} - (N+1)x^{N+1} + x}{(x-1)(x^{N+1}-1)}, \qquad (8)$$

$$\langle R_{11}^2 \rangle_s = \frac{N^2 X^{N+3} - (2N^2+2N-1) X^{N+2} + (N+1)^2 X^{N+1} - X^2 - X}{(X-1)^2 (X^{N+1}-1)} . \quad (9)$$

Now we return to hamiltonian (1). Following the theory of laser by Haken[3 4], one may obtain quantum Langevin equation for signal modes $\vec{E}_1$ and $\vec{E}_2$ in the cavity.

$$\dot{a}_1(t) = (-i\Delta_1 - \varkappa_1) a_1(t) - ig_1 J_{12}(t) + F_1(t), \quad (10)$$

$$\dot{a}_2(t) = (-i\Delta_2 - \varkappa_2) a_2(t) - ig_2 J_{12}(t) + F_2(t), \quad (11)$$

where

$$J_{12}(t) = \cos^2\varphi \, R_{12}(t) - \sin^2\varphi \, R_{21}(t) + \sin\varphi\cos\varphi \left(R_{22}(t) - R_{11}(t)\right) \quad (12)$$

it is easy to see that in the secular approximation the operators $R_{12}(t)$ and $R_{21}(t)$ are rapidly oscillating terms with frequency Ω whereas the operator $R_{22}(t) - R_{11}(t)$ is slowly varying in time; $\varkappa_1$, $\varkappa_2$ and $F_1(t)$, $F_2(t)$ are the cavity damping constants and noise operators for the modes $\vec{E}_1$ and $\vec{E}_2$ respectively. The noise operators $F_\lambda(t)$ (λ = 1,2) obey the relations[34]

$$\langle F_\lambda(t) \rangle_H = \langle F_\lambda^+(t) \rangle_H = 0$$

$$\langle F_\lambda^+(t) F_{\lambda'}^+(t') \rangle_H = \langle F_\lambda(t) F_{\lambda'}(t') \rangle_H = 0$$

$$\langle F_\lambda^+(t) F_{\lambda'}(t') \rangle_H = n_{th,\lambda}(T) \, 2\varkappa_\lambda \, \delta(t-t') \, \delta_{\lambda\lambda'} \quad (13)$$

$$\langle F_\lambda(t) F_{\lambda'}^+(t') \rangle_H = (n_{th,\lambda}(T)+1) 2\varkappa_\lambda \, \delta(t-t') \, \delta_{\lambda\lambda'},$$

where $\langle \ldots \rangle$ indicates the thermal average over the states of heat bath; $n_{th,\lambda}$ (T) is the number of thermal quanta at the temperature T.

Further, we shall discuss only the case when the signal modes $\vec{E}_1$ and $\vec{E}_2$ are located near the two sidebands of the collective resonance fluorescence (Fig.1), i.e.,

$$|\delta_1| ; |\delta_2| \ll \Omega, \quad (14)$$

where $\delta_1 = \Delta_1 - \Omega$; $\delta_2 = \Delta_2 + \Omega$.

Under the transformation

$$a_1(t) \to e^{-i\Omega t} \tilde{a}_1(t) ; \quad a_2(t) \to e^{i\Omega t} \tilde{a}_2(t)$$

$$R_{12}(t) \to e^{-i\Omega t} \tilde{R}_{12}(t) ; \quad R_{21}(t) \to e^{i\Omega t} \tilde{R}_{21}(t)$$

$$F_1(t) \to e^{-i\Omega t} \tilde{F}_1(t) ; \quad F_2(t) \to e^{i\Omega t} \tilde{F}_2(t)$$

and with the use of the secular approximation eqs. (10,11) reduce to

$$\dot{\tilde{a}}_1(t) = (-i\delta_1 - \varkappa_1) \tilde{a}_1(t) - iG_1 \tilde{R}_{12}(t) + \tilde{F}_1(t) \quad (15)$$

$$\dot{\tilde{a}}_2(t) = (-i\delta_2 - \varkappa_2) \tilde{a}_2(t) - iG_2 \tilde{R}_{21}(t) + \tilde{F}_2(t), \quad (16)$$

where $G_1 = \cos^2\varphi \, g_1$; $G_2 = -\sin^2\varphi \, g_2$.

For simplicity we consider only the case of $n_{th,\lambda}(T) = 0$, i.e., the temperature T = 0. In this case, as is easily seen from relations (12) and eqs. (15,16), the noise operators $\tilde{F}_{1,2}(t)$ cannot affect the normally ordered variance of the signal modes but they give commutators $[a_1, a_1^+]$ and $[a_2, a_2^+]$ additional values equal to $1 - e^{-2\varkappa_1 t} \xrightarrow[t\to\infty]{} 1$ and $1 - e^{-2\varkappa t} \xrightarrow[t\to\infty]{} 1$, respectively[3,4].

Missing the noise operator, one may obtain the stationary solutions of eqs. (15,16) in the form

$$\tilde{a}_1 = \frac{-iG_1 \tilde{R}_{12}}{+i\delta_1 + \varkappa_1} ; \quad \tilde{a}_2 = \frac{-iG_2 \tilde{R}_{21}}{+i\delta_2 + \varkappa_2} . \quad (17)$$

We shall consider the normally-ordered variance of fluctuation in the in-phase (b_1) and out-of-phase component b_2 of the mixture of signal modes a_1 and a_2.

$$b_1 = \tfrac{1}{2}(b^+ + b) \quad \text{and} \quad b_2 = \tfrac{-i}{2}(b^+ - b),$$

where $b = a_1 + a_2$; $b^+ = a_1^+ + a_2^+$.

By using solution (17) and steady-state density matrix (6), one finds the normally-ordered variance of fluctuation of the operators b_1 and b_2 in the form

$$\langle : (\Delta b_{1,2})^2 : \rangle = \langle : (b_{1,2} - \langle b_{1,2} \rangle)^2 : \rangle$$

$$= \frac{1}{2}\left\{ \frac{g_1^2}{\varkappa_1^2}\cos^4\varphi\,\langle R_{21}R_{12}\rangle_s + \frac{g_2^2}{\varkappa_2^2}\sin^4\varphi\,\langle R_{12}R_{21}\rangle_s \pm \frac{g_1 g_2}{\varkappa_1\varkappa_2}\sin^2\varphi\cos^2\varphi\left(\langle R_{21}R_{12}\rangle_s+\langle R_{12}R_{21}\rangle_s\right)\right\}, \qquad (18)$$

where

$$\langle R_{12}R_{21}\rangle_s = \langle R_{11}^2\rangle_s + (N+1)\langle R_{11}\rangle_s \qquad (19)$$

$$\langle R_{21}R_{12}\rangle_s = -\langle R_{11}^2\rangle_s + (N-1)\langle R_{11}\rangle_s + N \qquad (20)$$

here $\langle R_{11}\rangle_s$ and $\langle R_{11}^2\rangle_s$ can be found in relations (8,9).

In relation (18) and further, for simplicity we take $\delta_1 = \delta_2 = 0$. The symbol $\langle \ldots \rangle$ indicates the expectation value over the states of heatbath and atomic steady-state (4).

Taking into account the noise operators $F_{1,2}(t)$, one may obtain the commutator of the hermitian amplitude operators b_1 and b_2 in the form

$$\langle[b_1,b_2]\rangle = \frac{i}{2}\left(\frac{g_1^2\cos^4\varphi}{\varkappa_1^2} - \frac{g_2^2\cos^4\varphi}{\varkappa_2^2}\right)\langle R_{22}-R_{11}\rangle_s - i. \qquad (21)$$

The factor of squeezing of the operators b_1 and b_2 can be defined as[23]

$$F_{1,2} = \frac{\langle:(\Delta b_{1,2})^2:\rangle}{\frac{1}{2}\left|\langle[b_1,b_2]\rangle\right|}. \qquad (22)$$

We speak about squeezing if the factors F_1 or F_2 are less than zero[19-29]. By using the relations (18-22), one can see that :
(i) In the case of resonance $ctg^2\varphi = 1$ we have $\langle R_{12}R_{21}\rangle = \langle R_{21}R_{12}\rangle_s$ and

$$\langle:(\Delta b_{1,2})^2:\rangle = \frac{1}{2}\langle R_{12}R_{21}\rangle_s\left(\frac{g_1\cos^2\varphi}{\varkappa_1} \pm \frac{g_2\sin^2\varphi}{\varkappa_2}\right)^2 \geqslant 0$$

thus squeezing is absent in this case;
ii) In the case of $\frac{g_1}{\varkappa_1} \to 0$ or $\frac{g_2}{\varkappa_2} \to 0$ we have $\langle:(\Delta b_{1,2})^2:\rangle \geqslant 0$ thus the squeezing is absent for the separate mode $\vec{E}_1$ or $\vec{E}_2$.

The detailed behaviour of the factor of squeezing F_2 as a function of $\mathrm{ctg}^4\varphi$ when $\frac{g_1}{\varkappa_1} = \frac{g_2}{\varkappa_2} = 1$ is plotted in Fig. 2; and

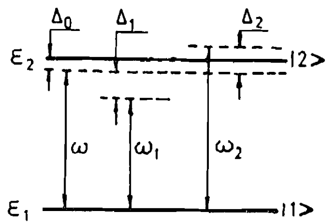
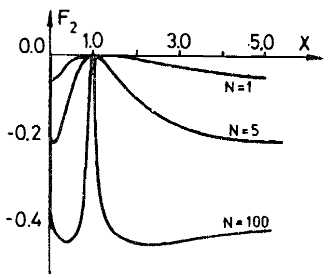


Fig. 1. Two-level atoms interacting with intense driving field $\vec{E}$ and with signal modes $\vec{E}_1$ and $\vec{E}_2$.

Fig. 2. Factor of squeezing F_2 as a function of the parameter $x = \mathrm{ctg}^4\varphi$ for the case of $g_1/\varkappa_1 = g_2/\varkappa_2 = 1$.



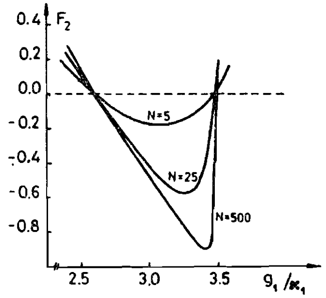


Fig. 3. Factor of squeezing F_2 as a function of the parameter $g_1/\varkappa_1$ for the case of $g_2/\varkappa_2 = 3$; $\mathrm{ctg}^4\varphi = 0.75$.

as a function of $\frac{g_1}{\varkappa_1}$ when $\mathrm{ctg}^4\varphi = 0.75$; $\frac{g_2}{\varkappa_2} = 3$, in Fig. 3. For the one atom case, as is seen from Fig. 2, the squeezing is small. For a large number of atoms and suitable values of parameters $g_1/\varkappa_1$, $g_2/\varkappa_2$ and $\mathrm{ctg}^4\varphi$ the substantial squeezing is presented (90% of squeezing is obtained for the case $N = 500$; Fig. 3). In the collective limit $N \to \infty$ the factor of squeezing tends to a limiting value $F_2 = -1$. To conclude, we have shown that the collective behaviour affects considerably the degree of squeezing. By using the system, as has been described above, one should obtain the intense field (the intensity is proportional to N) with substantial squeezing.

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Сжатие света в смеси двух мод, взаимодействующих с сильно возбужденными атомами

Обсуждено сжатие света в смеси двух мод, взаимодействующих с сильно возбужденными атомами. Наблюдено значительное сжатие света.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Squeezing in a Mixture of Two Modes Interacting with Strongly Driven Two-Level Atoms

The squeezing in a mixture of two modes interacting with strongly driven two-level atomis is discussed. The substantial squeezing is presented.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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