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INFLUENCE OF THE DEUTERON POLARIZABILITY ON RADIATIVE pd-CAPTURE AT ASTROPHYSICAL LOW ENERGIES

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1. Introduction

Experimental measurements of deuteron elastic scattering on recently performed[1] point to a significant deviation from the Rutherford scattering. This effect originates from a large magnitude of the deuteron polarizability arising in the nucleus Coulomb field. It can be shown that this phenomenon may be described by an extra effective polarization potential of attraction, introduced in addition to the Coulomb potential, decreasing at long distances r as r^{-4}.

A specific feature of the scattering in the potential, the sum of the Coulomb and polarizational ones, is a strong dependence on the energy of the elastic scattering amplitude in a low-energy region, and switching on the strong interaction does not practically change that dependence[2]. Thus, in studies[3,4] of the elastic pd-scattering at energies $E < 10$ keV it has been found, in particular[4] that the effective radius function suffers a discontinuity (Fig.1) at an energy about 2 keV. This singularity of the effective-radius function is caused by a strong energy dependence of the wave function of pd-system. In this paper, it is shown that this property of the wave function results in a rapid growth of the cross section $\sigma_{pd}(E)$ of radiative pd-capture at energies $E \le 2$ keV. A behaviour of that type of the cross section qualitatively differs from that predicted by the theory that does not take account of the polarizability of deuteron.

Hence it follows that the astrophysical S-factor of the reaction $p + d \to {}^3He + \gamma$

$$S_{12}(E) = E\, e^{2\pi\eta(E)}\, \sigma_{pd}(E) \qquad (1)$$

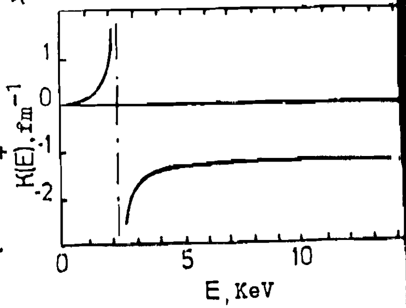

Fig. 1. Function of the effective radius $K(E) = C_0^2(\eta)\times q\,ctg\,\delta + 2\mu e^2 h(\eta)$ (for the notation see the text) for the case of doublet elastic pd-scattering[4].

($\eta = \mu e^2/q$ is the Sommerfeld parameter, μ is the reduced mass, $q = \sqrt{2\mu E}$) is not a smooth function of the energy, and therefore it cannot trivially be extrapolated to the region of astrophysical low energies from the existing experimental data. A further strong energy dependence of the cross section σ_{pd} gives rise to a sharp increase in the yield of ${}^3He$ *) nuclei and in the flux of "boron" neutrinos.

2. Calculation of the Cross Section of Reaction $p + d \to {}^3He + \gamma$

Assuming charge independence of nuclear forces we shall describe the effects of strong interaction in pd-system by the corresponding amplitude of radiative nd-capture. As is known, the radiative nd-capture at low energies proceeds mainly through the magnetic dipole transition whose amplitude $X_{nd}(E)$ is proportional to the overlapping integral

$$X_{nd}(E) \sim \langle \Psi_T | \Psi_{nd} \rangle ,$$

where $|\Psi_T\rangle$ is the tritium wave function, and $|\Psi_{nd}\rangle$ is the function of nd-system. It can be shown that the amplitude of S-wave nd-capture from a state with spin S may approximately be represented in the form

$$X^S_{nd}(E) = \frac{a^S}{b^2 + p^2}\left[1 + f^S_{nd}(p)(b + ip)\right], \quad E = p^2/2\mu , \qquad (2)$$

where f^S_{nd} - is the amplitude of elastic nd-scattering, b^2 is the tritium binding energy, a^S is the constant specifying spin properties of 3-body system. Using (2) we parametrize the half-mass-shell amplitude of nd-capture by the expression

$$X_{nd}(E, p) = \frac{a(E)}{b^2(E) + p^2}, \quad E \ne p^2/2\mu . \qquad (3)$$

As it is shown in ref.[5], at low energies the function $b(E)$ is a constant, and in the case of capture of S-wave neutrons it lies in the interval $0.22\ fm^{-1} \le b \le 0.25\ fm^{-1}$.

As amplitude (3) is defined in the plane-wave basis, the long-range interaction in pd-system may be taken into account with the help of the transformations:

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*) It is to be noted that the same reasonings are, obviously, valid if an incident proton is replaced by a tritium nucleus.

$$X_{pd}(E)=\int X_{nd}(E,p)\,\Psi(q,\vec{p})\frac{d\vec{p}}{(2\pi)^3}\,,\quad q^2=2\mu E \tag{4}$$

where $\Psi(q,\vec{p})$ is the wave function that takes account of the Coulomb and effective polarization potentials in pd -system, and $X_{pd}(E)$ is the amplitude of radiative pd -capture.

In the coordinate representation the S -wave component of the function $\Psi(q,\vec{p})$ obeys the equation

$$\Psi(q,r)=\frac{F_0(qr)}{qr}+4\pi\int_0^\infty r'^2dr'\,g_C(r,r',q)\,V_p(r')\,\Psi(q,r'). \tag{5}$$

where $F_0(qr)$ is a regular Coulomb function with the asymptotics $F_0(qr)\simeq\sin(qr-\eta\ln 2qr+\sigma_0)$, σ_0 is the Coulomb-scattering phase;

$g_C(r,r',q)$ is the Coulomb Green function:

$$g_C(r,r',q)=-i\frac{\mu}{2\pi}\cdot\frac{1}{qrr'}\Big\{F_0(qr)H_0(qr')\,\theta(r'-r)+H_0(qr)F_0(qr')\,\theta(r-r')\Big\},$$

$$\theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases},$$

$$H_0(qr)=F_0(qr)-iG_0(qr)\,,$$

$G_0(qr)$ is an irregular Coulomb function with the asymptotics:

$$G_0(qr)\simeq\cos(qr-\eta\ln 2qr+\sigma_0)\,,\quad r\to\infty$$

$V_p(r)=-\theta(r-r_0)\,\alpha e^2/2r^4$ is the polarization (6)

potential, $\alpha=0.70\pm0.05\ fm^3$ /1/, r_0 is the radius of the sphere outside of which the nuclear interaction may be neglected. Using expression (3) we rewrite amplitude (4) in the form

$$X_{pd}(E)=a(E)\int_0^\infty e^{-br}\,\Psi(q,r)\,r\,dr\,. \tag{7}$$

From equation (5) when $r\to\infty$ we get

$$\Psi(q,r)\simeq\frac{F_0(qr)}{qr}+iqf(q)\frac{H_0(qr)}{qr}\,, \tag{8}$$

where

$$f(q)=-\frac{2\mu}{q}\int_0^\infty r\,dr\,F_0(qr)\,V_p(r)\,\Psi(q,r). \tag{9}$$

At small q the amplitude $f(q)$ up to exponentially small terms is of the form /2/:

$$f(q)\simeq\frac{4}{15}\,\frac{\alpha}{\beta^2}\,q^4\,,\quad \beta^2=2\mu e^2. \tag{10}$$

For $r<r_0$ there is no polarization potential (6), which leads to a smooth dependence of the wave function on energy at short distances. A strong energy dependence resulting from a long-range nature of the polarization potential does appear when $r\gg r_0$, therefore approximation (8) can be made.

Substituting (8) into (7) and using (3) we obtain for $X_{pd}(E)$

$$X_{pd}(E)\simeq X_{nd}(E)(b^2+q^2)\big[I(q)+J(q)\big]\,,$$

where

$$I(q)=\frac{1}{q}\int_0^\infty e^{-br}F_0(qr)\,dr\,,$$
$$J(q)=i\,f(q)\int_0^\infty e^{-br}H_0(qr)\,dr\,. \tag{11}$$

The integral in (11) may be written in the form

$$\frac{1}{b}\int_0^\infty e^{-x}H_0\Big(\frac{q}{b}x\Big)dx\,, \tag{12}$$

whence it follows that in the energy region from 0 to 20 keV, where $q/b\ll1$, integral (12) is calculated with a good accuracy if the function $H_0(z)$ is approximated by

$$H_0(z)\simeq C_0(\eta)\,z(1+\eta z)-$$
$$-iC_0^{-1}(\eta)\big[1+2\eta z(\ln 2z+2\gamma-1+h(\eta)+\ln\eta)\big]\,, \tag{13}$$

$\gamma=0.57721$ is the Euler constant, $C_0^2(\eta)=2\pi\eta\,[e^{2\pi\eta}-1]^{-1}$, $h(\eta)=\mathrm{Re}\,\psi(i\eta)-\ln\eta$.

As a result, for $J(q)$ we find

$$J(q)=\frac{i}{b}\,f(q)\,C_0(\eta)\Big\{\Big(1+\frac{\beta}{b}\Big)\frac{q}{b}-iC_0^{-2}(\eta)\Big[1+\frac{\beta}{b}\Big(\ln\frac{\beta}{b}+h(\eta)+\gamma\Big)\Big]\Big\}.$$

In the same approximation for $I(q)$

$$I(q)=\frac{1}{\varepsilon^2}C_0(\eta)\left(1+\frac{\beta}{\varepsilon}\right).$$

And finally, the amplitude of radiative pd -capture is determined by the expression

$$X_{pd}(E)=C_0(\eta)X_{nd}(E)\left(1+q^2/\varepsilon^2\right)\Big\{1+\beta/\varepsilon+ \qquad (14)$$
$$+\frac{f(q)\beta}{C_0^2(\eta)}\left[i\frac{q}{\beta}C_0^2(\eta)\left(1+\beta/\varepsilon\right)+\varepsilon/\beta-\ln\varepsilon/\beta+h(\eta)+\gamma\right]\Big\}.$$

The total cross section of pd -capture (M1-transition) may be represented in the form

$$\sigma_{pd}=\frac{1}{3}\,{}^2\sigma_{nd}\left|\frac{{}^2X_{pd}}{{}^2X_{nd}}\right|^2+\frac{2}{3}\,{}^4\sigma_{nd}\left|\frac{{}^4X_{pd}}{{}^4X_{nd}}\right|^2, \qquad (15)$$

where ${}^{2,4}\sigma_{nd}$ and ${}^{2,4}X$ are cross sections and amplitudes of radiative Nd -capture from states with fixed spin, 1/2 and 3/2, respectively.

According to (14) the ratio X_{pd}/X_{nd} is spin-independent. This property of amplitudes results from spin independence of the Coulomb and polarization potentials. Then, from (15) we obtain

$$\sigma_{pd}=\sigma_{nd}\left|\frac{X_{pd}}{X_{nd}}\right|^2, \qquad (16)$$

where σ_{nd} is the total cross section of the neutron radiative capture, and the ratio of amplitudes is determined by expression (14).

3. The Problem of Solar Neutrino

From expressions (16) and (14) it follows that the function $S_{12}(E)$ in formula (1) is a constant only in the approximation that does not take account of the deuteron polarizability (α = 0). Account of the polarizability leads to a strongly energy-dependent term in the amplitude X_{pd} proportional to the quantity

$$\frac{f(q)\beta}{C_0^2(\eta)}\simeq\frac{4}{15\pi}\cdot\frac{\alpha}{\beta^2}q^5e^{\pi\frac{\beta}{q}}. \qquad (17)$$

The solid curve in Fig.2 represents the energy dependence of the astrophysical factor $S_{12}(E)$ calculated by formulae (1), (14), (16) at a value of σ_{nd} taken from experiments/6/ on thermal neutrons and extrapolated by the law $1/v$ into the region of energies of several keV. The dashed line is obtained in the approximation α = 0. As is seen from the Figure, both the curves coincide in a wide energy region up to 4 keV. However, at $E<3$ keV the contribution of the polarization potential becomes significant, therefore at E = 2 keV the difference is 1.5, at E= 1.5 keV - 8, and at the energy E = 1.3 keV corresponding to the temperature at the sun centre $T_c\simeq 1.5\ 10^7$K/7/ the difference reaches 50.

In Fig.3 we show the total cross section of radiative pd -capture in the energy interval from 0.3 to 45 keV (in the lab.system). Both the curves calculated with and without deuteron polarizability well describe the experimental data/8/, however, at energies of an order of 2 keV, where there are no experimental data, the cross sections become qualitatively differing*).

A sharp growth of the cross section of radiative pd -capture leads to a considerable increase of the yield of ${}^3He$ nuclei. The rate of production of these nuclei is proportional to the quantity $\langle\sigma_{pd}v\rangle$, where averaging is made over the Maxwell distribution:

$$\langle\sigma_{pd}v\rangle=\left(\frac{\mu}{2\pi kT}\right)^{3/2}\int d\vec{v}\,e^{-\frac{\mu}{2kT}(\vec{v}-\vec{v}_0)^2}\sigma_{nd}(E)v\left|\frac{X_{pd}(E)}{X_{nd}(E)}\right|^2,$$

Here v_0 is the mean velocity.

Let us estimate the quantity $\langle\sigma_{pd}v\rangle$ at temperatures close to T_c and compare it with an analogous quantity calculated in the standard theory that does not take account of the deuteron polarizability/9/.

Considering that the width ΔE of the Maxwell distribution near $T\simeq T_c$ amounts to $\Delta E\sim 0.3$ eV for the ratio

$$R(T)=\langle\sigma_{pd}v\rangle/\langle\sigma_{pd}v\rangle_{\alpha=0} \qquad (18)$$

we obtain the values reported in the Table.

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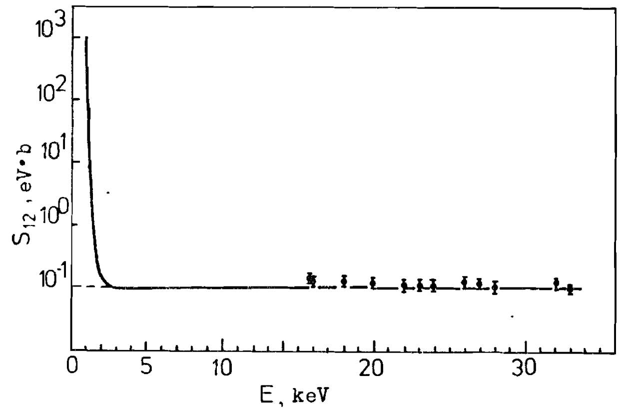
*)The cross section of pd -capture turns to infinity because the cross section of elastic pd -scattering when the polarization potential is present also turns to infinity at E = 0/3,4/.



F i g. 2. The energy dependence of the astrophysical factor S_{12}. The solid curve corresponds to the value $\alpha = 0.70$ fm^3, the dashed line-to $\alpha = 0$. Experimental data are taken from ref./8/.

T a b l e. The ratio (18) of yields of ^3He nuclei in reaction $p + d \rightarrow {}^3He + \gamma$.

T(kev)	I	I	1.2	1.3	1.4	1.5	1.6	3
R'	5.7 10^3	0.89 10^3	1.9 10^2	51.3	18.2	8.2	4.5	1.05

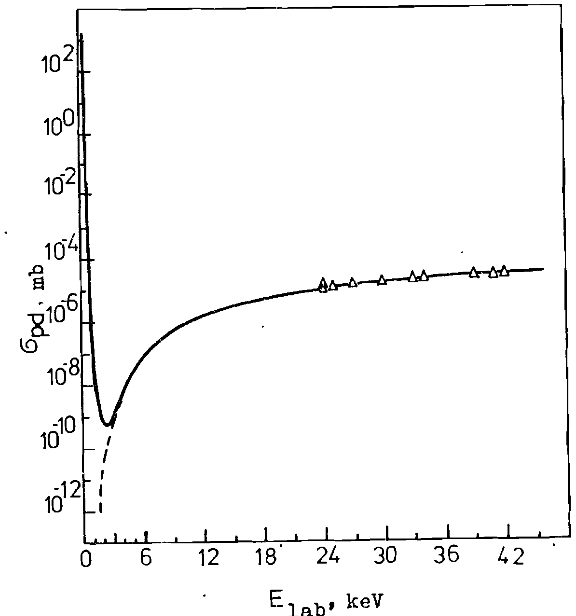
As is seen from the Table at the temperature T = 1.3 keV accepted in the standard model of the sun for its central part the yield of ^3He nuclei is 50 times as large as the value used in calculating the neutrino flux R_ν from the sun/9/. Since

$$R_\nu \sim \langle \sigma_{pd} v \rangle$$

the obtained disastrous growth of the yield should result in the "boron" neutrinos flux $\Phi \simeq 350$ SNU !

It is obvious that an accurate description of the kinetics of all involved processes becomes very important.

Note that from all the above considerations one may conclude about increasing the cross section of reaction $^3He + {}^3He \rightarrow {}^4He + 2p$ in the region of several keV, i.e., about decreasing the "boron" neutrino flux from the sun.



F i g. 3. The cross section of radiative pd -capture as a function of energy. The notation is the same as in Fig. 2. Experimental data are taken from ref./8/.

The predicted increase of the yield of nuclei in reactions of radiative pd -capture (and the corresponding feasible increase of the neutrino flux) has a simple qualitative interpretation. The attractive polarization potential at long distances diminishes the action of the Coulomb barrier between proton and deuteron . Due to the long-range nature of both the potentials the above compensation is strongly dependent on energy. As is seen from Fig.1, at the energy of $\sim$2 keV the elastic pd -scattering amplitude turns into zero, i.e., the interaction at that energy in an initial state vanishes, which just promotes the increase of the capture amplitude.

However, it should be remembered that the above estimates have been performed under the assumption for the deuteron polarizability to be energy-independent. Nevertheless, from a physical point of view it is clear that at sufficiently low collision energies (e.g., in a thermal region) the deuteron can be considered as an absolutely rigid system , i.e., the influence of polarizability may be neglected.

Thus, we are faced urgent necessity of a more thorough theoretical analysis and experimental study of the radiative *pd* -capture at relative energies of colliding particles from 1 to 3 keV.

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Влияние поляризуемости дейтрона на радиационный pd -захват при астрофизических низких энергиях

Обсуждается роль поляризуемости дейтрона, возникающей в кулоновском поле протона в процессе радиационного pd -захвата в области энергий от 1 до 30 кэВ. Показано, что при энергиях $E \leq 2$ кэВ сечения реакций $p + d \to {}^3He + \gamma$ рассчитанные с учетом и пренебрежением поляризуемостью дейтрона сильно различаются. При астрофизических низких энергиях S-фактор оказывается быстро растущей функцией энергии, что приводит к резкому увеличению выхода ядер ^3He /"гелиевая катастрофа"/. Согласно стандартной модели Солнца это должно привести к возрастанию потока "борных" нейтрино, R_ν. Дана оценка величины R_ν для температур, соответствующих энергиям, меньшим 2 кэВ. Рассчитанное в работе сечение /и S -фактор/ совпадает с экспериментальным в той области низких энергий, где измерения приведены.

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Influence of the Deuteron Polarizability on Radiative pd-Capture at Astrophysical Low Energies

The role of deuteron polarizability is discussed that arises in the Coulomb field of the proton in the process of radiative pd -capture in the region of energies from 1 to 30 keV. It is shown that at energies $E \leq 2$ keV the cross sections of reaction $p + d \to {}^3He + \gamma$ calculated with and without the deuteron polarizability are very different. At astrophysical low energies the S-factor appears to be a rapidly growing function of energy, which leads to a sharp growth of the yield of ^3He nuclei("Helium catastrophe"). According to the standard theory of the sun this should increase the "Boron"-neutrino flux R_ν. The quantity R_ν is estimated for temperatures corresponding to energies lower than 2 keV. The calculated cross section (and S-factor) coincides with the experimental one in the low-energy region available for measurements.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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