

**ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА**

**E4-86-684**

**N.N.Bogolubov, Jr., A.S.Shumovsky, Tran Quang**

**COLLECTIVE SPECTRAL PROPERTIES  
OF RAMAN SCATTERING**

Submitted to "Physica A"

**1986**

## I. INTRODUCTION

During the last years the collective effects in the interaction of atoms and molecules with a laser field and the vacuum of radiation have attracted considerable interest. Many theoretical and experimental studies of superfluorescence<sup>/1-10/</sup>, collective resonance fluorescence<sup>/11-16/</sup>, optical bistability<sup>/17-19/</sup>, etc., were carried out since the early work on superradiance by Dicke<sup>/1/</sup>.

The recent publications deal with collective effects in double optical resonance<sup>/20/</sup> and the resonant Raman scattering<sup>/21,22/</sup>.

In this paper the theory of collective Raman scattering (Fig.1) has been developed by using the quantum-mechanical master-equation approach and secular approximation<sup>/12,22/</sup>. In contrast with paper<sup>/22/</sup>, we consider the collective Raman scattering with only one transition strongly driven (Fig.1) and investigate the influence of the frequency detuning of resonance on the collective spectral properties of the Stokes lines.

## II. MASTER EQUATION

The  $N$  three-level atoms concentrated in a region small compared to the wavelength of all the relevant radiation modes (Dicke model) interact with a monochromatic driving field of a frequency  $\omega$  and with a field of radiation (Fig.1). Let us label the ground state by  $|1\rangle$ , the real excited state by  $|3\rangle$  and the resonant intermediate state by  $|2\rangle$  with energies  $\omega_1$ ,  $\omega_3$  and  $\omega_2$ , respectively (the system of  $\hbar = 1$ ). The real excited state  $|3\rangle$  may be a low-lying vibrational or rotational excitation from the ground state. To keep the discussion general, we will not specify these states but say that the intermediate state  $|2\rangle$  can be connected via the electromagnetic interaction Hamiltonian with both the states  $|1\rangle$  and  $|3\rangle$  (in the dipole approximation) but the states  $|3\rangle$  and  $|1\rangle$  are not connected by

the dipole Hamiltonian because of parity consideration. The transition  $|3\rangle - |1\rangle$  is caused by an atomic reservoir and assumed to be nonradiative<sup>/23/</sup>.

In treating the external field classically and using the Born and Markov approximation with respect to the coupling of the system with the vacuum field and atomic reservoir, one can obtain a master equation for the reduced density matrix  $\rho$  for the system alone in the form<sup>/2,22/</sup>

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -i [H_{coh}, \rho] \\ & - \gamma_{21} (J_{21} J_{12} \rho - J_{12} \rho J_{21} + H.C.) \\ & - \gamma_{23} (J_{23} J_{32} \rho - J_{32} \rho J_{23} + H.C.) \\ & - \gamma_{31} (J_{31} J_{13} \rho - J_{13} \rho J_{31} + H.C.) = L \rho, \end{aligned} \quad (1)$$

where  $2\gamma_{21}$  and  $2\gamma_{23}$  are radiative spontaneous transition probabilities per unit time for a single atom to change from level  $|2\rangle$  to  $|1\rangle$  and from  $|2\rangle$  to  $|3\rangle$ , respectively;  $2\gamma_{31}$  is the nonradiative rate for atomic transition from  $|3\rangle$  to  $|1\rangle$ .

The coherence part of hamiltonian  $H_{coh}$  in the interaction picture has the form

$$H_{coh} = \frac{\Omega}{2} (J_{22} - J_{11}) + G (J_{21} + J_{12}) - \Omega_3 J_{33}.$$

Here  $\Omega_3 = \omega_{23} - \frac{\omega_{21}}{2}$  (where  $\omega_{ij} = \omega_i - \omega_j$ );  $\delta = \omega_{21} - \omega$  is the frequency detuning of resonance;  $G = -d_{21} E_0$  is the matrix element of the driving field and atom interaction;

$$J_{ij} = \sum_{k=1}^N |i\rangle_k \langle j|_k \quad (i, j = 1, 2, 3)$$

are the collective angular momenta of the atoms. They satisfy the commutation relation

$$[J_{ij}, J_{i'j'}] = J_{i'j'} \delta_{ji'} - J_{ij} \delta_{ij'}.$$

The atomic coherence phenomena can be illustrated with greater lucidity by introducing the Schwinger representation for angular momentum<sup>/24/</sup>

$$J_{ij} = C_i^+ C_j \quad (i, j = 1, 2, 3),$$

where  $C_i$  obey boson commutation relation  $[C_i, C_j^+] = \delta_{ij}$ .

Further, we investigate only the case of an intense external field or much detuning  $\delta$  so that

$$\Omega = \frac{1}{2} \left( \frac{1}{4} \delta^2 + G^2 \right)^{1/2} \gg N \gamma_{21}; N \gamma_{23}; N \gamma_{31}. \quad (2)$$

After performing the canonical transformation

$$\begin{aligned} C_1 &= Q_1 \cos \phi + Q_2 \sin \phi \\ C_2 &= -Q_1 \sin \phi + Q_2 \cos \phi \\ C_3 &= Q_3, \end{aligned} \quad (3)$$

where

$$\tan 2\phi = 2G/\delta,$$

one can find that the Liouville operator  $L$  appearing in equation (1) splits into two components  $L_0$  and  $L_1$ . The component  $L_0$  is slowly varying in time whereas  $L_1$  contains rapidly oscillating terms at frequencies  $2\Omega$  and  $4\Omega$ . For the case when relation (2) is fulfilled, we make the secular approximation, i.e., retain only a slowly varying part<sup>/12,20/</sup>. Correction of the results obtained in this fashion will be of an order of

$$(\gamma_{21} N/\Omega)^2; (\gamma_{23} N/\Omega)^2 \quad \text{or} \quad (\gamma_{31} N/\Omega)^2.$$

Making the secular approximation, one can find the stationary solution of the master equation

$$\tilde{\rho} = U \rho U^+ = A^{-1} \sum_{R=0}^N X^R \sum_{N_1=0}^R Z^{N_1} |R, N_1\rangle \langle N_1, R|, \quad (4)$$

where  $U$  is the unitary operator representing the canonical transformation (3)

$$X = \frac{\gamma_{31}}{\gamma_{23}} \tan^2 \phi$$

$$Z = c \tan^4 \phi$$

$$A = \frac{Z}{Z-1} \frac{(XZ)^{N+1} - 1}{XZ - 1} - \frac{1}{Z-1} \frac{X^{N+1} - 1}{X - 1},$$

$|R, N_1\rangle$  is an eigenstate of the operator  $R = R_{11} + R_{22}$ ,  $R_{11}$  and the operator of number of atoms

$$\hat{N} = R_{11} + R_{22} + R_{33} = J_{11} + J_{22} + J_{33};$$

here  $R_{ij} = Q_i^\dagger Q_j \quad (i, j = 1, 2, 3).$

The operators  $Q_i$  satisfy the boson commutation relation

$$[Q_i, Q_j^\dagger] = \delta_{ij} \quad (5)$$

so that

$$[R_{ij}, R_{i'j'}] = R_{ij} \delta_{i'j} - R_{i'j} \delta_{ij'} \quad (6)$$

For the case of resonance the solution (4) reduces to

$$\tilde{\rho} = B^{-1} \sum_{R=0}^N P^R \sum_{N_1=0}^R |R, N_1\rangle \langle N_1, R|, \quad (7)$$

where

$$P = \gamma_{31} / \gamma_{23}$$

$$B = \frac{(N+1)P^{N+2} - (N+2)P^{N+1} + 1}{(P-1)^2}$$

The solution (7) has the same form as in our previous paper for collective double resonant process<sup>/20/</sup> and resonant Raman scattering in intense driving and scattered light<sup>/22/</sup>. As in ref.<sup>/25/</sup>, for simplicity we introduce the characteristic function

$$\chi_{R_{11}, R}(\beta, \xi) = \langle e^{i\beta R_{11} + i\xi R} \rangle_S =$$

$$= A^{-1} \left[ \frac{\gamma_2}{\gamma_2 - 1} \frac{(\gamma_1 \gamma_2)^{N+1} - 1}{\gamma_1 \gamma_2 - 1} - \frac{1}{\gamma_2 - 1} \frac{\gamma_1^{N+1} - 1}{\gamma_1 - 1} \right], \quad (8)$$

where

$$\gamma_1 = x e^{i\xi}$$

$$\gamma_2 = z e^{i\beta}$$

Here  $\langle B \rangle_S$  denotes the expectation value of an operator  $B$  in the steady-state (4).

Once the characteristic function is known, it is easy to calculate the statistical moments

$$\langle R_{11}^n R_{11}^m \rangle_S = \frac{\partial^n}{\partial (i\xi)^n} \frac{\partial^m}{\partial (i\beta)^m} \chi_{R_{11}, R}(\beta, \xi) \Big|_{\substack{i\beta=0 \\ i\xi=0}} \quad (9)$$

In particular, we have

$$\langle R \rangle_S = A^{-1} \left[ \frac{z}{z-1} f_1(x, z) - \frac{1}{z-1} f_1(x) \right] \quad (10)$$

$$\langle R^2 \rangle_S = A^{-1} \left[ \frac{z}{z-1} f_2(x, z) - \frac{1}{z-1} f_2(x) \right] \quad (11)$$

$$\langle R_{11} \rangle_S = A^{-1} \left[ \frac{z}{z-1} f_1(x, z) - \frac{z}{(z-1)^2} f_0(x, z) + \frac{z}{(z-1)^2} f_0(x) \right] \quad (12)$$

$$\langle R_{11}^2 \rangle_S = A^{-1} \left[ \frac{z}{z-1} f_2(x, z) - \frac{2z}{(z-1)^2} f_1(x, z) + \frac{z^2+z}{(z-1)^3} f_0(x, z) - \frac{z^2+z}{(z-1)^3} f_0(x) \right] \quad (13)$$

$$\langle RR_{11} \rangle_S = A^{-1} \left[ \frac{z}{z-1} f_2(x, z) - \frac{z}{(z-1)^2} f_1(x, z) + \frac{z}{(z-1)^2} f_1(x) \right], \quad (14)$$

where

$$f_0(\alpha) = (\alpha^{N+1} - 1) / (\alpha - 1)$$

$$f_1(\alpha) = (N\alpha^{N+2} - (N+1)\alpha^{N+1} + \alpha) / (\alpha - 1)^2$$

$$f_2(\alpha) = \frac{(N^2\alpha^{N+3} - (2N^2+2N-1)\alpha^{N+2} + (N+1)\alpha^{N+1} - \alpha^2 - \alpha)}{(\alpha - 1)^3}$$

In the case of resonances  $\text{ctg}^2 \theta = 1$  and  $\frac{\gamma_{31}}{\gamma_{23}} = 1$  relations (10-14) reduce to

$$\langle R \rangle_S = 2 \langle R_{11} \rangle_S = \frac{2}{3} N \quad (15)$$

$$\langle R^2 \rangle_S = 2 \langle R R_{11} \rangle_S = \frac{1}{2} N(N + \frac{1}{3}) \quad (16)$$

$$\langle R_{11}^2 \rangle_S = \langle R_{22}^2 \rangle_S = \frac{1}{6} N(N + \frac{1}{2}). \quad (17)$$

### III. COLLECTIVE SPECTRAL PROPERTIES OF SCATTERED LIGHT

As in ref.<sup>/23/</sup>, the steady state spectrum of the spontaneous emission corresponding to transition  $|2\rangle \rightarrow |3\rangle$  (Stokes line) is proportional to the Fourier transform of the atomic correlation function

$$\langle J_{23}(\tau) J_{32} \rangle_S = \lim_{t \rightarrow \infty} \langle J_{23}(t+\tau) J_{32}(t) \rangle.$$

Using the secular approximation and the quantum regression theorem<sup>/26/</sup>, one can find the equation of motion for the correlation functions  $\langle R_{ij}(\tau) J_{32} \rangle_S$  ( $i, j = 1, 2, 3$ ) in the form

$$\frac{d}{d\tau} \langle R_{13}(\tau) J_{32} \rangle_S = i(\Omega_3 - \Omega) \langle R_{13}(\tau) J_{32} \rangle_S - \langle \Gamma_{13}(\tau) R_{13}(\tau) J_{32} \rangle_S \quad (18)$$

$$\frac{d}{d\tau} \langle R_{23}(\tau) J_{32} \rangle_S = i(\Omega_3 + \Omega) \langle R_{23}(\tau) J_{32} \rangle_S - \langle \Gamma_{23}(\tau) R_{23}(\tau) J_{32} \rangle_S, \quad (19)$$

where

$$\begin{aligned} \Gamma_{13}(\tau) = & \gamma_{21} \sin^2 \zeta + 2\gamma_{23} \sin^2 \zeta + \nu_{31} \sin^2 \zeta \\ & + \gamma_{21} (\sin^2 \zeta - \cos^2 \zeta) R_{22}(\tau) + (\nu_{31} \sin^2 \zeta - \gamma_{23} \cos^2 \zeta) R_{22}(\tau) \\ & + (\gamma_{23} \sin^2 \zeta - \nu_{31} \cos^2 \zeta) (R_{33}(\tau) - R_{11}(\tau)) \end{aligned} \quad (20)$$

$$\begin{aligned} \Gamma_{23}(\tau) = & \gamma_{21} \cos^2 \zeta + 2\gamma_{23} \cos^2 \zeta + \nu_{31} \cos^2 \zeta \\ & + \gamma_{21} (\cos^2 \zeta - \sin^2 \zeta) R_{11}(\tau) + (\nu_{31} \cos^2 \zeta - \gamma_{23} \sin^2 \zeta) R_{11}(\tau) \\ & + (\gamma_{23} \cos^2 \zeta - \nu_{31} \sin^2 \zeta) (R_{33}(\tau) - R_{22}(\tau)). \end{aligned} \quad (21)$$

For the one-atom case one can use the well-known operator relation<sup>/25/</sup>

$$R_{ij} R_{ke} = R_{ie} \delta_{kj}$$

and eqs.(18,19) reduce to

$$\begin{aligned} \frac{d}{d\tau} \langle R_{13}(\tau) J_{32} \rangle_S = & i(\Omega_3 - \Omega) \langle R_{13}(\tau) J_{32} \rangle_S \\ & - \beta_{13} \langle R_{13}(\tau) J_{32} \rangle_S \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{d}{d\tau} \langle R_{23}(\tau) J_{32} \rangle_S = & i(\Omega_3 + \Omega) \langle R_{23}(\tau) J_{32} \rangle_S \\ & - \beta_{23} \langle R_{23}(\tau) J_{32} \rangle_S, \end{aligned} \quad (23)$$

where

$$\beta_{13} = \gamma_{21} \sin^2 \zeta + \gamma_{23} \sin^2 \zeta + \nu_{31} \quad (24)$$

$$\beta_{23} = \gamma_{21} \cos^2 \zeta + \gamma_{23} \cos^2 \zeta + \nu_{31}. \quad (25)$$

The value  $\beta_{13}$  and  $\beta_{23}$  can be considered as the widths of two spectrum components of Stoke line located at the frequencies  $\omega_{23} - \frac{\Omega}{2} - \Omega$  and  $\omega_{23} - \frac{\Omega}{2} + \Omega$  respectively. It is easy to show that the values  $\beta_{13}$  and  $\beta_{23}$  in relations (24,25) coincide with the results of the previous papers of the one-atom Raman scattering<sup>/23/</sup>.

It is also easy to see that in the case of resonance  $\text{ctg}^2 \zeta = 1$  and  $\frac{\nu_{31}}{\gamma_{23}} = 1$  the operators  $\Gamma_{13}(\tau)$  and  $\Gamma_{23}(\tau)$  in eqs.(20,21) become the  $C$ -numbers and spectrum widths of the stokes line for the collective case are the same as in the single-atom spectrum.

For the general case by analogy with the papers<sup>/11,22/</sup>, we factorize

$$\begin{aligned} \langle \Gamma_{13}(\tau) R_{13}(\tau) J_{32} \rangle_S = & \langle \Gamma_{13} \rangle_S \langle R_{13}(\tau) J_{32} \rangle_S \\ \langle \Gamma_{23}(\tau) R_{23}(\tau) J_{32} \rangle_S = & \langle \Gamma_{23} \rangle_S \langle R_{23}(\tau) J_{32} \rangle_S \end{aligned} \quad (26)$$

as in references<sup>/11,22/</sup>. By using the relations (10-14) one can show that in the case of large  $N$  the factorization (26) yields a small error (with an order higher than  $1/\sqrt{N}$ ) in the calculation of the steady-state fluorescent intensity spectrum.

Using the factorization (26) and solution (4), one can find the

solution of eqs.(18,19) and write the atomic correlation function in the form

$$\langle J_{23}(\tau) J_{32} \rangle_S = \sin^2 \mathcal{G} \langle R_{13} R_{31} \rangle_S e^{i(\Omega_3 - \Omega)t - \langle \Gamma_{13} \rangle_S t} + \cos^2 \mathcal{G} \langle R_{23} R_{32} \rangle_S e^{i(\Omega_3 + \Omega)t - \langle \Gamma_{23} \rangle_S t}, \quad (27)$$

where

$$\langle R_{13} R_{31} \rangle_S = (N+1) \langle R_{11} \rangle_S - \langle R R_{11} \rangle_S \quad (28)$$

$$\langle R_{23} R_{32} \rangle_S = (N+1) (\langle R \rangle_S - \langle R_{11} \rangle_S) - \langle R^2 \rangle_S + \langle R R_{11} \rangle_S. \quad (29)$$

The values  $\langle R \rangle_S$ ,  $\langle R_{11} \rangle_S$ ,  $\langle R^2 \rangle_S$  and  $\langle R R_{11} \rangle_S$  can be found in relations (10-14). Expression (27) yields the two-peaked structure of the Stoke spectrum. The spectrum components located at the frequencies  $\omega_{23} - \frac{\delta}{2} - \Omega$  and  $\omega_{23} - \frac{\delta}{2} + \Omega$  have the intensities

$$I_{-\Omega} = \sin^2 \mathcal{G} \langle R_{13} R_{31} \rangle_S \quad \text{and}$$

$$I_{+\Omega} = \cos^2 \mathcal{G} \langle R_{23} R_{32} \rangle_S$$

and the widths  $\langle \Gamma_{13} \rangle_S$  and  $\langle \Gamma_{23} \rangle_S$ , respectively. For the case of resonance  $\text{ctg}^2 \mathcal{G} = 1$  the intensities and widths of two-spectrum components are equal (i.e.,  $I_{-\Omega} = I_{+\Omega}$  and  $\langle \Gamma_{13} \rangle_S = \langle \Gamma_{23} \rangle_S$ ).

Using relations (15-17), it is easy to show that in the case of  $\text{ctg}^2 \mathcal{G} = 1$ ;  $\nu_{31}/\nu_{23} = 1$  the peak intensity of each spectrum component of the Stoke line in (27) varies as  $N^2$  while, as has been mentioned before, the width of each component is the same as in the single-atom spectrum. The spectrum picture changes for the case of  $\text{ctg}^2 \mathcal{G} \neq 1$ . By using the relations (27-29) and (10-14), one can show that for the case of  $\text{ctg}^2 \mathcal{G} \neq 1$  or  $\nu_{31}/\nu_{23} \neq 1$  and the number of atoms  $N$  large enough (take for example for the case of  $X = \frac{\nu_{31}}{\nu_{23}} \text{ctg}^2 \mathcal{G} > 1$ ;  $XZ = \frac{\nu_{31}}{\nu_{23}} \text{ctg}^2 \mathcal{G} > 1$  it is necessary  $N \gg 1$  so that  $X^N \gg 1$  and  $(XZ)^N \gg \frac{\nu_{23}}{\nu_{31}}$ ) the intensities and widths of all spectrum components of the Stoke line are proportional to  $N$ .

The detailed behaviour of intensities per atom of the two Stoke spectra  $I_{-\Omega}/N^2$  and  $I_{+\Omega}/N^2$  as a function of  $\text{ctg}^2 \mathcal{G}$ , where  $\frac{\nu_{31}}{\nu_{23}} = 1$  is shown in fig.2, and as a function of  $\frac{\nu_{31}}{\nu_{23}}$ , where  $\text{ctg}^2 \mathcal{G} = 1$  is shown in fig.3. For all finite values of  $N$  one observes a smooth variation of functions  $I_{-\Omega}/N^2$  and  $I_{+\Omega}/N^2$  with the parameters

$\text{ctg}^2 \mathcal{G}$  or  $\frac{\nu_{31}}{\nu_{23}}$ . For large  $N$  the intensities of spectrum components are large only in the around vicinity of the point  $\text{ctg}^2 \mathcal{G} = 1$ ,  $\nu_{31}/\nu_{23} = 1$  (see figs. 2,3). For the cooperative limit  $N \rightarrow \infty$  the peak intensities per atoms  $I_{-\Omega}/N^2$  and  $I_{+\Omega}/N^2$  have a discontinuous behaviour analogous to a typical nonequilibrium first-order phase transition /14,15;20,22/.

In conclusion we want to note that the characteristics for spectra of the Rayleigh line can be obtained using an analogous approach.

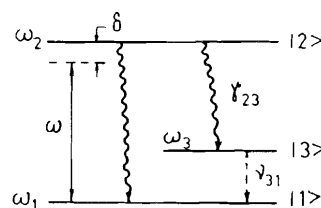


Fig.1. Three-level system of atoms interacting with the monochromatic applied field and with radiation.

Fig.2. Peak intensities per atom  $I_{-\Omega}/N^2$  (dashed curves) and  $I_{+\Omega}/N^2$  (Solid curves) as a function of  $\text{ctg}^2 \mathcal{G}$  when  $\nu_{31}/\nu_{23} = 1$ . The dotted curve indicates the behaviour as  $N \rightarrow \infty$ .

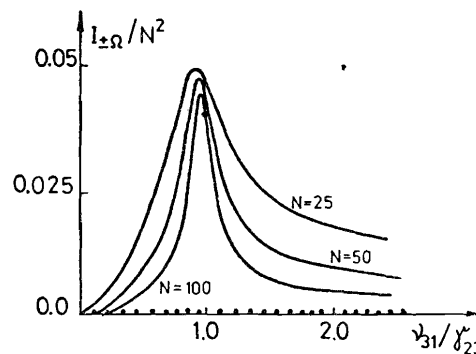
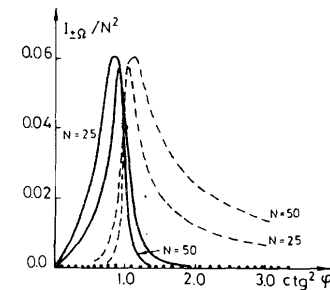


Fig.3. Peak intensities per atom  $I_{\pm}/N^2$  as a function of  $\nu_{31}/\nu_{23}$  when  $\text{ctg}^2 \mathcal{G} = 1$ . The dotted curve indicates the behaviour as  $N \rightarrow \infty$

REFERENCES

1. Dicke R.H., Phys.Rev. 93 (1954) 99.
2. Agarwal G.S., in: Quantum optics (Springer, Berlin, 1974).
3. MacGillivray J.C., Phys.Rev. A14 (1976) 1169.
4. Bonifacio R. and Lugiato L.A., Phys.Rev. A11 (1975) 1507, A12 (1975) 587.
5. Polder P., Shurmans M.F.H., Vrehan Q.H.F., Phys.Rev.A19(1979),1192.
6. Haake F., King H., Schroeder G., Haus J., Glauber R., Phys.Rev. A20 (1979) 2047.
7. Gibbs H.M., Vrehan Q.H.F., Hikspoors H.M.J., Phys.Rev.Lett. 39 (1977) 547.
8. Skribanowitz N., Herman J.P., MacGillivray J.C., Feld M.S., Phys. Rev.Lett. 30 (1973) 309.
9. Florian R., Schwan L.O. and Schmid O., Phys.Rev. A29 (1984) 2709.
10. Moi L., Goy P., Gross M., Raimond J.M., Fabre C., Haroch S., Phys. Rev. A27 (1989) 2043 and A27 (1983) 2065.
11. Compagno G., Persico F., Phys.Rev. A25 (1982) 3138.
12. Agarwal G.S., Narducci L.M., Feng P.H., Gilmore R., Phys.Rev.Lett., 42 (1979) 1260.
13. Narducci L.M., Feng P.H., Gilmore R., Agarwal G.S., Phys.Rev. A18 (1978) 1571.
14. Puri R.R., Lawande S.V., Phys.Lett. 72A (1979) 200.
15. Drummond P.D., Phys.Rev. A22 (1980) 1179.
16. Bogolubov(jr) N.N., Shumovsky A.S., Tran Quang, JINR (1986), E4-86-347, Dubna.
17. Bonifacio R., Lugiato L.A., Phys.Rev., A18 (1978) 1129.
18. Drummond P.D., Walls D.F., J.Phys. A13 (1980) 725.
19. Drummond P.D., Walls P.F., Phys.Rev. A23 (1981) 2563.
20. Bogolubov N.N.(jr), Shumovsky A.S., Tran Quang, Phys.Lett., 112A (1985) 323.
21. Raymer M.G., Walmsley I.A., Mostowski J., Sobolevska B., Phys. Rev. A32 (1985) 332.
22. Bogolubov N.N.(jr), Shumovsky A.S., Tran Quang, JINR (1985) E17-85-679; J.of Phys.B 1986 (to be published).
23. Agarwal G.S., Sudhashu S.Jha., J.of Phys.B, 1979, 12, 2655.
24. Schwinger J.V. in: Quantum Theory of Angular Momentum ed. by L.C.Biedenharm and H.VanDam (Academie press, New-York, 1985).
25. Louisell W.H., Radiation and Noise in Quantum Electronics (Mc. Graw-Hill Book Company, New-York).
26. Lax M., Phys.Rev. 172 (1968) 350.

Received by Publishing Department  
on October 11, 1986.

WILL YOU FILL BLANK SPACES IN YOUR LIBRARY?

You can receive by post the books listed below. Prices - in US \$,  
including the packing and registered postage

- |               |  |       |
|---------------|--|-------|
| D3,4-82-704   | Proceedings of the IV International School on Neutron Physics. Dubna, 1982   | 12.00 |
| D11-83-511    | Proceedings of the Conference on Systems and Techniques of Analytical Computing and Their Applications in Theoretical Physics. Dubna, 1982.                            | 9.50  |
| D7-83-644     | Proceedings of the International School-Seminar on Heavy Ion Physics. Alushta, 1983.   | 11.30 |
| D2,13-83-689  | Proceedings of the Workshop on Radiation Problems and Gravitational Wave Detection. Dubna, 1983.   | 6.00  |
| D13-84-63     | Proceedings of the XI International Symposium on Nuclear Electronics. Bratislava, Czechoslovakia, 1983.  | 12.00 |
| E1,2-84-160   | Proceedings of the 1983 JINR-CERN School of Physics. Tabor, Czechoslovakia, 1983.  | 6.50  |
| D2-84-366     | Proceedings of the VII International Conference on the Problems of Quantum Field Theory. Alushta, 1984.  | 11.00 |
| D1,2-84-599   | Proceedings of the VII International Seminar on High Energy Physics Problems. Dubna, 1984.   | 12.00 |
| D17-84-850    | Proceedings of the III International Symposium on Selected Topics in Statistical Mechanics. Dubna, 1984. /2 volumes/.<br>Dubna, 1984. /2 volumes/.                     | 22.50 |
| D10,11-84-818 | Proceedings of the V International Meeting on Problems of Mathematical Simulation, Programming and Mathematical Methods for Solving the Physical Problems, Dubna, 1983 | 7.50  |
|               | Proceedings of the IX All-Union Conference on Charged Particle Accelerators. Dubna, 1984. 2 volumes.   | 25.00 |
| D4-85-851     | Proceedings on the International School on Nuclear Structure. Alushta, 1985.   | 11.00 |
| D11-85-791    | Proceedings of the International Conference on Computer Algebra and Its Applications in Theoretical Physics. Dubna, 1985.  | 12.00 |
| D13-85-793    | Proceedings of the XII International Symposium on Nuclear Electronics. Dubna, 1985.  | 14.00 |

Orders for the above-mentioned books can be sent at the address:  
Publishing Department, JINR  
Head Post Office, P.O.Box 79 101000 Moscow, USSR

**SUBJECT CATEGORIES  
OF THE JINR PUBLICATIONS**

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condensed matter
18.	Applied researches
19.	Biophysics

Боголюбов Н.Н./мл./, Шумовский А.С. Чан Куанг E4-86-684  
Коллективные спектральные свойства рассеяния  
Рамана

Развита теория коллективного рассеяния Рамана на основе применения квантового уравнения типа "Master Equation" и секулярного приближения. Исследованы влияния расстройки резонанса и других параметров на коллективные свойства рассеянного света.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Bogolubov N.N., Jr., Shumovsky A.S., Tran Quang E4-86-684  
Collective Spectral Properties of Raman  
Scattering

The theory of collective Raman scattering has been developed by using the quantum-mechanical master-equation approach and secular approximation. The influence of the frequency detuning of resonance and other parameters on the collective spectral properties of scattered light is investigated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1986