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INFLUENCE OF PAIRING CORRELATIONS ON THE PROBABILITY AND DYNAMICS OF TUNNELLING THROUGH THE BARRIER IN FISSION AND FUSION OF COMPLEX NUCLEI

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1. INTRODUCTION: THE DYNAMICALLY INDUCED ENHANCEMENT OF SUPERFLUIDITY IN TUNNELLING THROUGH THE FISSION BARRIER

Intensive experimental and theoretical studies performed during the last two decades have led to a considerable progress in clearing up the prominent role of nuclear shell effects in the fission process /1,2/. The shell structure of nuclei -- large-scale nonuniformities in the energy distribution of the individual particles, especially near the Fermi energy /2/ -- has been demonstrated to be a factor which influences in many important ways the probability and dynamics of low-energy fission and, in particular, substantially enhances the stability of nuclei against spontaneous fission. In comparison with the shell structure effects, the role of residual nuclear correlations in large-scale cold rearrangements of nuclei and, first of all, the role of the nucleon pairing correlations of superconducting type remains still to be much less clear both experimentally and theoretically.

At the same time, simple and quite reliable theoretical estimates indicate /2-10/ that the pairing correlations (whose intensity is usually specified by the magnitude of the pairing gap parameter Δ) can noticeably modify both the potential energy $V(q,\Delta)$ and the effective mass $M(q,\Delta)$ associated with a large-scale subbarrier rearrangement of a nucleus, i.e. the main ingredients of the action integral

$$S(L) = 2\int_{q_1}^{q_2}\left\{\frac{2}{\hbar^2}\left[V(q,\Delta) - E\right]\ M(q,\Delta)\right\}^{1/2} \qquad (1)$$

and thus they can strongly affect the penetrability of the potential barrier

$$p = \left\{1 + \exp\left[S(L_{min})\right]\right\}^{-1} \qquad (2)$$

and, in particular, the spontaneous fission half-life

$$T_{sf}\ [\text{years}] = \frac{\ln 2}{np} \simeq 10^{-28}\exp\left[S(L_{min})\right], \qquad (3)$$

where $n \simeq 10^{20.4}s^{-1}$ /4/ is the number of assaults of the nucleus on the fission barrier per unit time, and p is the probability of tunnelling through the barrier for a given assault. Expressions (1) and (2) describe, in the WKB approximation, the penetrability of a one-dimensional potential barrier along some effective trajectory L given in a multidimensional space of deformations α_i $(i=1,2,\ldots,m)$; the parameter q specifies the position of a point on the trajectory L, with q_1 and q_2 corresponding to the classical turning points at which $V(q,\Delta)=E$, E is the total energy of the system ($E=E_0\simeq 0.5$ MeV for spontaneous fission /4/), and L_{min} is the least action trajectory determined by the variational condition

$$\delta S(L) = 0. \qquad (4)$$

The effective mass associated with motion along the trajectory L has the form [2,5]

$$M(q,\Delta) \equiv M_{qq}(q,\Delta) = \sum_{i=1}^{m}\sum_{j=1}^{m} M_{\alpha_i\alpha_j}(\alpha_1(q),\alpha_2(q),\ldots,\alpha_m(q),\Delta)\frac{d\alpha_i}{dq}\cdot\frac{d\alpha_j}{dq}. \quad (5)$$

where $M_{\alpha_i\alpha_j}$ are components of the (symmetric) mass tensor which correspond to the deformation parameters α_i and α_j.

For a qualitative discussion of global relative changes in the barrier penetrability that are associated with the presence of pairing correlations it will be convenient to employ expressions for $V(q,\Delta)$ and $M(q,\Delta)$ obtained in the "uniform" single-particle model, i.e. assuming uniform level spacings in single-particle energy spectra. Although the use of this assumption may lead to certain quantitative distorsions in details, its great advantage lies in the possibility of deriving a number of transparent formulae disclosing principal features of the penetrability pattern.

In the uniform model, the potential energy will have, in quadratic approximation, the following form [9]:

$$V(q,\Delta) \simeq V_0(q,\Delta_0) + g(\Delta - \Delta_0)^2. \quad (6)$$

Here $V_0(q,\Delta_0)$ is the potential energy for $\Delta = \Delta_0$, Δ_0 being the stationary solution of the pairing gap equation

$$\frac{\partial \langle H\rangle}{\partial \Delta} = 0, \quad (7)$$

where $\langle H\rangle$ is the expectation value of the pairing Hamiltonian, and g is the total density of the doubly degenerate single-particle levels inclusive of protons and neutrons.

For estimating the Δ dependence of the effective mass one can use the well-known result of the adiabatic cranking model [2-6]:

$$M(q,\Delta) \approx \frac{F(q)}{\Delta^2} + \eta, \quad (8)$$

where the second term, which is approximately constant and generally very small compared to the first one, provides the correct limiting form of eq. (8) at large Δ values. The approximate eq. (8) adequately reproduces the Δ dependence of $M(q,\Delta)$ obtained from numerical cranking calculations (see, e.g., Fig. IX-3 in [2]) and is valid for $\Delta \gg G$, G being the pairing matrix element customarily approximated as $G = \frac{const}{A}$; in the order of magnitude $\frac{G}{\Delta} \sim 0.10-0.15$ [2,4].

It follows from eqs. (6) and (8) that, in fact, variations of Δ can noticeably influence the magnitude of the action integral. As was noted long ago [2-6], this influence results mainly from the strong dependence of the effective mass upon Δ, so that

$$S \propto \Delta^{-1}. \quad (9)$$

So far we implied that the pairing gap parameter is determined at each deformation within the standard BCS approach by requiring the expectation value of the Hamiltonian to be stationary (a minimum) with respect to small variations in Δ [2-5], i.e. by solving eq. (7). For the uniform single-particle model, the thus found gap parameter (Δ_0) will be practically independent of deformation, if the pairing

matrix element G=const does not depend on the nuclear surface area (only the surface-independent pairing is considered in the present paper). For a realistic single-particle model, the gap parameter may change with deformation even at G=const; it is expected to fluctuate around some average value [2-5].

At the same time, the barrier penetrability problem is known to be essentially dynamical [2,5]. Therefore, as has first been stressed by Moretto and Babinet [9], it would be more appropriate to determine Δ in this problem by minimizing the action rather than the expectation value of the Hamiltonian. This means that in searching for the least action trajectory the gap parameter Δ should be treated as a dynamical variable similar to the α_i variables. Such a treatment of pairing correlations that considers Δ as a free variable determined from minimization of the action integral in the space $\{\alpha_1,\alpha_2,\ldots,\alpha_m,\Delta\}$ we will term "the dynamical treatment", in contradistinction to the statical (BCS) consideration based on eq. (7).

Following Moretto and Babinet [9], we shall consider a single deformation coordinate $\alpha_1 \equiv q$ and, by using the uniform approximation, determine the function $\Delta(q)$ which minimizes the action integral (1). On accounting for eqs. (5) and (6), the action integral takes the form

$$S = 2\int_{q_1}^{q_2}\left\{\frac{2}{\hbar^2}\left[V_0(q,\Delta_0) - E + g(\Delta-\Delta_0)^2\right]\left[M_{qq} + 2M_{q\Delta}\frac{d\Delta}{dq} + M_{\Delta\Delta}\left(\frac{d\Delta}{dq}\right)^2\right]\right\}^{1/2} dq. \quad (10)$$

In the case of two dynamical variables (q and Δ) the symmetric mass tensor consists of three components, but, as argued in [9], at least in the uniform model, the main contribution to the effective mass gives the M_{qq} term defined by eq. (8). If we keep this term alone, eq. (10) will not contain the derivatives of Δ with respect to q and thus the variational condition (4) will be reduced to the algebraic equation:

$$\frac{d}{d\Delta}\left\{\left[V_0(q,\Delta_0) - E + g(\Delta-\Delta_0)^2\right]\left[\frac{F(q)}{\Delta^2} + \eta\right]\right\} = 0. \quad (11)$$

That is just the new pairing gap equation relevant to the dynamical problem. For $\eta = 0$ it has an especially simple solution:

$$\frac{\Delta(q)}{\Delta_0} = 1 + \frac{V_0(q,\Delta_0) - E}{g\Delta_0^2}. \quad (12)$$

Hence it follows that the q dependence of Δ will be reminiscent of the barrier profile (Fig. 1a). At $q=q_1$, where $V_0(q_1,\Delta_0)=E$, the gap parameter is $\Delta = \Delta_0$. As the system deepens into the barrier, Δ increases substantially, so that at the saddle point ($q=q_s$) we obtain $\Delta_s \simeq 2.2\,\Delta_0$, after taking the parameter values $V_0(q_s,\Delta_0)-E = B_f = 6$ MeV, $g=9$ MeV^{-1} and $\Delta_0=0.75$ MeV for spontaneous fission of a nucleus with A=250 *). For $q > q_s$ the gap progressively

*) The gap parameter given by eq. (12) is somewhat overestimated because we have assumed $\eta = 0$ in eq. (11); to "compensate" this, we use throughout this paper the value of $g\Delta_0^2 \simeq 5.1$ MeV being 10-15% higher than the "normal" one.

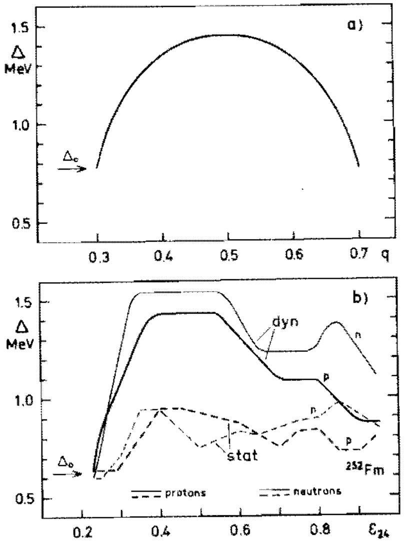

Fig. 1. Enhancement of the gap parameter Δ in tunnelling through the fission barrier: a) a calculation by Moretto and Babinet [9] for a model spontaneously fissioning nucleus with Δ_o=0.775 MeV and a parabolic fission barrier of B_f=6 MeV, see [9]; b) calculations by Staszczak et al. [10] for neutron and proton subsystems of the spontaneously fissioning nucleus ^{252}Fm, carried out in the statical and in the dynamical treatments of pairing correlations. For further details see the text.

narrows and at the $q=q_2$ turning point it is $\Delta=\Delta_o$ again. Thus, the gap equation (12) derived by Moretto and Babinet [9] predicts a dramatic enhancement of nuclear superfluidity in tunnelling through a potential barrier. As has been qualitatively noted in [9], this superfluidity enhancement must have a substantial effect on the spontaneous fission half-lives.

The above conclusions by Moretto and Babinet have recently been confirmed by more realistic calculations carried out by Staszczak et al. [10]. These latter, in contrast to [9], have been performed taking into account two kinds of nucleons and using the Nilsson single-particle potential [4] to calculate the ingredients of the action integral. The shell correction method [2] has been applied to determine the potential energy, whereas the mass parameters have been computed within the cranking model [2,6]. The minimization of the action integral for spontaneous fission of the Fm isotopes with N=134-164 has been performed in the three-dimensional space $\{\varepsilon_{24}, \Delta_n, \Delta_p\}$, where the deformation parameter $\varepsilon_{24}=[\varepsilon, \varepsilon_4(\varepsilon)]$ describing a "symmetric" path to fission is defined as in [11] and Δ_n and Δ_p are the neutron and proton pairing gaps, respectively; in this case all (five) components of the mass tensor have been computed and taken into account. Figure 1b shows that the Δ_n and Δ_p values found by minimizing the action integral substantially exceed those obtained from the standard BCS calculations. Although this enhancement of pairing correlations leads to an increase in the barrier height, on the average, by 20%, it reduces twice the effective mass on the dynamical trajectory in the space $\{\varepsilon_{24}, \Delta_n, \Delta_p\}$. As a result, the action integral for spontaneous fission of the Fm isotopes turns out to be 5-15 units smaller along the dynamical trajectory than along the static (traditional) path corresponding to the BCS treatment of pairing correlations. For ^{250}Fm or ^{252}Fm, a decrease in the action by 15 units causes a reduction in the calculated T_{sf} values by 6-6.5 orders of magnitude.

Thus, although further calculations of the kind described in [10] are required, which would be performed for a more complete deformation space including, in particular, non-axial and reflection-asymmetric shapes, an important fact lies in that the main prediction of the simple model by Moretto and Babinet is clearly confirmed by realistic microscopic calculations. Therefore, the use of this analytically solvable model for a more detailed analysis of the role of pairing correlations in the tunnelling process proves to be fully justified. Such an analysis is accomplished in the present paper.

In Sections 2-4 we demonstrate that the dynamically induced enhancement of pairing correlations in tunnelling leads to a variety of new and important consequences for deeply subbarrier fission and fusion of nuclei. Also, the outcomes of the dynamical treatment of pairing correlations are contrasted here with those of the traditional approach, and possibilities of empirical verification of the ideas about the dynamical superfluidity enhancement are examined. In Section 5 we propose particular experiments which could essentially clarify the role of pairing correlations in large-scale subbarrier rearrangements of nuclei. Concluding remarks are given in Section 6.

2. THE SPONTANEOUS FISSION HALF-LIFE AS A FUNCTION OF THE INTENSITY OF PAIRING CORRELATIONS IN THE INITIAL STATE AND OF THE BARRIER HEIGHT

By substituting eq. (12) for $\Delta(q)$ in eqs. (6) and (8) with $\eta=0$ and using the thus found ingredients $V(q,\Delta)$ and $M(q,\Delta)$ in eq. (1) we obtain the following expression for the minimum value of the action integral corresponding to the dynamical treatment of pairing correlations:

$$S_{dyn} \equiv S_{dyn}^{min} = 2\int_{q_1}^{q_2}\left\{\frac{2}{\hbar^2}\,\frac{\left[V_o(q,\Delta_o)-E\right]M(q,\Delta_o)}{\left[1+\dfrac{V_o(q,\Delta_o)-E}{g\Delta_o^2}\right]}\right\}^{1/2}dq. \qquad (13)$$

The above integral can be calculated explicitly by using F_o/Δ_o^2 for $M(q,\Delta_o)$, with $F_o \equiv \langle F(q)\rangle_q$ being independent of deformation, and approximating the potential by the inverted parabola

$$V_o(q,\Delta_o) - E = B_f\left[1-\nu^2(q-q_s)^2\right] \qquad (14)$$

with $\nu^2=4/(q_2-q_1)^2$ and $q_s=(q_1+q_2)/2$. For spontaneous fission, the calculation yields

$$S_{dyn}(\varkappa_o) = S_o\varkappa_o f(\varkappa_o), \qquad (15)$$

where

$$S_o = \pi(q_2-q_1)(F_o g/2\hbar^2)^{1/2} \qquad (16)$$

$$\varkappa_o = (B_f/g\Delta_o^2)^{1/2} \qquad (17)$$

$$f(\varkappa_o) = (4/\pi)\varkappa_o^{-2}(1+\varkappa_o^2)^{1/2}\left[E(k) - \frac{K(k)}{(1+\varkappa_o^2)}\right] \qquad (18)$$

with K(k) and E(k) being the complete elliptic integrals [12] of the 1st and 2nd kind, respectively, which are thoroughly tabulated in [13]. The modulus of the elliptic integrals is

$$k = \left[\varkappa_0^2/(1+\varkappa_0^2) \right]^{1/2}. \qquad (19)$$

In the standard treatment of pairing correlations, under the same assumptions about the potential and effective mass, we would obtain from eq. (1) the well-known formula

$$S_{stat}(\varkappa_0) = S_0\varkappa_0 = \pi(q_2 - q_1)(B_f F_0/2\hbar^2\Delta_0^2)^{1/2}. \qquad (20)$$

Thus

$$S_{dyn}(\varkappa_0) = f(\varkappa_0)S_{stat}(\varkappa_0) \qquad (21)$$

and, consequently,

$$\lg(T_{sf}^{dyn}/T_{sf}^{stat}) = 0.434\, S_{stat}(\varkappa_0)\left[f(\varkappa_0) - 1\right]. \qquad (22)$$

To estimate the effect for ^{250}Fm or ^{252}Fm, let us assume B_f=6 MeV and $g\Delta_0^2$=5.1 MeV. Then $f(\varkappa_0)\simeq 0.74$ and using for $S_{stat}(\varkappa_0)$ the empirical values of the action integral $S_{emp}\simeq 65$-70, extracted from experimental T_{sf} values by means of eq. (3), we find that the dynamical treatment of pairing correlations leads to a reduction in T_{sf} by 7.5-8 orders of magnitude. Note that eqs.(15) and (20) will yield the same result at a fixed $\varkappa_0$ value, if in eq. (15) the value of S_0 (i.e. $(q_2-q_1)F_0^{1/2}$) is chosen to be $1/f(\varkappa_0)$ times larger than in eq. (20). According to eq. (22), the T_{sf} decrease due to the dynamical enhancement of pairing correlations is proportional to the magnitude of the action integral; hence the decrease is expected to be much stronger for ^{238}U ($S_{emp}\simeq 101$) than for ^{260}106 ($S_{emp}\simeq 42$). As for the quantity $g\Delta_0^2$ entering $\varkappa_0$, in the uniform model it is expected to be approximately constant within the region of known spontaneously fissioning nuclei, since $\Delta_0 \simeq 12A^{-1/2}$ and $g=3\tilde{a}/\pi^2$, with $\tilde{a}\sim A^{-1}$ being the level density parameter [14]. However, in a realistic single-particle model $g\Delta_0^2$ will undergo strong variations around magic Z or N values (see, e.g., [14] and references therein).

Thus, the estimates made in the simple model yield a factor of the "dynamical" reduction in T_{sf}, which in the order of magnitude is close to that following from the realistic calculations [10]. On the other hand, this reduction substantially exceeds the inaccuracy characteristic of modern calculations of absolute T_{sf} values. For example, the calculations [11,15,16] reproduce the systematics of experimental T_{sf} values for even-even nuclei with accuracy within a factor of about 50, on the average, and even maximum discrepancies rarely exceed 10^3. Therefore, in the dynamical treatment of pairing correlations, to obtain an agreement between the theoretical and experimental values of T_{sf} will apparently require a renormalization of one of the parameters of the problem, most probably, a renormalization of (an increase in) the average value of the effective mass along the least action trajectory. Since the effective mass is a rather complicated and so far only crudely understood characteristic, it seems to be not too difficult to find a theoretical justification for such a renormalization. (Note that a systematic underestimating by

the cranking model of the effective mass parameters for small-amplitude collective nuclear motions is a well-known fact [17,18]; see also [15]). Unfortunately, the magnitude of the effective mass associated with large-scale rearrangements of nuclei cannot be checked experimentally. Therefore, the question as to whether the dynamical superfluidity enhancement actually takes place in tunnelling is very difficult to solve by considering absolute T_{sf} calculations and comparing these with experimental data. For empirical verification of the effect in question, it is necessary to find out a more straightforward way, with no absolute calculations required. Important clues concerning this point will be provided by the analysis of the patterns of change in the barrier penetrability (or in the action integral) with respect to the main parameters (Δ_0, B_f, E) of the problem, which analysis is given below for two different approaches to the role of pairing correlations.

In the standard approach (within the uniform model) the pairing gap value $\Delta = \Delta_0$ characterizing superfluidity properties of a fissioning nucleus in the initial state (at $q \leq q_1$) will remain virtually unchanged at any other deformation, in particular, at the saddle-point deformation $q=q_s$, that is

$$\Delta_s^{stat} \simeq \Delta_0. \qquad (23)$$

In contradistinction to this, in the dynamical treatment of pairing, the saddle-point gap value Δ_s^{dyn} will be given, according to eq. (12), by

$$\Delta_s^{dyn} \simeq \Delta_0 + B_f/g\Delta_0 \qquad (24)$$

if spontaneous fission is considered. At a fixed value of B_f/g, the minimum of Δ_s^{dyn}, equal to $2(B_f/g)^{1/2}$, is reached at $\Delta_{0\,min}=(B_f/g)^{1/2}$, so that in the transuranium region $\Delta_0 \lesssim \Delta_{0\,min}$. This means that if we attenuate the initial pairing correlations by choosing $\Delta_0' < \Delta_0$, then, in tunnelling through the barrier, the system guided by the least action principle restores the pairing correlations at the saddle point at least up to the level $\Delta_s'^{dyn} \simeq \Delta_s^{dyn}$, to which it enhances them starting with Δ_0 (see Fig. 2). Moreover, considering eq. (24) literally, we observe, for $\Delta_0 < \Delta_{0\,min}$, an inverse response of the fissioning system to the attenuation of pairing correlations in the initial state: the weaker the pairing at $q \leq q_1$, the higher the level to which it enhances at $q=q_s$. If we also take into account the dependence of Δ_s^{dyn} upon B_f, it becomes clear that the presence of the second term in the right-hand side of eqs. (12) or (24) will lead to essentially novel patterns of change in the barrier penetrability with respect to the parameters Δ_0, B_f and E.

However, eqs. (15)-(20) show that in fact there is only one parameter in our penetrability problem -- the dimensionless parameter $\varkappa_0=(B_f/g\Delta_0^2)^{1/2}=(B_f/2E_{cond})^{1/2}$, with E_{cond} being the so-called condensation energy [14] (note that one should use $\varkappa = \left[(B_f-E)/g\Delta_0^2\right]^{1/2}$ instead of $\varkappa_0$ if $E \neq 0$, see Section 3). As $f(\varkappa_0) \leq 1$ (Fig.3), then $S_{dyn}(\varkappa_0) < S_{stat}(\varkappa_0)$ for all $\varkappa_0 > 0$; at $\varkappa_0 \approx 1.0$-1.2 corresponding to spontaneous fission of transuranium nuclei the difference between $S_{stat}(\varkappa_0)$ and

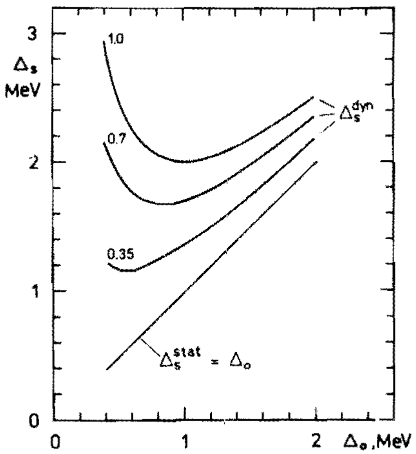

Fig. 2. The saddle-point gap parameter Δ_s versus the initial pairing gap Δ_o, for two different treatments of pairing correlations in spontaneous fission; B_f/g values (in MeV^2) are indicated near the Δ_s^{dyn} curves, see the text and eq. (24).

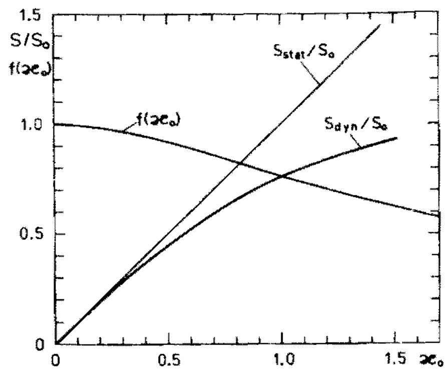

Fig. 3. The $\varkappa_o$ dependence of the action integrals S_{stat} and S_{dyn} (in units of S_o). Also shown is the universal function $f(\varkappa_o)$. For further details see the text and footnote on this page.

$S_{dyn}(\varkappa_o)$ reaches 30-40%. As shown in Fig.3, the function $S_{dyn}(\varkappa_o)$ is an essentially nonlinear and much weaker one than $S_{stat}(\varkappa_o)$ *). It is therefore obvious that the dependence of S_{dyn} upon Δ_o will also be considerably weaker than that of $S_{stat} \backsim \Delta_o^{-1}$, this divergence being the greater the smaller Δ_o (Fig.4).

Rather unusual turns out to be the dependence of S_{dyn} on the barrier height B_f, exemplified for spontaneous fission of ^{240}Pu in Fig.5. As compared with the conventional curve $S_{stat} \backsim B_f^{1/2}$, the B_f dependence of S_{dyn} is much more gentle, especially in the region of $B_f > 3$ MeV. The physical reason for such a difference in the B_f dependences of S_{stat} and S_{dyn} is clear from eq. (24) and Fig.2: the higher

*) It follows from eqs. (15)-(20) that for $\varkappa_o \rightarrow \infty$ the function $f(\varkappa_o)$ tends to zero while $S_{dyn}(\varkappa_o)$ tends to the constant limit $4S_o/\pi$. However we remind that eq. (6) for $V(q,\Delta)$ is valid only in the quadratic approximation with respect to $(\Delta-\Delta_o)$, whereas the $\eta=0$ assumption in eq. (8) distorts the $M(q,\Delta)$ behaviour the more the larger Δ value; at very small Δ's, $\Delta \lesssim G$, eq.(8) is invalid at all /2-5/. Therefore eqs. (11)-(13), too, are valid only within certain limits, so that eq. (15) for $S_{dyn}(\varkappa_o)$ cannot be used for $\varkappa_o \gg 1$. In this paper all considerations are confined to the range $0 \leq \varkappa_o \lesssim 1.5$ and to such Δ_o and B_f values for which the approximations adopted are quite justified.

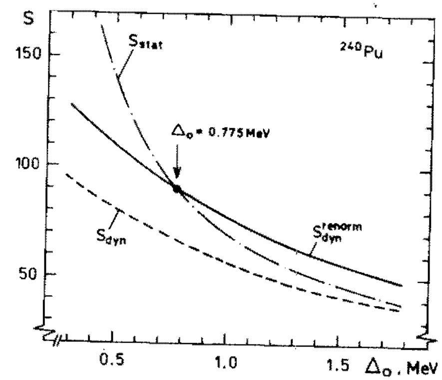

Fig.4. The dependence of the action integral upon the initial pairing gap Δ_o, calculated for spontaneous fission of ^{240}Pu in the statical (S_{stat}) and in the dynamical (S_{dyn}) treatments of pairing correlations. The calculations are performed for a constant barrier height, B_f=6 MeV, with g=9 MeV^{-1}. The S_{dyn}^{renorm} curve is obtained by renormalizing the S_{dyn} curve using the empirical value of the action integral, S_{emp}=90, at Δ_o=0.775 MeV; the S_{stat} curve is also normalized using S_{emp}=90 at Δ_o=0.775 MeV.

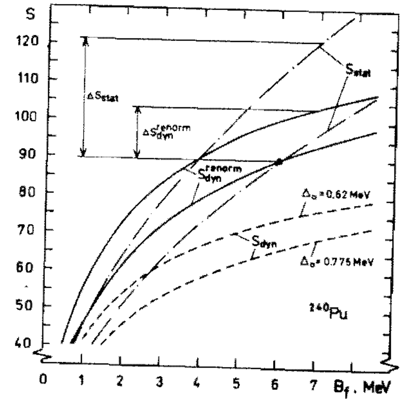

Fig.5. The dependence of the action integral upon the barrier height B_f, calculated for spontaneous fission of ^{240}Pu in the statical (S_{stat}) and in the dynamical (S_{dyn}) treatments of pairing correlations. The S_{dyn}^{renorm} curves are obtained by renormalizing the S_{dyn} curves using S_{emp}=90 at B_f=6 MeV and Δ_o=0.775 MeV. For each of the three pairs of curves, the lower curve corresponds to Δ_o=0.775 MeV while the upper one corresponds to Δ_o=0.62 MeV. Also shown are the action increments ΔS_{stat} and ΔS_{dyn}^{renorm} associated with an increase in B_f by 1 MeV and a simultaneous decrease in Δ_o by 20%. For further details see caption to Fig.4.

the barrier, the stronger the dynamical enhancement of superfluidity in tunnelling; the decrease in S_{dyn} due to this extra enhancement of superfluidity compensates to a considerable extent for the S_{dyn} growth caused directly by increasing B_f. Of course, this interesting "feedback" does not occur in the traditional consideration of pairing correlations. Calculations demonstrate that for B_f=6 MeV and $\Delta_o \simeq 0.78$MeV the value of $\partial S_{dyn}/\partial B_f$ is approximately a factor of 2.5 smaller than $\partial S_{stat}/\partial B_f$. Another important feature is that at a fixed B_f value the Δ_o decrease leads to a noticeable growth of $\partial S_{stat}/\partial B_f$, whereas $\partial S_{dyn}/\partial B_f$ remains practically unchanged (see Fig.5).

Thus, the dynamical treatment of pairing correlations substantially alters the traditional understanding of the probability of deeply subbarrier fission. The fact that the barrier penetrability becomes a much weaker function of B_f and Δ_0 allows us, in particular, to give a more adequate interpretation of the empirical systematics of the spontaneous fission half-lives for odd-A and odd-odd nuclei.

It is known that, as regards spontaneous fission, the odd nuclei are more stable than their even-even neighbours, typically by a factor of 10^5, although the magnitude of this irregular hindrance varies in fact between 10^1 and 10^{10}. Such an enhanced stability is usually explained as being due either to a barrier increase /19-21/ caused by the presence of one or two odd (unpaired) particles or to an increase in the effective inertia /3,6/, caused by the same fact. Thus, according to Newton /19/ and Wheeler /20/, the so-called specialization energy should be associated with spontaneous fission of odd nuclei -- an increment in the barrier height caused by the requirement that the spin projection K onto the symmetry axis of the nucleus should be conserved in tunnelling. The specialization energy is expected to be strongly dependent on quantum numbers of the initial state /20-24/; for odd-A nuclei it amounts, on the average, to 0.5-1 MeV, sometimes reaching 2.5 MeV /22-24/ (see also Section 5). At the same time, the ground states of odd nuclei are one- or two-quasiparticle (q-p) states and, due to the blocking effect /25-28/, pairing correlations are considerably reduced here, as compared with even-even nuclei whose ground states correspond to a q-p vacuum. The reduction of pairing correlations (i.e., a Δ_0 decrease, on the average, by 10-30% for one-q-p states and 20-40% for two-q-p states /25-28/) leads, according to eq. (8), to a noticeable increase in the effective inertia associated with fission. Microscopic estimates /3-6/ show that the addition of an odd nucleon to an even-even system increases the effective inertia by about 30%, on the average. This in turn results in a hindrance factor of the order of 10^5 for spontaneous fission of odd-A nuclei with $Z \lesssim 101$. In addition, the shell-correction calculations /7,8/ predict that the blocking effect leads also to a perceptible increase in the barrier -- on the average, by 0.3-0.4 MeV. The net increase in the barrier height of an odd-A nucleus due to the specialization and blocking effects easily provides a hindrance factor of the order of 10^5 or greater.

Thus, either of the two reasons -- an increase in the barrier or that in the effective inertia -- may entirely account for the order of magnitude of empirical hindrance factors for ground-state spontaneous fission of odd nuclei; a similar situation occurs also for spontaneously fissioning isomers /29,30/. Generally, both reasons are equally well justified and both should necessarily be taken into account, but then the theoretical hindrance factors turn out to be (on the average) many orders of magnitude larger than the empirical ones. The dynamical treatment of pairing correlations enables us to overcome the difficulty in a natural way: in this treatment the hindrance conditioned by each of the two reasons is drastically lowered (see Figs. 3-5), whereas taking both reasons into account results in hindrance factors close to empirical ones in the order of magnitude. For example, let us

assume that in going from ^{240}Pu to ^{241}Pu the barrier increases by 1 MeV and the initial pairing gap decreases by 15%; this corresponds to a $\varkappa_0$ increase from 1.09 to 1.34. Then in the statical consideration of pairing correlations the hindrance factor will amount to about 10^9, while in the dynamical one, to $2\cdot10^4$, in excellent agreement with the empirical value $4\cdot10^4$.

We can now conclude that the analysis of the typical values of the hindrance factors associated with spontaneous fission of odd nuclei provides a direct and clear evidence in favour of the dynamical treatment of pairing correlations in tunnelling. It is also important to note that this treatment of pairing effects leads to a novel orientation in estimating the reliablity of T_{sf} predictions for unknown heavy and, in particular, superheavy nuclei. Since inaccuracies in the barrier heights affect the scale of uncertainties in T_{sf} far less than it is generally thought, the emphasis should be put on more reliable determination of other ingredients of the problem, in the first place, the effective inertia associated with fission and its dependence upon pairing properties.

3. THE INFLUENCE OF PAIRING CORRELATIONS ON THE ENERGY DEPENDENCE OF THE FISSION BARRIER PENETRABILITY

In the dynamical treatment of pairing correlations, we obtain, by generalizing eq. (15), the following formula for the energy dependence of the penetrability of a parabolic fission barrier:

$$P_{dyn}(E) = \left\{1 + \exp\left[2\pi(B_f - E)/\hbar\omega_{dyn}\right]\right\}^{-1}, \qquad (25)$$

where

$$\hbar\omega_{dyn} = \hbar\omega/f(\varkappa) \qquad (26)$$

$$\hbar\omega = \left[8B_f\hbar^2\Delta_0^2/F_0(q_2-q_1)^2\right]^{1/2} \qquad (27)$$

$$\varkappa = \left[(B_f - E)/g\Delta_0^2\right]^{1/2} \qquad (28)$$

and the function $f(\varkappa)$ is defined by the same eq. (18) like $f(\varkappa_0)$. In the standard approach, the penetrability $p_{stat}(E)$ will be described by the Hill-Wheeler formula, i.e. by eq. (25) for $\varkappa=0$.

We see that, all other things being equal, $\hbar\omega_{dyn} > \hbar\omega$ for all $E < B_f$; in the transuranium region $\hbar\omega_{dyn} \approx 1.35\,\hbar\omega$ for $B_f - E = 6$ MeV. Therefore in the dynamical approach the energy dependence of the penetrability turns out to be weaker. Another new feature lies in that the dependence of $\hbar\omega_{dyn}$ on the energy of the initial state, E, takes place even if Δ_0 and F_0 are considered to be independent of E. For the ^{240}Pu nucleus, a comparison of the functions $p_{dyn}(E)$ and $p_{stat}(E)$ is given in Fig.6; here $\hbar\omega$ is chosen so that the magnitude of $p_{stat}(E=0)$ corresponds to the experimental value of $T_{sf}=1.3\cdot10^{11}$y whereas $\hbar\omega_{dyn}$ is found from eq. (26). It can be seen that in the region of $E \sim B_f$ the difference between the curves $p_{stat}(E)$ and $p_{dyn}(E)$ is small, but in the deeply subbarrier region the curves

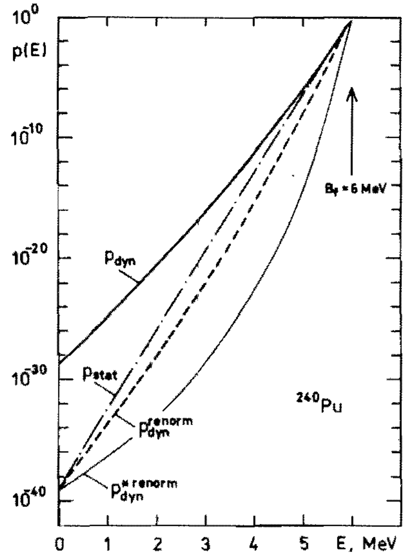

Fig. 6. The energy dependence of the fission barrier penetrability calculated for ^{240}Pu within two different treatments of pairing correlations. For explanation of the curves see the text and caption to Fig. 4.

diverge considerably and for $B_f - E$ = = 4-5 MeV the difference in penetrability reaches $10^5 - 10^8$. It would, however, be reasonable to renormalize the "dynamical" curve so that the magnitude of $p_{dyn}(E=0)$ should correspond to the experimental T_{sf} value. This can be achieved by increasing by a factor of about 1.8 the average value of the effective mass associated with fission. We note that this renormalization does not lead to contradictions with any empirical data if, of course, it is appropriately performed for all fissioning nuclei rather than for ^{240}Pu alone (see also [15,17,18]). After the renormalization we obtain the curve $p_{dyn}^{renorm}(E)$ shown in Fig.6, which is interesting to compare again with the $p_{stat}(E)$ curve reflecting the traditional understanding of the penetrability of the one-humped parabolic barrier. It should, however, be stressed that the traditional understanding is far from requiring the necessary coincidence of the $\hbar\omega$ parameters deduced from spontaneous fission half-lives and from near-threshold fission cross sections though some similarity of $\hbar\omega$ values is usually expected for the two cases [30].

Unfortunately, the excitation energy dependence of the effective mass associated with fission so far remains rather unclear. Generally, there are no grounds to believe that the effective mass does not vary as the initial excitation energy of a fissioning nucleus increases from zero to $E \approx B_f$, at least for the reason that with excitation of the nucleus there appear quasiparticles and, according to theoretical expectations, pairing correlations progressively attenuate [14]. For example, Fig.6 illustrates changes in $p_{dyn}^{renorm}(E)$ if $\hbar\omega_{dyn}$ is calculated from eq. (26) using, instead of Δ_o, the average correlation function Δ_{ave} that depends on excitation energy according to the BCS prescriptions for a system with the uniform spectrum of doubly degenerate single-particle levels [14]. This schematic calculation did not take into account the discrete character of q-p excitations and, in addition, it has been assumed that the effective mass $M \propto \Delta_{ave}^{-2}$ depends on excitation energy only through the E dependence of Δ_{ave}. The calculational result

(the $p_{dyn}^{*renorm}(E)$ curve in Fig.6) has therefore a purely illustrative value and is presented only to stress the sensitivity of the penetrability to assumptions about the energy dependence of the effective mass. At the same time, all the present-day considerations of subbarrier fission phenomena actually neglect the possible dependence of the action integral ingredients upon excitation energy. The development of theory along this line is certainly called for, whereas experimental studies of the energy dependence of the probability of deeply subbarrier fission undoubtedly remain to be still a challenging task.

It would seem that the considerable differences between $p_{stat}(E)$ and $p_{dyn}(E)$ in the deeply subbarrier region can be used for an empirical verification of the predictions concerning the enhancement of superfluidity in tunnelling. However, on this route one encounters substantial difficulties, especially at energies E above the threshold of 2 or 4 q-p excitations. First, as has just been noted, there is no necessary information about the behaviour of the effective mass and other pertinent quantities with excitation. Second, for almost all actinide nuclei the fission barrier profile is known to be double-humped. The latter, of course, in no way changes the very conclusion about the superfluidity enhancement as eqs. (11)-(13) are valid for a one-dimensional barrier of an arbitrary profile. However, in the region of $E > E_i$, where E_i is the energy of the isomeric minimum,there arises a wealth of structures and effects [1,2,29-33] caused by the presence of two peaks in the fission barrier. It is conceivable that the dynamical treatment of pairing correlations may noticeably change the customary understanding of the phenomena associated with the double-humped barrier such as, e.g., the isomeric shelf [31-33]. Analyzing the role of pairing correlations in the penetration of the double-humped barrier lies, however, beyound the scope of this paper. So, we shall only note that the abundance of diverse and complex physical effects associated with the energy region $B_f^{max} \gtrsim E \gtrsim E_i$ will sooner prevent than aid the identification of another new effect -- the dynamical enhancement of superfluidity in tunnelling. Much more suitable for the purpose in view looks the region $E < E_i$ in which the treatment of penetrability phenomena is radically simplified and can virtually be done in terms of a single-humped barrier. Possible manifestations of the superfluidity effects in tunnelling at energies $E < E_i$ will be considered in Section 5.

4. PAIRING CORRELATIONS AND SUBBARRIER FUSION OF COMPLEX NUCLEI

Fission of a nucleus into two fragments and fusion of two nuclei into one whole system, particularly, cold fission and subbarrier fusion of heavy nuclei, are known to be highly similar processes showing many important common features [1]. Therefore if the dynamical enhancement of pairing correlations substantially increases the probability of subbarrier fission, there are good grounds to expect a similar effect in the subbarrier fusion of complex nuclei. Unusually large fusion cross sections at energies below the Coulomb barrier were observed in many experiments carried out during recent years. Although to date a large number of various explanations have been proposed for the enhanced fusability of nuclei at subbarrier energies, the problem remains still open to a considerable extent (see, e.g., [34] and references therein). Below we shall estimate the scale of the relative increase of subbarrier fusion cross sections due to the dynamical enhancement of superfluidity in a system of fusing nuclei.

The fusion cross section is usually presented [35] via the partial wave summation

$$\sigma_{fus}(E_{cm}) = \pi \lambda\!\!\!^{-2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_\ell(E_{cm}), \qquad (29)$$

where E_{cm} is the center-of-mass energy, $\lambda\!\!\!^{-}$ the energy-dependent reduced de Broglie wavelength, and $T_\ell(E_{cm})$ the transmission coefficient for the ℓ-th partial wave; equation (29) is written down for the case of non-identical partners. As in the case of subbarrier fission, it is reasonable to assume that the tunnelling through a (generally, multidimensional) potential barrier associated with fusion is governed by the least action principle and that the transmission coefficient can be determined in the quasiclassical WKB approximation. By extending the analogy to fission still further, we shall presume that the effective potential V_{fus} and the effective mass M_{fus} associated with fusion are characterized (qualitatively) by the same properties as those taken in Sections 1-3 for the corresponding quantitites associated with subbarrier fission. In other words, we make use of an expression of type (25) for the transmission coefficient $T_\ell(E_{cm})$. Then, by converting the sum in eq. (29) to an integral, we obtain

$$\sigma^{dyn}_{fus}(E_{cm}) = \pi \lambda\!\!\!^{-2} \int_0^{\infty} \frac{(2\ell + 1)d\ell}{1 + \exp\{(2\pi/\hbar\omega_\ell)[B_{fus}(\ell) - E_{cm}]\, f(\varkappa_\ell)\}}, \qquad (30)$$

where $B_{fus}(\ell)$ is the height of the ℓ-dependent effective fusion barrier, $\hbar\omega_\ell$ the curvature of the effective barrier near its top ($q = q_\ell$)

$$\hbar\omega_\ell = \left[M^{-1}_{fus} \left. \frac{\partial^2 V_{fus}}{\partial q^2} \right|_{q=q_\ell} \right]^{1/2} \qquad (31)$$

and $f(\varkappa_\ell)$ defined by eq. (18) is the function of the ℓ-dependent parameter

$$\varkappa_\ell = \left\{ \left[B_{fus}(\ell) - E_{cm} \right] / g\Delta_0^2 \right\}^{1/2}. \qquad (32)$$

In the following we shall use the common parametrization [35,36] for the ℓ-dependent barrier

$$B_{fus}(\ell) = B_{fus} + (\hbar^2/2\mu q_\ell^2)\, \ell(\ell + 1) \qquad (33)$$

and the standard assumptions [35,36] that

$$\hbar\omega_\ell \simeq \hbar\omega_0, \quad q_\ell \simeq q_0, \qquad (34)$$

where B_{fus}, $\hbar\omega_0$ and q_0 are the height, the curvature parameter and the top posi-

tion of the effective fusion barrier for $\ell = 0$, respectively, while μ is the reduced mass. Now, passing in eq. (30) from integrating over ℓ to that over $\varkappa^2_\ell$ and then expanding the power of the exponent in an appropriate Taylor series, we obtain, in the first approximation, the following result:

$$\sigma^{dyn}_{fus}(E_{cm}) = \frac{\hbar\omega_0 q_0^2}{2E_{cm}} \cdot \frac{(1 + \varkappa^2)^{1/2}}{2E(k)}\, \ln\left\{1 + \exp\left[\frac{2\pi(E_{cm} - B_{fus})}{\hbar\omega_0} f(\varkappa)\right]\right\}, \qquad (35)$$

where $\varkappa$ is $\varkappa_\ell$ for $\ell = 0$ and $E(k)$ the complete elliptic integral of the 2nd kind with $k = \varkappa/(1 + \varkappa^2)^{1/2}$. By performing calculations analogous to those outlined above, yet employing the statical treatment of pairing correlations (i.e. using for $T_\ell(E_{cm})$ the standard Hill-Wheeler formula) we would obtain for the fusion cross section, $\sigma^{stat}_{fus}(E_{cm})$, the well-known Wong formula [36], i.e. eq.(35) for $\varkappa = 0$. Then the relative increase in the subbarrier fusion cross section due to the dynamical superfluidity enhancement in tunnelling can be evaluated as

$$\frac{\sigma^{dyn}_{fus}}{\sigma^{stat}_{fus}} \approx \frac{(1 + \varkappa^2)^{1/2}}{2E(k)} \cdot \exp\left\{\frac{2\pi(B_{fus} - E_{cm})}{\hbar\omega_0}\left[1 - f(\varkappa)\right]\right\}. \qquad (36)$$

The magnitude as well as the behaviour of $\sigma^{dyn}_{fus}/\sigma^{stat}_{fus}$ are almost completely determined by the exponential factor. As follows from Fig.7, in the region of $(B_{fus} - E_{cm}) \approx 5$-$8$MeV the dynamical treatment of pairing correlations leads to an increase in fusability by several orders of magnitude as compared with the results of standard approach.

In spite of the large magnitude of the effect predicted, unambiguous identification of its manifestations in experimental subbarrier fusion cross sections turns out, unfortunately, to be a rather difficult task. First, while there is accumulated a lot of data about subbarrier fusion, until now its mechanism is poorly understood, especially for heavy systems, since there are remained to be largely open even such fundamental questions as what effective potential V_{fus} does control the process, what is the effective mass M_{fus} associated with fusion and what are its properties. (Let us, however, note that in evaluating the relative enhancement of fusion cross sections, $\sigma^{dyn}_{fus}/\sigma^{stat}_{fus}$, we have not used any too specific assumptions concerning V_{fus} or B_{fus}. As for the effective mass M_{fus}, it is qualitatively clear that this

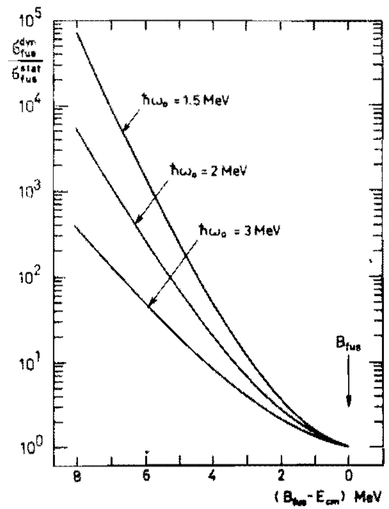

Fig. 7. The relative increase in subbarrier fusion cross sections calculated by using eq. (36) with $g\Delta_0^2 \simeq 5.1$ MeV for the three $\hbar\omega_0$ values indicated.

quantity is far from being necessarily identical to the reduced mass μ; as in fission, M_{fus} can, on the average, significantly exceed μ, as indicated by theoretical considerations (e.g., /37,38/) and, perhaps, by some empirical data (e.g., /39/), too. We are just in line with these expectations since we have used μ instead of M_{fus} only in estimating angular-momentum corrections which are relatively unimportant.) Second, a wide variety of pertinent effects has already been proposed to explain the unexpectedly high subbarrier fusion cross sections (see, e.g., /34,39/ and references therein). The superfluidity enhancement in tunnelling appears to be highly probable and strong yet not the only possible effect facilitating deeply subbarrier fusion. Finally, the superfluidity effect can occur in cooperation with other effects (i.e., in a more involved way) and for different fusing systems its "partial" contribution can be quite different.

5. SPONTANEOUS FISSION FROM QUASI-PARTICLE ISOMERIC STATES AS A PROBE OF THE ROLE OF PAIRING CORRELATIONS IN TUNNELLING

In Section 2 we have established the fact that two different treatments of the role of pairing correlations in tunnelling yield substantially different magnitudes of the average hindrance factors associated with ground-state spontaneous fission of odd nuclei; from this fact, using comparisons with the corresponding empirical data, we have obtained an important (a posteriori) evidence in favour of the dynamical treatment of pairing correlations. Below we shall demonstrate that the situation characteristic of the ground-state spontaneous fission of (doubly) odd nuclei takes place also for spontaneous fission from q-p isomeric states in even-even actinide nuclei; this (truly a priori) prediction can be effectively used for further testing of the ideas about the dynamical enhancement of superfluidity in tunnelling.

Quasi-particle or K isomers are expected to occur when breaking up of one or several pairs of nucleons in an even-even nucleus and appropriate recoupling of the spins of the nucleons lead to the formation of relatively low-lying ($E^*\gtrsim 2\Delta_0$) states having high values of the quantum number K. The high K values cause a strong retardation of γ-transitions, which, in turn, favours searches for a spontaneous fission branch in the decay of the K-isomeric states in heaviest nuclei. By now, quite a number of such isomers have been found in the region of even-even nuclei with $Z \geq 92$ /40-42/; a famous example is provided by the two-neutron (2n) state of ^{244}Cm with $K^{\pi}=6^+$, Nilsson configuration $\frac{5}{2}^+[622]_n$, $\frac{7}{2}^+[624]_n$, excitation energy E^*=1042 keV and $T_{1/2}$ = 34 ms /40/.

The appearance of even a single pair of quasiparticles in the neutron (n) or proton (p) subsystem of a nucleus will lead, due to the blocking effect /25-28,14/, to a noticeable weakening of pairing correlations, i.e. to a decrease in the gap parameter in the q-p state

$$\Delta^*_{0n(p)} = \beta_{n(p)}\Delta_{0n(p)}, \qquad (37)$$

where $\beta_{n(p)} < 1$ is the blocking factor. (Here and below starred quantities are those relevant to an isomeric q-p state.) According to theoretical estimates /25-28,14/, the value of $\beta_{n(p)}$ is rather sensitive to details of structure of a particular state; for 2 q-p states $\beta_{n(p)}$ is predicted to be about 0.6-0.8, on the average. Similar $\beta_{n(p)}$ values follow from analyses of empirical spectroscopic information, too (see, e.g., /25-28/).

From the viewpoint of the spontaneous fission probability it is important that isomeric q-p states are characterized not only by the weakened pairing correlations, but also by rather high K values. Both these causes act towards increasing the fission barrier /7,8,19-24/, so that its height for an isomeric q-p state can turn out to be appreciably larger than that for the ground state. The reaction of a fissioning system to both the increase in the barrier and the decrease in the initial pairing gap will be essentially dependent on the role of pairing correlations in tunnelling: in the statical treatment of pairing correlations the increase in fission stability, caused by each of the two effects, is predicted to be much higher than in the dynamical approach (see Figs. 3-5). Thus, high values of K significantly aid in strengthening the difference in T^*_{sf} predictions for q-p isomeric states that is associated with employing two different treatments of pairing correlations.

With possible changes in the pre-exponent factor in eq. (3) being neglected, the relative variation in the spontaneous fission half-life, associated with passing from the ground state to an isomeric q-p state, will be

$$\delta T_{sf} = \lg(T^*_{sf}/T_{sf}) = 0.434\, S_{emp}\left(\frac{S^*}{S} - 1\right), \qquad (38)$$

where S_{emp} is the empirical magnitude of the action integral for the ground-state spontaneous fission.

To evaluate S^*/S, it is necessary at first to estimate the barrier increment for the isomeric state, which is expected to be caused mainly by the specialization effect /19-24/. In Section 2 we have already indicated both the rough value of the specialization energy for a nucleus with one unpaired particle and the strong dependence of this value on the quantum numbers of the initial state from which fission proceeds. For a 2q-p initial state, conservation of the quantum numbers of each of the two unpaired particles individually will lead to a net barrier increment equal to the sum of the individual specialization energies, if the unpaired particles are assumed to move completely independently; however, the complete independence can hardly be realized, so that, most likely, there should be observed a considerable mixing of 2 q-p configurations with the same total values of K /30/.

All other things being equal, the magnitude of the specialization energy depends on how "good" is the quantum number K. If the non-axial deformations with $\gamma \approx 10^0$-20^0 do occur in the region of the inner fission saddle point, as has been predicted for actinide nuclei theoretically (see, e.g., /7,8,22/) and supported by some empirical hints (see, e.g., /1,30/), then the K number will be conserved here only approximately and the single-particle wavefunctions of given K will show **mixing** of components with $K \pm 2$ [22]. As a result, the specialization effect will be attenuated. However, all the available hints concerning the violation of axial symmetry in fission are obtained from analyses of measured excitation functions for near-threshold fission. At the same time, calculations [11,16] show that the triaxial shapes give rise to a large effective inertia and therefore in spontaneous or deeply subbarrier fission the least action trajectory, unlike the static path, lies much closer to the region of axially symmetric shapes with $\gamma=0$; then the specialization effect can be expected to operate almost in full measure.

Having mentioned a number of the pros and cons of the "specialization", for the following relative estimates we shall assume that the concurrent influence of the specialization and blocking effects on the potential energy of deformation will at least compensate the effect of the energy gain $\delta E = E^* \approx 1.1\text{-}1.3$ MeV associated with passing from the ground state to an isomeric 2 q-p state. Then, by using eqs.(15)-(20), (25)-(28) and (38), one can easily see that $\delta T_{sf}^{stat} > 0$ and $\delta T_{sf}^{dyn} \gtrsim 0$. In other words, either of the considered treatments of pairing correlations, the stability against spontaneous fission for 2 q-p isomeric states is expected to be not lower than that for the ground states.

Of primary importance is the fact that the following difference

$$\delta T_{sf}^{stat} - \delta T_{sf}^{dyn} \equiv \lg \frac{T_{sf}^{*stat}}{T_{sf}^{*dyn}} = 0.434\, S_{emp} \frac{S^*_{stat}}{S_{stat}} \left[1 - \frac{f(\varkappa^*)}{f(\varkappa_0)} \right] \qquad (39)$$

proves to be essentially positive, since $\varkappa^* = \left[(B_f^* - E^*)/g\Delta_0^{*2} \right]^{1/2} \gtrsim \varkappa_0/\beta$ significantly exceeds $\varkappa_0 = (B_f/g\Delta_0^2)^{1/2}$ and hence $f(\varkappa^*) < f(\varkappa_0)$. For $S_{emp}=65$ (^{250}Fm), for example, T_{sf}^{*stat} will be higher than T_{sf}^{*dyn} by a factor of $10^2\text{-}10^3$, if, allowing for the two-component composition of the nucleus, one takes for the blocking factor β the value of 0.85, in approximate correspondence to the average value $\beta_{n(p)}=0.7$ for 2 q-p excitations in a given subsystem of the nucleus. Thus, there is predicted a very strong, "logarithmic" excess of T_{sf}^{*stat} over T_{sf}^{*dyn}. Therefore experimental determination of the partial half-lives T_{sf}^* for several 2 q-p isomers and their comparison with realistic microscopic calculations carried out within both the statical and the dynamical approach to the role of pairing correlations would make it possible to decide which of the two approaches is more adequate. It is important that to decide the issue requires relative quantities T_{sf}^*/T_{sf} (or S^*/S) rather than absolute T_{sf}^* values to be calculated. The possible inaccuracy of the T_{sf}^*/T_{sf} calculations, characteristic of the present-day theory [11,15], is obviously much lower that the difference between $T_{sf}^{*stat}/T_{sf}^{stat}$ and $T_{sf}^{*dyn}/T_{sf}^{dyn}$ estimated from eqs. (38)-(39).

Quasi-particle isomers can occur not only in the first, but also in the second potential well. In fact, for a number of even-even Pu and Cm isotopes there were identified two spontaneously fissionable states with anomalously short half-lives [29,30]. On the basis of a variety of empirical indications, the shorter-lived

states with $T_{sf}^{(m)} \approx 5\cdot10^{-12}s - 5\cdot10^{-9}s$ are interpreted as the "ground" states in the second well, while the longer-lived states with $T_{sf}^{*(m)} \approx 5\cdot10^{-9}s - 2\cdot10^{-7}s$, as 2 q-p excitations in the second well [29,30]. However, so far it is not completely clear [29] whether all the measured $T_{sf}^{*(m)}$ values refer directly to spontaneous fission from 2 q-p states or there takes place at first a K-forbidden γ-transition to the bottom of the second well (K=0) and then spontaneous fission occurs in a more short time ($T_{sf}^{(m)}$) compared to the retarded γ-transition. Therefore the empirical values of $\delta T_{sf}^{(m)} = \lg(T_{sf}^{*(m)}/T_{sf}^{(m)}) \simeq 1.1\text{-}4.3$ seem to be considered rather as the lower limits to the hindrance factors for spontaneous fission from 2 q-p states in the second well. Obviously, for 2 q-p states in the second well the difference given by eq. (39) will be substantially smaller than for those in the first well. Nevertheless, since the highly sensitive empirical $\delta T_{sf}^{(m)}$ values are known for five nuclei [29,30], whereas the ratio $T_{sf}^{*(m)stat}/T_{sf}^{*(m)dyn}$ expected from eq. (39) is still evaluated, roughly speaking, by a value of 10 or more, it would be important to perform thorough realistic calculations of $\delta T_{sf}^{(m)}$ involving the two different approaches to the role of pairing correlations.

Eventually, one can put a question on searching for spontaneous fission from higher-lying q-p states in the first well, e.g., from 4 q-p states of the (2n,2p) type, whose energy can often be still below the bottom of the second well. Four q-p isomers have not yet been observed in the actinide region, but, as in the Hf region [27,28,40,43,44], their occurence here is quite probable; for example, theory predicts the (2n,2p) states with $K^{\pi}=13^-$ and $E^* \approx 2.5$ MeV in ^{254}Cf, and with $K^{\pi}=13^+$ and $E^* \approx 2$ MeV in ^{248}Cf [27]. Although for such cases all estimates turn out to be less reliable, it should be qualitatively expected that for high-K (2n,2p) states the difference in T_{sf}^*/T_{sf} associated with the two different treatments of pairing correlations will be by several orders of magnitude higher than that for (2n) or (2p) states.

Experimentally, spontaneous fission from q-p isomeric states in the first potential well has never been observed. An attempt to detect it for the 2 q-p isomeric state in ^{244}Cm [40] was done by Vandenbosch et al. [45], but, due to insufficient sensitivity of the experiment, their result, $T_{sf}^*/T_{sf} \gtrsim 10^{-5}$, does not allow any conclusions about the role of pairing correlations to be made. Obviously, the most appropriate objects for detecting spontaneous-fission decay from q-p isomeric states are expected to be the heaviest even-even nuclei showing spontaneous fission as a predominant or quite probable decay mode of their ground-states. Interesting examples are provided by the isomeric states with $T_{1/2}^* = 1.8 \pm 0.1$ s in ^{250}Fm and with $T_{1/2}^* = 0.28 \pm 0.04$ s in $^{254}102$, which have been observed in [41]; although energies, spins and parities of these isomers are not yet established experimentally, their interpretation [41] as 2 q-p states with $K^{\pi}=7^-$ or 8^- is fully confirmed by semimicroscopic calculations [42]. In the $^{249}Cf + {}^4He$ and $^{208}Pb + {}^{48}Ca$ reactions, both these isomers can be produced with rather high yields. We have seen, however, that for any plausible role of pairing correlations the stability of 2 q-p isomers against spontaneous fission is expected to be at least not lower than that of the ground states; thus, to obtain experimental results critical in terms of choosing the most adequate treatment of pairing correlations, a very high sensitivity of experiments is needed which would enable spontaneous fission from q-p isomeric states to be observed even if T^*_{sf} exceeds T_{sf} by some orders of magnitude.

Of far reaching importance would be searches for new q-p (and other types of) isomers in nuclides with $Z \gtrsim 100$, the existence of which is predicted theoretically /27,42/, as well as spectroscopic studies of the structure of the heaviest nuclei on the whole. Particularly great urgency of such a research is due to its close connection with the problem of synthesizing new elements and elucidating the pattern of nuclear stability near the limits of the Mendeleev Periodic Table. In the region of $Z \gtrsim 102$, the ground-state spontaneous fission half-lives prove to be rather short and thus the range of T_{sf} is expected to overlap that of typical half-lives for K-forbidden γ-transitions. Therefore two or even more spontaneous fission activities of different half-lives can obviously be associated here with the same nuclide, without necessarily requiring, however, that fission should directly proceed from an isomeric state. If now T_{sf} turns out to be much shorter than the total half-life of the isomeric state, $T^*_{1/2}$, and, at the same time, it turns out to fall below the limit of the detection speed of experimental device, then the comparatively large value of $T^*_{1/2}$ can imitate a high ground-state stability against spontaneous fission, although in fact the latter is much lower. These points are to be taken into account when setting experiments on the synthesis and identification of new spontaneously fissionable nuclides of the heaviest elements. There are presently known many unidentified spontaneous fission activities produced in irradiating targets of the actinide elements by different heavy ion beams (see, e.g., /46,47/). It is quite probable that the origin of some of these activities is associated with the presence of certain isomeric states in the known heaviest nuclei rather than with the ground-state spontaneous fission of nuclides being not yet identified.

6. CONCLUDING REMARKS

Thus, the nucleon pairing correlations of superconducting type strongly affect the probability and dynamics of tunnelling through the barrier in fission and fusion of complex nuclei. Generally, the presence of pairing correlations **assists** in increasing the barrier penetrability, yet all the quantitative and even some qualitative conclusions about the role of pairing correlations are essentially dependent on the choise of a particular approach to their treatment. As compared with what follows from the standard (BCS) approach, the allowance made in the framework of the least action principle for the coupling of the pairing vibrations with the fission mode results in a large enhancement of superfluidity in the subbarrier region of deformations, as was first shown by Moretto and Babinet /9/ and recently confirmed by more realistic calculations /10/. In Sections 2-5, we have demonstrated that this dynamically induced enhancement of superfluidity leads to a variety of important consequences for deeply subbarrier fission and fusion of nuclei. The most essential effect we predict lies in that the tunnelling probability turns out to be, in the dynamical treatment of pairing correlations, a considerably weakened function of the main parameters of the problem - Δ_o, B_f, and E. Within the framework of our consideration, the three parameters are in fact assembled into a single one -- the dimensionless parameter $\varkappa=(\Delta E/E_{cond})^{1/2}$, with $\Delta E=B_f-E$ being the deficit of the initial energy and $E_{cond}=g\Delta_o^2/2$ the condensation energy associated with the presence of the monopole pairing interaction in nuclei /14/; it is the magnitude of $\varkappa$ that governs the tunnelling probability. Although the new predictions may seem to be somewhat peculiar, these do not contradict any empirical evidence and, moreover,allow a more adequate explanation of some empirical facts to be given, e.g., that of the average value of the hindrance factors associated with ground-state spontaneous fission of odd nuclei.

While aiming to discuss the physics of the subbarrier processes in terms of quantitative estimates of <u>relative</u> nature, we have used a variety of approximations which enabled us to obtain results in the transparent analytic form. Some of these approximations, e.g., using the single-humped parabolic curve for the barrier profile, are not so essential and may easily be avoided by turning to numerical calculations. More significant distortions could be associated with applying the uniform model and assuming the dominance of the M_{qq} term in the effective mass. However, since the calculations /10/ not involving the above assumptions fully confirm the main conclusion by Moretto and Babinet /9/ represented by the gap equation (12), there is good reason to believe that our results obtained by employing eq. (12) provide a physically correct picture for the barrier penetration probability in fission and fusion of complex nuclei. Evidently, clarifying this picture via realistic numerical calculations like those performed in /10/ would present a task far from being simple, even when considering only a single deformation coordinate. At the same time, realistic microscopic calculations carried out for an extended range of nuclei and for a sufficiently complete deformation space are certainly called for.

Does the dynamically induced enhancement of superfluidity really occur in large-scale subbarrier rearrangements of nuclei? Our analysis of the average empirical values of the hindrance factors associated with ground-state spontaneous fission of odd nuclei provides clear indications for this important question to be answered positively. An effective tool for obtaining further empirical information on the superfluidity issue has been shown, in Section 5, to be given by studying the probability of spontaneous fission from q-p isomeric states of the heaviest nuclides. Although our discussion has been confined to q-p isomers of even-even nuclei, many points of this paper remain to be valid and can be applied in analyzing the probability of spontaneous fission from isomeric states in odd nuclei, e.g., from those of the (n,2p) or (p,2n) type /27/; the involvement of odd isomeric species would significantly extend the possibilities of studying nuclear structure effects in deeply subbarrier fission.

The original cause of the dynamical enhancement of superfluidity in tunnelling through the fission barrier is a strong, of the $1/\Delta^2$ type, dependence of the effective mass M upon the pairing gap parameter Δ , or, generally, the fact that the derivative $\frac{\partial M}{\partial \Delta}$ is an essentailly negative and large quantity; this dependence expresses, perhaps, the most definite of all the theoretical predictions for the effective mass and it emerges not only in the cranking model /2-6/ but in more advanced approaches, too (see, e.g., /48/). Thus, experimental verification of the ideas of the enhancement of superfluidity in tunnelling would mean, in fact, an empirical test of one of the major properties of the effective mass associated with large-scale subbarrier rearrangements of complex nuclei. Being an important dynamical characteristic of both fission and fusion, the effective mass is known to be not accessible to direct measurement; therefore a chance to gain any empirical information concerning its properties appears to be quite unique. The possibility of empirical checking of the property $\frac{\partial M}{\partial \Delta} < 0$ is even more valuable, since it is predicted to cause interesting and strong effects not only in the stage of tunnelling through the barrier but also in the stage of descent of a fissioning nucleus to the scission point /49/.

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Лазарев Ю.А. E4-86-577
Влияние парных корреляций на вероятность и динамику
туннельного проникновения через барьер
при делении и слиянии сложных ядер

В рамках модели, допускающей аналитические решения, получены следствия для проницаемости потенциального барьера, к которым приводит принцип наименьшего действия при рассмотрении параметра щели Δ как динамической переменной. Установлено, что по сравнению с традиционным (БКШ) подходом динамическое рассмотрение парных корреляций ведет к значительному ослаблению зависимости проницаемости барьера деления от его высоты, а также от параметра щели (Δ_0) и энергии начального состояния. Дано более адекватное объяснение средней величины факторов запрета на спонтанное деление нечетных ядер из основного состояния. Показано, что динамическое усиление парных корреляций при туннелировании может быть одной из причин сильного повышения сечений подбарьерного слияния сложных ядер. В результате анализа устойчивости квазичастичных изомерных состояний в тяжелых четно-четных ядрах найдено, что отношение парциальных периодов T^*_{sf}/T_{sf} спонтанного деления из изомерного двухквазичастичного и основного состояний ядра существенно зависит от того, имеет место или нет динамическое повышение сверхтекучести при туннелировании; следовательно, измерения T^*_{sf}/T_{sf} дают уникальную возможность для получения эмпирической информации о свойствах эффективной инерции, связанной с большими подбарьерными перестройками ядер.

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Lazarev Yu.A. E4-86-577
Influence of Pairing Correlations on the Probability
and Dynamics of Tunnelling through the Barrier in Fission
and Fusion of Complex Nuclei

An analytically solvable model is used to study the barrier penetrability pattern in the case when the pairing gap Δ is treated as a dynamical variable governed by the least action principle. It is found that, as compared to the standard (BCS) approach, the dynamical treatment of pairing results in a considerably weakened dependence of the fission barrier penetrability on the intensity of pairing correlations in the initial state (Δ_0), on the barrier height, and on the energy of the initial state. On this basis, a more adequate explanation is proposed for typical order-of-magnitude values of the empirical hindrance factors for ground-state spontaneous fission of odd nuclei. It is also shown that a large enhancement of superfluidity in tunnelling - the inherent effect of the dynamical treatment of pairing - strongly facilitates deeply subbarrier fusion of complex nuclei. Finally, an analysis is given for the probability of spontaneous fission from K-isomeric quasi-particle (q-p) states in even-even heavy nuclei. The relative change of the partial spontaneous fission half-life in going from the ground-state to a high-spin q - p isomeric state, T^*_{sf}/T_{sf}, is found to be strongly dependent on whether or not there takes place the dynamically induced enhancement of superfluidity in tunnelling. Measurements of T^*_{sf}/T_{sf} provide thus a unique possibility of verifying theoretical predictions about the strong, inverse-square Δ dependence of the effective inertia associated with large-scale subbarrier rearrangements of nuclei.

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