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**RESONANT FORMATION** OF  $dd_{\mu}$  MESIC MOLECULES

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#### 1. INTRODUCTION

The phenomenon of the mesic molecule resonant formation has been established experimentally for the first time in the works of V.P.Dzhelepov with the collaborators.<sup>1)</sup> In contrast to the ddµ molecule non-resonant formation, when the binding energy of the formed mesic molecule is transferred by the conversion electron:

 $d\mu + D_2 \rightarrow \left[ (dd\mu) de \right]^+ + e$ (1) the vory essense of the resonant mechanism of ddu molecule formation suggested by E.A.Vesman in 1967<sup>2</sup>) is the following. Mesic atom dµ with the kinetic energy  $\varepsilon_p$  when approaching one of the nuclei of  $D_2$  molecule forms with it mesic molecule ddµ (to be more precise, the mesic molecular ion  $(dd\mu)^+$ ) in a loosely bound rotational-vibrational state (J = v = 1), which becomes a "heavy nucleus" of mesic molecular complex  $[(dd\mu)dee]$ :

$$d\mu + D_2 \rightarrow \left[ (dd\mu) dee \right] \cdot$$
 (2)

The released binding energy  $|\epsilon_{JV}| = |\epsilon_{11}|$  of ddµ mesic molecule is transmitted to the excitation of vibrational-rotational states (YK) of mesic molecular complex  $[(dd\mu)dee]_{YK}$  (see Fig. 1).

If  $|\epsilon_{11}| \approx 2$  eV, the rate of reaction (2) at deuterium temperatures  $T \gtrsim 100$  K is  $\lambda_{dd\mu} \sim 10^6$  s<sup>-1</sup>, that exceeds the rate of non-resonant process (1) in tens times. The existence of a loosely bound state (J = v =1)<sup>3)</sup> was firmly established in theoretical works in the period 1973 -1983. It made possible to carry out the first well-grounded calculation of the ddy molecule formation rate  $\lambda_{ddu}(T)$  as a function of medium temperature  $T^{4)}$ . But on the whole this calculation was of illustrative charactor because the value of  $\epsilon_{11}$  was known at that time with an accuracy of  $\sim$  0.1 eV only and was determined just from the comparison of theoretical curve  $\lambda_{dd\mu}$  (T) with experiment<sup>5)</sup> at T = 300 K. Besides, the calculation<sup>4)</sup> did not take into account the spin structure of  $d\mu$ atom and ddy molecule energy levels and the energy levels rotational structure of D2 molecules and [(ddu)dee] complexes. Some peculiarities of the kinetics of the mesic molecular processes in deuterium, namely, the desintegration ("back decay") and stabilization of the mesic molecular complexes [(ddµ)dee], fusion and cascade transitions in ddµ mesic molecule, the process of dµ atom spin-flip in collisions, etc., were not induced into consideration, as well.

All the effects listed above are taken into account in the present paper.\* The energy of the (J = v = 1) state of  $dd\mu$  mesic molecule has

\* The first attempt to include the hyperfine structure of energy levels of  $(d\mu)_F$  and  $(dd\mu)_S$  was presented in paper<sup>6</sup>.

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объсякие слави институт ядерных за разлотото БИБИ 20 дала been calculated in recent years with a high accuracy  $(10^{-3} \text{ eV}), 7, 8)$  the rates of fusion  $\lambda_{f}^{Jv}$  and de-excitation  $\lambda_{dex}^{Jv}$  to the state (J = 1, v = 0) have been also found for  $dd\mu$  mesic molecule. Besides, in papers 11-13) the new refined expression has been obtained for the rate  $\lambda_{ddu}(T)$  (see also<sup>14,15)</sup>). Basing on the results of the listed papers, we present here the theoretical calculation of rates  $\lambda_{ddu}(T)$  for reactions

 $d\mu + DA \rightarrow \lceil (dd\mu)aee \rceil$ ,

(2a)

where A = D,H,T; a = d,p,t. This calculation does not involve any free parameters and additional hypotheses. The calculated dependence  $\lambda_{ddu}(T)$ fits well all the available experimental data on reaction (2) which is taken in the following as an example to demonstrate our method of the resonant rate calculations.

### 2. SCHEME OF THE CALCULATION OF RATES $\lambda_{ddu}$ (T)

In collisions of mesic atoms  $(d\mu)_F$  in spin state F with molecule  $(D_2)_{\mathbf{y}_i K_i}$  in vibrational-rotational state  $(\mathbf{y}_i K_i)$  the excited mesic molecular complexes  $[(dd_{\mu})dee]_{\gamma_{fK_{f}}}$  are formed in vibrational-rotational state  $(\mathcal{Y}_{f}K_{f})$ :

 $(d\mu)_{F} + (D_{2})_{j_{1}K_{1}} - [(dd\mu)_{S}^{Jv}dee]_{y_{f}K_{f}}$  (3) with rate  $\lambda_{FK_{1},SK_{f}}$  where S is the total spin of ddu mesic molecule. The formed complexes either decay with rate  $\Gamma_{SF}$ , into initial fragments (in the general case  $F' \neq F$ ,  $\gamma'_i \neq \gamma_i$  and  $K'_i \neq K_i$ ), or are stabilized with rate  $\lambda_{dex}$  due to de-excitation of ddu molecule:

$$\left[ (dd\mu)_{S}^{J\mathbf{v}} dee \right] \xrightarrow{\Gamma_{SF}} (d\mu)_{FF} + (D_{2})_{Y_{1}}^{K_{1}} K_{1}^{F}$$
(4a)  
$$\left[ (dd\mu)_{S}^{J\mathbf{v}} de \right]^{*} + e.$$
(4b)

The nuclear fusion reactions with the rates  $\lambda_{f}$  also prevent the complexes desintegration (4a):



The reaction channels probabilities are equal  $\beta = 0.58$  and  $1 - \beta = 0.42$ , respectively,<sup>16,17</sup> the muon sticking probability in the first channel being  $\omega_d = 0.122^{17,18}$ ,  $\lambda_f^{dd}$  is the rate of nuclear reaction.

The rate of reaction (3) equals 11, 13) \*

$$\Lambda_{\Gamma K_{i}, SK_{f}} = \lambda_{FK_{i}, SK_{f}} \varphi$$

$$\lambda_{FK_{i}, SK_{f}} = 2\pi N_{o} W_{FS} \int d^{3} p f(\epsilon_{p}, T) |V_{fi}|^{2} \delta(E_{i} - E_{f}),$$
(6)

where N<sub>o</sub> = 4.25  $\cdot$  10<sup>22</sup> cm<sup>-3</sup> is the liquid hydrogen density,  $\Psi$  = N/N<sub>o</sub> is the douterium density,  $\vec{p}$  is the momentum of relative motion of  $d\mu$  atom and D<sub>2</sub> molecule,  $f(\epsilon_{\rm p},T) = 2(\epsilon_{\rm p}/\pi)^{1/2}T^{-3/2}\exp\{-\epsilon_{\rm p}/T\}$  is the Maxwellian distribution over the relative collision energy  $\hat{\ensuremath{\varepsilon_{\text{D}}}}$  at given temperaturo T\*\*, V<sub>f</sub>; is the coordinate matrix element of <sup>P</sup>the transition from the initial state i to the final state f:

$$|V_{fi}|^{2} = 1/(2K_{i}+1)\sum_{M_{K_{i}},M_{K_{f}},M_{J}} |\langle \Psi^{(f)}|\hat{v}|\Psi^{(i)}\rangle|^{2}, \quad (7)$$

 ${\rm M}_{\rm K_i},~{\rm M}_{\rm K_f}$  and  ${\rm M}_{\rm J}$  are third projections of angular momenta  ${\rm K}_{\rm i},{\rm K}_{\rm f}$  and J. Overlapping between the spin functions of initial (F) and final (S) states is given by the factor<sup>11)</sup>

$$W_{FS} = 2(2S+1) \begin{cases} 1/2 & 1 & F \\ 1 & S & 1 \end{cases}^{2},$$
(8)

where  $\begin{cases} S_{\mu} & S_{d} & F \\ S_{d} & S & I \end{cases}$  is the Wigner 6j-symbol, I = 1 is the total spin of

deuterium nuclei in ddu molecule (identity of nuclei in ddu molecule being taken into account).

Coordinate wave functions and transition operator are

$$\Psi^{(i)} = \Psi_{d\mu}(\vec{r}_{1}) \Psi_{D2}^{\lambda} K_{i} (\vec{\rho}_{1}) e^{i\vec{p}\vec{\rho}_{2}}$$

$$\Psi^{(f)} = \Psi_{dd\mu}^{Jv}(\vec{r},\vec{R}) \Psi_{MD}^{\lambda fKf}(\vec{\rho})$$

$$\hat{V} = \vec{\rho} \cdot \vec{d} \langle \rho \cdot \partial W(\rho) / \partial \rho , \qquad (9)$$

 $\vec{r}_1$  and  $\vec{\rho}_1$  are intrinsic coordinates of dµ atom and D<sub>2</sub> molecule,  $\vec{\rho}_2$ is their relative coordinate,  $\vec{r}$  and  $\vec{R}$  are Jacobi coordinates of ddu molocule,  $\vec{\rho}$  is an internuclear coordinate of mesic molecular complex  $MD = \left[ (dd\mu)dee \right]$ ,  $\vec{d}$  is the ddµ molecule dipole moment. (The choice of **OXPRESSIONS** (6)-(9) is justified in papers 11-15.)

\* Another expression for (6) with  $\delta(E_i - E_f)$  substituted by the **Broit-Wigner** factor is suggested in paper<sup>19)</sup>. For the discussion of this quostion see  $^{13,20)}$ .

\*\* The cross-section  $S_{e\ell} \approx 2 \cdot 10^{-19} \text{ cm}^2$  <sup>34</sup>) of the elastic scattering  $d\mu$  + dis large enough for the Maxwell distribution establishing in times ~ 3.10<sup>-8</sup>  $\varphi^{-1}$  s. See paper<sup>45)</sup>, where the kinetics in D<sub>2</sub> + T<sub>2</sub> mixture at small  $\Psi$  is considered.

The initial  $(E_i)$  and final  $(E_f)$  energies for reaction (3) equal, respectively,

$$E_{i} = E_{F}(d\mu) + E_{y_{i}K_{i}}(D_{2}) + \varepsilon_{p}$$

$$E_{f} = \varepsilon_{S}(dd\mu) + E_{y_{f}K_{f}}(MD)$$

$$E_{F}(d\mu) = E_{1s}(d\mu) + \Delta E_{F}$$

$$\varepsilon_{S}(dd\mu) = \varepsilon_{Jv}(dd\mu) + \Delta \varepsilon_{S}, \qquad (10)$$

where  $E_{1S}(d\mu)$  and  $\Delta E_F$  are the nonrelativistic energy and relativistic hyperfine splitting of  $d\mu$  atom 1s state, corresponding to the total spin F,  $\varepsilon_{Jv}(dd\mu)$  and  $\varepsilon_S$  are the corresponding quantities for (Jv) state of  $dd\mu$  mesic molecule with the total spin  $S^{21,22}$ . The resonance condition  $E_f = E_i$  can be presented as  $\varepsilon_D = \varepsilon_{if}$ , where

$$\varepsilon_{if} = \varepsilon_{11} + \Delta \varepsilon_{FS} + \Delta E_{\mathbf{y}_{i}K_{i}}, \mathbf{y}_{f}K_{f} = \varepsilon_{o} + \Delta \varepsilon_{FS} + \Delta E_{if} \quad (11)$$

$$\begin{split} \varepsilon_{0} &= \varepsilon_{11} + \Delta E_{y} \text{ is the resonance defect, } \Delta E_{y} &= \Delta E_{y_{1}\hat{0}_{y}y_{f}\hat{0}} = E_{y_{f}\hat{0}}(MD) - \\ E_{y_{1}\hat{0}(D_{2})}; \varepsilon_{11} &= E_{11}(dd\mu) - E_{1s}(d\mu) \text{ the nonrelativistic energy } \varepsilon_{Jv} \text{ of the state } (J = v = 1), \quad \Delta \varepsilon_{FS} = \Delta \varepsilon_{S} - \Delta E_{F} \text{ is the spin splitting of } \varepsilon_{11}, \\ \Delta E_{y_{1}K_{1}}, y_{f}K_{f}^{=} E_{y_{f}K_{f}}(MD) - E_{y_{1}K_{1}}(D_{2}) = \Delta E_{y} + \Delta E_{if} \end{split}$$

$$\Delta \mathbf{E}_{if} = \left\{ \mathbf{E}_{\mathbf{y}_{f}\mathbf{K}_{f}}(\mathbf{MD}) - \mathbf{E}_{\mathbf{y}_{f}\mathbf{0}}(\mathbf{MD}) \right\} - \left\{ \mathbf{E}_{\mathbf{y}_{i}\mathbf{K}_{i}}(\mathbf{D}_{2}) - \mathbf{E}_{\mathbf{y}_{i}\mathbf{0}}(\mathbf{D}_{2}) \right\} = \varepsilon_{f} - \varepsilon_{i}.$$

Energies  $E_{y_1 K_1}(D_2)$  and  $E_{y_f K_f}(MD)$  of rotational-vibrational states of  $D_2$  molecule and complexes MD have been calculated with an accuracy of  $\sim 10^{-4}$  eV in paper<sup>23</sup>.

The general scheme of the ddµ-resonant formation is presented in Fig. 1. This process is possible if the sum  $\varepsilon_p + |\varepsilon_{11}|$  of the dµ atom kinetic energy  $\varepsilon_p$  and ddµ molecule binding energy  $|\varepsilon_{11}|$  is equal to the transition energy  $\Delta E_{\gamma}$  from the ground state  $\gamma_i = 0$  of D<sub>2</sub> molecule to the excited vibrational state  $\gamma_f = 7$  of the complex [(ddµ)dee].

The scheme of splitting of energy levels of initial and final states is shown in Figs. 2 and 3. In our calculations were used the quantities  $\varepsilon_{11} = -1.964 \text{ eV}, \star \Delta \varepsilon_{\text{FS}}$  and  $\Delta E_{\text{if}}$ , which can be found in papers<sup>8</sup>, 21-25).

\* The used value of  $\varepsilon_{11} = \varepsilon_{11}^{nrel} + \Delta \varepsilon_{11}^{rel} + \Delta \varepsilon_{11}^{dim}$  is a sum of the nonrelativistic energy for point-like nuclei  $\varepsilon_{11}^{nrel} = -1.9750 \text{ eV}^8$ , relativistic corrections  $\Delta \varepsilon_{11}^{rel} = 0.0097 \text{ eV}^{22}$  and corrections to the finite size of ddµ mesic molecule in the mesic molecular complex  $\Delta \varepsilon_{11}^{dim} =$ = 0.0010 eV<sup>24</sup><sub>1</sub>,25) therefore  $\varepsilon_{11} = -1.9643$  eV with the possible error



Fig. 1. Scheme of the resonance formation of ddu mesic molecules: the reaction is allowed if the resonance condition  $\epsilon_{\rm D} + |\epsilon_{11}| = \Delta E_{\rm Y}$  is fullfilled.

For reaction (1)  $\gamma_i = 0$ ,  $\gamma_f = 7$ , the resonance defect is  $\varepsilon_0 = 0.0337 \text{ eV}$ , and values of  $\Delta \varepsilon_{FS}$  equal:

 $\Delta \varepsilon_{1/2} \ _{1/2} = 0.0163 \text{ eV} \qquad \Delta \varepsilon_{1/2} \ _{3/2} = 0.0403 \text{ eV}$  $\Delta \varepsilon_{3/2} \ _{1/2} = -0.0322 \text{ eV} \qquad \Delta \varepsilon_{3/2} \ _{3/2} = -0.0082 \text{ eV}. (12)$ 

The rate  $\lambda_{\rm FS}$  of the  $({\rm dd}\mu)_{\rm S}$  mesic molecule formation from the initial states of mesic atoms  $({\rm d}\mu)_{\rm F}$ , averaged over the rotational states of  $({\rm D}_2)_{\rm K_1}$  molecule and summed over the final states of complex (MD)<sub>Kf</sub> equals

 $\lambda_{\text{FS}} = \sum_{K_i, K_f} \omega(K_i) \lambda_{\text{FK}_i, \text{SK}_f} \Theta(\epsilon_{if}),$ 

whore

$$\Theta(\mathbf{x}) = \begin{cases} 1 & \text{at } \mathbf{x} > 0 \\ 0 & \text{at } \mathbf{x} < 0 \\ \omega(K_{i}) = \xi(K_{i}) & Z_{i}^{-1}(2K_{i} + 1) \exp\{-\varepsilon_{i}/T\}, \end{cases}$$
(13b)

(13a)

 $\begin{array}{l} {\bf t}_{j} = \mathbb{E}_{y_{j} K_{1}} \left( \mathbb{D}_{2} \right) - \mathbb{E}_{y_{1} 0} \left( \mathbb{D}_{2} \right) \text{ is the rotational energy of molecule } \mathbb{D}_{2}, \\ {\bf w}(K_{1}) \quad \text{is the Boltzmann distribution of molecules } \mathbb{D}_{2} \text{ over rotational} \\ \text{degrees of freedom, } \mathbb{Z}_{1} \quad \text{is the statistical sum over rotational} \end{array}$ 

 $<sup>\</sup>delta \epsilon_{11} = \pm 0.0010$  eV which is comparable with the fine structure splitting ( 0.0006 eV) in ddu molecule<sup>22</sup>. (See Fig. 2)



Fig. 2. Scheme of hyperfine and fine splitting of energy levels of atoms  $(d\mu)_F$  and mesic molecules  $(dd\mu)_S^{U}$ :  $\vec{F} = \vec{S}_{\mu} + \vec{S}_d$  is a total spin of  $d\mu$  atom,  $\vec{S} = \vec{F} + \vec{S}_d = \vec{S}_{\mu} + \vec{I}$  and  $\vec{J} = \vec{S} + \vec{J}$  are total spin and total angular momentum of  $dd\mu$  molecule, respectively,  $\Delta E_Y = E_{Y_10} - E_{Y_10} = 1.9980 \text{ eV}$ ,  $\epsilon_0 = \Delta E_Y - |\epsilon_{11}| = 0.0337 \text{ eV}$ . Transition energies are given at vertical arrows showing transitions (F - S), their resonance energies  $\epsilon_0 + \Delta \epsilon_{FS}$  are given at the beginning, while corresponding weights  $W_{FS}$  at the end of the arrows.

states of  $D_2$  molecule at given temperature T,

$$\boldsymbol{\xi}(K_{i}) = \begin{cases} 2/3 & \text{for even } K_{i} \\ 1/3 & \text{for odd } K_{i} \end{cases}$$
(13c)

(for the case of equilibrium population of ortho- and para-states of  $D_2$  molecule).

Resonance process (2) is possible only if  $\varepsilon_{if} > 0$ . At temperatures  $T < 10^3$  K rotational states  $K_i < 10$  are excited, i.e., in resonance condition (11) ~100 various values of  $\Delta E_{if}$  are possible, corresponding to various combinations ( $K_i, K_f$ ) and 209 terms in sum (13), limited only by a condition  $\varepsilon_{if} > 0$ . In reality, at given temperature T an essential contribution to sum (13) comes only from 2 - 3 terms, for which  $\varepsilon_{if} < T$ .

The rate  $\Gamma_{SF}$ , of complex back decay (4a) equals\*

$$\mathbf{F}_{SF'} = \sum_{\mathbf{K}'_{1}, \mathbf{K}_{f}} \mathbf{F}_{SK'_{f}, F'K'_{1}} \boldsymbol{\omega}^{(K_{f})} \boldsymbol{\xi}^{(K'_{1})}$$
(14)

\* Rates of the rotational relaxation ( $\sim 10^{13} \varphi s^{-1} 2^{6}$ ) of the formed complexes noticeably exceed rates  $\lambda_f, \lambda_{dex}$  and  $\Gamma_{SF}$ , hence, during the

$$\omega(K_{f}) = Z_{f}^{-1} (2K_{f} + 1) \exp \left\{-\varepsilon_{f}/T\right\}$$

$$Z_{f} = \sum_{K_{f}} (2K_{f} + 1) \exp \left\{-\varepsilon_{f}/T\right\}$$
(14)

where  $\Gamma_{SK_{F},FK_{1}}$  is defined by the expression  $^{13,26,27}$ 



Fig. 3. Scheme of resonance transitions in reaction (3) with taken into account the rotational level splitting of molecule  $(D_2) \cdot v_i K_i$  and complex  $[(du)dec] \cdot v_{fKf}$ . On the left are shown values  $\varepsilon_{11} + \Delta \varepsilon_{FS} + \varepsilon_i$ , on the right  $\Delta E_y + \varepsilon_f$ , the energies being reckonned from the energy of the molecule  $(D_2) \cdot v_{j:=0,K_i=0}$  ground state.

lifetime of complex  $[(dd \mu)dee]$  the Boltzmann distribution over rotational states  $K_f$  is established.

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The details of calculations of  $\lambda_{\rm FS}$  and  $\Gamma_{\rm SF}$  are given in papers<sup>11-13</sup>, the dependence of these rates on temperature T is presented in Figs. 4 and 5. The observed rate of ddµ molecule formation  $\lambda_{\rm ddµ}$  is expressed through  $\lambda_{\rm FS}$  and  $\Gamma_{\rm SF}$ , the rates of fusion  $\lambda_{\rm f}^{\rm Jv}$ (5) and Auger transitions  $\lambda_{\rm dex}^{\rm Jv}$  (4), the rate  $\lambda_{\rm nr}$  of nonresonance formation of ddµ molecules in reaction (1) and rates  $\lambda_{\rm FF}$ , of dµ atom spin-flip in collisions with deuterium nuclei. To find adequate expressions for  $\lambda_{\rm ddµ}$ , one should consider the kinetics of muon catalysis processes in deuterium.

#### 3. KINETICS OF MUON CATALYSIS PROCESSES IN DEUTERIUM

During the time  $t_0 \sim 10^{-12} \varphi^{-1} s^{29}$  muons with energy  $\sim 10 \text{ keV}$ in deuterium form mesic atoms dµ in highly excited states  $n \ge 14^{30}$ . During  $t_a \sim 10^{-11} \varphi^{-1}$  s they go down to the ground 1s-state of dµ atom<sup>31</sup> and statistically populate sublevels F = 1/2 and F = 3/2 of its hyper-



**l'ig. 5. Temperature** dependence of complex back decay rates in reaction (4a). Approximate equalities are valid.  $\sum_{F} \Gamma_{3/2}F \approx \sum_{F} \Gamma_{1/2}F \approx \text{const.}$ 

fino structure<sup>32)</sup> (which subsequently will be denoted by indices 1 and 2, rospectively) with weights  $\dot{\gamma}_1 = 1/3$  and  $\dot{\gamma}_2 = 2/3$ . After this processes (3) and (17) of dd  $\mu$  molecule formation take place as well as **spin-flip** processes<sup>33,34</sup>:

$$(d_{\mu})_{F=3/2} + d_{\lambda_{12}}^{\lambda_{21}} (d_{\mu})_{F=1/2} + J$$
 (16)

the rates of direct and inverse processes (16) being connected by the dotailed balance relation

$$\lambda_{12} = \mathbf{\hat{y}} \cdot \lambda_{21}, \quad \mathbf{\hat{y}} = 2 \exp(-\Delta E/T)$$
$$\Delta E = \Delta E_{d\mu}^{hfs} = 0.0485 \text{ eV}. \quad (17)$$

at c ≠ S

The general case of muon catalysis kinetics in hydrogen isotope mixtures is rather complicated and is considered in papers<sup>35,36)</sup>. The Bohome of processes in pure deuterium is depicted in Fig. 6, and the Pystem of equations describing the kinetics of these processes has the form

$$|\mathbf{N}_{\mathrm{I}}/1t = -(\lambda_{0} + \lambda_{\mathrm{BF}} + \lambda_{\mathrm{FF}})\mathbf{N}_{\mathrm{F}} + \lambda_{\mathrm{FF}} \mathbf{N}_{\mathrm{F}} + \sum_{\alpha} \left\{ \Gamma_{\alpha}\mathbf{F}^{+} + \frac{\Gamma(1-\omega)\lambda_{\mathbf{f}}^{(\alpha)}}{\mathbf{f}} \right\} \mathbf{N}_{\alpha},$$

$$\mathbf{F}' \neq \mathbf{F}$$

$$|\mathbf{N}_{\alpha}/11 = -(\lambda_{0} + \lambda_{\mathbf{f}}^{(\alpha)} + \Gamma_{\alpha} + \sum_{\alpha'} \lambda_{\alpha \alpha'})\mathbf{N}_{\alpha} + \sum_{\alpha' \neq \alpha} \lambda_{\alpha'} \mathbf{N}_{\alpha'} + \sum_{\mathbf{F}} \lambda_{\mathrm{F}\alpha} \mathbf{N}_{\mathrm{F}}$$

$$|\mathbf{N}_{\mathrm{I}}/11 = \mathbf{\beta} \sum_{\alpha} \lambda_{\mathbf{f}}^{(\alpha)} \mathbf{N}_{\alpha}$$

$$\lambda_{\mathrm{F}} = \sum_{\mathrm{S}} \lambda_{\mathrm{FS}}, \quad \Gamma_{\mathrm{S}} = \sum_{\mathrm{F}} \Gamma_{\mathrm{SF}}, \quad \Gamma_{\alpha}\mathbf{F} = 0, \quad \lambda_{\mathrm{B}r} = \sum_{\alpha\neq \mathrm{S}} \lambda_{\mathrm{F}\alpha}$$

$$(18)$$

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Fig. 8. Kinetics of u-catalysis processes in deuterium. Rates  $\lambda_{FS}$ ,  $\Gamma_{SF}$ ,  $\lambda_{FF}$ , and  $\lambda_{f}$  are defined with formulas (13a), (14), (24) and (22) at  $\Psi = 1$ .

with initial conditions

 $N_{\rm F}(0) = Z_{\rm F}$   $N_{\alpha}(0) = N_{\rm n}(0) = 0.$  (19)

Here the notation is introduced:  $N_F$  is the number of mesic atoms  $(d\mu)_F$ ,  $N_{\alpha}$  is the number of  $(dd\mu)_{\alpha}$  mesic molecules in state  $\alpha = \{S, Jv\}$  (for the resonance state (J = v = 1) we put in the following  $\alpha = S$ ),  $N_n$  is the number of neutrons emitted in reaction (5),  $\omega = \omega_d$  is the total muon sticking probability in reaction (5),  $\lambda_0 = 0.455 \cdot 10^6 \text{ s}^{-1}$  is the muon decay rate,  $\lambda_f^{(\alpha)}$  is the nuclear fusion rate for the state  $(dd\mu)_{\alpha}$ ,  $\lambda_{nr}$ is the rate of the nonresonance ddu molecule formation in reaction (1) in various states  $\alpha = (S, Jv)$  (except the state (J = v = 1)), for which  $\Gamma_{\alpha F} = 0$ ,  $\lambda_{\alpha \alpha}$ , are the rates of Auger transitions (4b),  $\lambda_{FF}$ , is the rate of spin-flip reactions (16). All the rates  $\lambda_F$ ,  $\lambda_{FF}$ ,  $\lambda_{F\alpha}$  and  $\lambda_{nr}$  are normalized to the liquid hydrogen density  $N_0 = 4.25 \cdot 10^{22} \text{ cm}^{-3}$ . In the calculations below we use the following values of rates:

$$\lambda_{FF} = 4.7 \cdot 10^{7} \text{ s}^{-1} 34) \qquad \lambda_{nr} = 0.4 \cdot 10^{5} \text{ s}^{-1} 4.28)$$

$$\lambda_{f}^{(S)} = 0.43 \cdot 10^{9} \text{ s}^{-1} 9) \qquad \lambda_{f}^{(\alpha)} = 1.5 \cdot 10^{9} \text{ s}^{-1} 9) \qquad (20)$$

$$\lambda_{S\alpha} = 0.08 \cdot 10^{9} \text{ s}^{-1} 10) \qquad \omega = 0.071^{-17} 17,18).$$

Rates  $\lambda_{FS}(T) \approx 10^6 \div 10^7 \text{ s}^{-1}$  and  $\Gamma_{SF} \approx 10^8 \div 10^9 \text{ s}^{-1}$  are calculated in the present paper (see Figs. 4 and 5).

Taking into account inequalities  $\{\Gamma_{SF}, \lambda_{f}^{(\alpha)}\} \gg \{\lambda_{FS}, \lambda_{0}\}$  it follows from equations (18) that at  $t \gg (\lambda_{f}^{(\alpha)} + \Gamma_{\alpha})^{-1} \sim 0.5 \cdot 10^{-9}$  s the quasistationary regime is achieved, i.e. the following relations are fullfilled

$$dN_{\alpha}/dt \approx 0$$
,  $N_{\alpha} \approx \sum_{F} \lambda_{F\alpha} / (\lambda_{f}^{(\alpha)} + \Gamma_{\alpha}) N_{F} \ll 1$  (21)

$$N_{g} \approx \Psi \sum_{F} \lambda_{FS} / (\mathcal{K}_{f} + \Gamma_{S}) N_{F}$$

$$\lambda_{a} \lambda_{f}^{(a)} N_{a} = \sum_{F} (\lambda_{F} + \lambda_{nr}) \Psi N_{F} - \sum_{S} \Gamma_{S} N_{S}$$

$$\lambda_{f} = \lambda_{f}^{(S)} + \sum_{a} \lambda_{Sa}.$$
(21)

In this case the system of equations (18) is reduced to more simple one within the accuracy  $\{\lambda_{FS}, \lambda_0\} / \{\Gamma_{SF}, \lambda_f^{(\alpha)}\} \sim 10^{-2}$ :  $(1N_{\Gamma}/1) t = -(\lambda_0 + \lambda_F \varphi + \lambda_{FF}, \varphi) N_F + \tilde{\lambda}_{F}, F \varphi N_F, + \tilde{\lambda}_F^{(1-\omega)} \tilde{\lambda}_F^{N}_{dd\mu}$   $(1N_{\pi}/1) t = \beta \tilde{\lambda}_f N_{dd\mu}$ ,  $N_{1d\mu} = \sum_{\alpha} N_{\alpha} = \tilde{\lambda}_f^{-1} \varphi_F^{(\alpha)} \lambda_F^{N} N_F$ (22)

to which corresponds the scheme of the processes presented in Fig. 7. The effective rates in this scheme are given by:

$$\begin{aligned}
\widetilde{\lambda}_{F} &= \lambda_{nr} + \sum_{S} \lambda_{FS} \widetilde{\lambda}_{f} / (\widetilde{\lambda}_{f}^{+} \Gamma_{S}) \\
\widetilde{\lambda}_{FF} &= \lambda_{FF} + \sum_{S} \lambda_{FS} \Gamma_{SF} / (\widetilde{\lambda}_{f}^{+} \Gamma_{S}) \\
\overline{\lambda}_{f} &= \sum_{\alpha} \lambda_{f}^{(\alpha)} N_{\alpha} / \sum_{\alpha} N_{\alpha} \\
\end{aligned}$$
(23)

and  $\overline{\lambda}_{f} \approx \lambda_{f}^{(S)}$  within accuracy  $\sim \lambda_{nr} / \widetilde{\lambda}_{F}$ .



Fig. 7. The principal scheme of  $\nu$ -catalysis processes in deuterium, valid at  $t \gg (\tilde{\lambda}_f + \Gamma_S)^{-1} \approx 0.5 \cdot 10^{-9} \text{ s.}$ 

It is seen from (23) that the rates  $\lambda_{\rm FS}$  of the dd  $\mu$  molecule resonance formation is about three times as small at the value of the rates (20) than that calculated by formulas (13) which do not take into unuount the back decay of mesic molecular complexes (4a). (The importance of this process was first pointed out in papers<sup>26,27)</sup>, see<sup>37)</sup>, as well).

For the same reason the effective rate  $\lambda_{FF}$ , of spin-flip process differs from the rate  $\lambda_{FF}$ , of reaction (16), as the process (4a) changes the populations of the spin states  $(d\nu)_F$  and that is equivalent to the additional depolarization (F = 3/2)  $\rightarrow$  (F = 1/2). The second term for  $\tilde{\lambda}_{\rm FF}$ , in (23) describes this process. The more detailed discussion of this effect is performed in<sup>38)</sup>.

At t >>  $(\lambda_{21} \varphi)^{-1}$  the dynamical equilibrium of the spin-flip processes (16) gives as a result the quasistationary populations  $P_F$  of the spin states  $(d \mu)_F$ . In this case the time distributions  $N_F(t)$  become universal:

$$N_{F}(t) = P_{F} N_{d\mu}(t)$$

$$\sum_{F} P_{F} = \sum_{F} 2_{F} = 1.$$
(24)

Taking into account these relations we obtain from (22):

$$d N_{d\mu} / dt = -\lambda_{c} N_{d\mu}$$

$$d N_{n} / dt = \beta \lambda_{dd\mu} \varphi N_{d\mu} = \beta \lambda_{dd\mu} \varphi e^{-\lambda_{c}t}$$

$$\lambda_{c} = \lambda_{0} + \omega \lambda_{dd\mu} \varphi, \lambda_{dd\mu} = \sum_{i} P_{F} \lambda_{F},$$
(25a)

where the populations  $\ensuremath{\mathbb{P}_{\rm F}}$  are equal to

$$P_{1} = (1 + \tilde{\gamma})^{-1}, P_{2} = 1 - P_{1}, \\ \tilde{\gamma} = (\tilde{\lambda}_{12} + \tilde{\gamma}_{2}\tilde{\lambda}_{1}) / (\tilde{\lambda}_{21} + \tilde{\gamma}_{1}\tilde{\lambda}_{2}).$$
(25b)

Neglecting the terms of order  $\sim \omega^2$  it is possible to represent the solution of the system (22) in the analytical form:

$$N_{F}(t) = P_{F}e^{-\lambda_{c}t} + Q_{F}e^{-\Lambda t}$$

$$Q_{F} = \mathbf{2}_{F} - P_{F}$$

$$\Lambda = \lambda_{0} + (\widetilde{\lambda}_{21} + \widetilde{\lambda}_{12} + \mathbf{2}_{1}\widetilde{\lambda}_{2} + \mathbf{2}_{2}\widetilde{\lambda}_{1}) \mathbf{\varphi}.$$
(26)

Taking into account relations (22)-(25) we finally obtain:

 $dN_n/dt = \beta \varphi \left\{ \lambda_{dd\mu} e^{-\lambda_c t} + (\lambda_2 - P_2)(\lambda_2 - \lambda_1) e^{-\Lambda t} \right\}$ The multiplicity of muon catalysis  $\chi_c$  (the number of neutrons per one muon) is equal to

$$x_{c} = \beta \varphi \left[ \lambda_{dd \mu} \lambda_{c}^{-1} + (\beta_{2} - P_{2}) (\widetilde{\lambda}_{2} - \widetilde{\lambda}_{1}) \Lambda^{-1} \right].$$
(28)

At low deuterium temperatures 10  $\precsim$  T  $\precsim$  100 K, when the inequalities are valid:

$$\begin{array}{c} \lambda_{3/2} \gg \lambda_{1/2}, \ \lambda_{21} \gg \left\{ \lambda_{12}, \ \lambda_{1}, \ \lambda_{2} \right\}, \ \widetilde{\lambda}_{1} \approx \lambda_{nr}, \\ P_{1} \approx 1, \ P_{2} \approx \widetilde{\gamma} \approx 2/3 \ \lambda_{nr} / \lambda_{21} \approx 10^{-2}, \ \lambda_{ddu} \approx 1.05 \ \lambda_{nr} \end{array}$$

the time distribution of neutron is simplified:

$$dN_{n}/dt \approx \beta \varphi e^{-\lambda_{0} t} (\widetilde{\lambda_{1}} \exp \left\{ -\omega \widetilde{\lambda_{1}} \varphi t \right\} + 2/3 \widetilde{\lambda_{2}} \exp \left\{ -\widetilde{\lambda_{21}} \varphi t \right\}) (29)$$
Just this distribution was used at SIN experiment<sup>42</sup> for the

measurement of the ratio  $\widetilde{\lambda}_{3/2}$  /  $\widetilde{\lambda}_{1/2}$  .

#### 4. RESULTS AND DISCUSSION

It is clear from the previous consideration that the calculated values  $\lambda_{\rm dd\,\mu}(T)$  in the framework of accepted calculation scheme

depend on two parameters only: the binding energy  $|\epsilon_{11}|$  of the state (1 • v • 1) and the effective nuclear fusion rate  $\widetilde{\lambda}_f$  in the ddµ moleoule.

In Fig. 8 there are presented theoretical values of  $|\epsilon_{11}| \pm \delta\epsilon_1$ and  $\lambda_{\mu} \pm \delta \lambda_{\mu}$ , as well as the range of their allowed values, following from experiments<sup>17</sup> and<sup>42</sup>. The lower range corresponds to pairs of  $|\epsilon_{11}|$  and  $\lambda_{\mu}$  for which the values  $\lambda_{dd\mu}$  calculated by (23) - (26) fall on the boundaries of the interval of measured value<sup>17</sup>

$$\lambda_{dd\mu}(T = 298 \text{ K}) = (2.76 \pm 0.08) \cdot 10^6 \text{ s}^{-1}$$



Hig. II. The range of allowed values of  $|\epsilon_{11}| \pm \delta\epsilon_{11}$  and  $\tilde{\lambda}_f \pm \delta\tilde{\lambda}_f$  following from emperiments<sup>17)</sup> and<sup>42)</sup> (shadowed). The cross represents the theoretical willow of  $|\epsilon_{11}|$  and  $\tilde{\lambda}_f$  with their possible uncertainties.

The upper range corresponds to the measured ratio<sup>42)</sup>

$$\tilde{\lambda}_{3/2} / \tilde{\lambda}_{1/2}$$
 (T = 34 K) = 79.5 ± 8.0

of the rates of the ddu molecule formation from states F = 3/2 and I' - 1/2 of  $(d\mu)_{\rm F}$  mesic atom. It is easily seen that the shadowed area of [111] and  $\tilde{\lambda}_{\rm f}$  values compatible with results of both experiments<sup>17</sup>)



- Fig. 9. The dependence of reaction (1) rate  $\lambda_{dd\mu}$  (T) on temperature T.Experimental points:  $\mathbf{1}$  Dubna<sup>5)</sup>,  $\mathbf{1}$  Gatchina<sup>17)</sup>,  $\mathbf{1}$  Los Alamos<sup>42)</sup> (Dubna data are renormalized to the value  $\lambda_{dd\mu}$  (T = 298 K) measured at Gatchina). A solid line presents the calculation at  $\mathbf{e}_{11} = -1.964 \text{ eV}$ ,  $\tilde{\lambda}_f = 0.41 \cdot 10^9 \text{ s}^{-1}$ , à dashed line was calculated at  $\mathbf{e}_{11} = -1.964 \text{ eV}$ ,  $\tilde{\lambda}_f = 0.51 \cdot 10^9 \text{ s}^{-1}$ .
- and<sup>42)</sup> is within theoretical uncertainties of  $|\epsilon_{11}|$  and  $\tilde{\lambda}_{f}$ :  $|\epsilon_{11}| = 1.964\pm0.001 \text{ eV}, \quad \tilde{\lambda}_{f} = (0.51\pm0.10)10^9 \text{ s}^{-1}.$  (30a) \* The solid line in Fig. 9 presents  $\lambda_{dd\mu}(T)$  calculated at  $|\epsilon_{11}| = 1.964 \text{ eV}$  and  $\tilde{\lambda}_{f} = 0.41\cdot10^9 \text{ s}^{-1}$  (30b)

taken from the shadowed area in Fig. 8. It can be easily seen that these values of  $\varepsilon_{11}$  and  $\lambda_{\rm f}$  provide a good fit of all the experiments<sup>5,17,39</sup>) (Data from paper<sup>5)</sup> are multiplied by the normalization factor B = 3.6, which equals the ratio of rates  $\lambda_{\rm dd\,\mu}$  from<sup>17)</sup> and<sup>5)</sup> at T = 298 K). The supplementary analysis in respect to criterion  $\chi^2$  confirms that the best fit of theory and experiment is achieved at these very values (30b).

.The dashed line in Fig. 9 represents the calculated rates  $\lambda_{dd\mu}(T)$  at the theoretical values (30a) of  $\epsilon_{l\,l}$  and  $\widetilde{\lambda}_{f}$ . It is seen that the deviations of dashed line from solid one do not exceed ~ 20%. Since the accuracy of calculation scheme<sup>13)</sup> is estimated by ~ 1%, they are mainly caused by the uncertainties in the nuclear rate  $\widetilde{\lambda}_{f}$ , which today is known with the accuracy ~ 20 ÷  $30x^{9}$ .

The presented calculation of  $\lambda_{dd\,\mu}^{}(T)$  is carried out for the equilibrium mixture of ortho- and para-states of  $D_2$  molecules, corresponding to values of  $\xi(K_1)$  (13c). The curves for the cases of pure ortho- and para-states of  $D_2$  differ from each other not more than by 5% (Fig. 10, see also paper<sup>43</sup>), specially devoted to this question).



Fig. 10. The comparison of the  $\lambda_{\rm dd\,\mu}$  molecules formation rates in pure ortho and para deuterium.

It follows from our calculations<sup>4</sup>) that at T = 34 K  $\lambda_{dd\,\mu} \simeq \tilde{\lambda}_{1/2} \simeq \lambda_{nr} \simeq 0.4 \cdot 10^5 \text{ s}^{-1}$  (31) that noticeably differs from experimental values (0.76 ± 0.15)  $\cdot 10^5 \text{ s}^{-1}$  40) and (1.03 ± 0.04)  $\cdot 10^5 \text{ s}^{-1}$  41). A possible reason of such a discrepanoy may be the method of data analysis: in particular, to the neutron time distribution (27) two exponentials should be included.

In such a situation it seems necessary to make a new measurement of  $\lambda_{|\downarrow||\mu}$  in the liquid deuterium because they allow one to calculate the absolute value of  $\lambda_{3/2}$  from the measured ratio  $\lambda_{3/2} / \lambda_{1/2}$ . The rate  $\lambda_{4/2}$  being extremely sensitive to the value of  $\varepsilon_{11}$  (see the Table and Fig. 0), the level energy  $\varepsilon_{11}$  can be found with a high accuracy (~10<sup>-4</sup> eV). At low deuterium temperatures practically only one term duntributes to sum (13) for partial rates  $\lambda_{\rm FS}$ , namely, the one corresponding to the dipole transition (F = 3/2, K\_1 = 0) - (S = 1/2, K\_f = 1) with resonance energy  $\varepsilon_{\rm if}$  = 5.9 meV = 69 K. In this case  $\omega({\rm K}_{\rm i}) \approx 1$  and dependence (6), (13) of  $\lambda_{\rm dd\,\mu}({\rm T})$  at T  $\lesssim$  100 K can be put in the analytical form.

$$\lambda_{1,1,\mu}(T) \approx \lambda_{3/2-1/2}(T) \approx 4\sqrt{\pi}N_0W_{3/2-1/2} |V_{if}|^2 \sqrt{\varepsilon_{if}} T^{-3/2} \exp\{-\varepsilon_{if}/T\} = 2.5 \cdot 10^{10} \cdot T^{-3/2} \exp\{-69/T\} s^{-1}, \qquad (32)$$

Note that the shape of the curve for  $\lambda_{dd\,\mu}(T)$  is very sensitive to the value of  $m_{1}$ , hence accurate measurements of the temperature

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dependence of the ddµ molecule resonance formation rate will allow experimental studies of relativistic effects in a three-body system <sup>44</sup>. The degree of sensitivity of  $\lambda_{dd\mu}(T)$  to  $\epsilon_{ll}$  is demonstrated in Fig. 11:  $\epsilon_{ll}$  variation of 10 meV twice changes the value of  $\lambda_{dd\mu}$  at T ≤ 100 K.



Fig. 11. The dependence of the shape of  $\lambda_{dd\mu}(T)$  curve for reaction (1) on the value  $|\epsilon_1|$  at  $\lambda_f = 0.41 \cdot 10^9 \text{ s}^{-1}$ .

In Fig. 12 there are presented spin-flip rates (F = 3/2) - (F = 1/2) (16) without  $(\lambda_{21})$  and with  $(\widetilde{\lambda}_{21})$  formation and desintegration of  $\left[ (dd \mu) dee \right]$  complexes taken into account. It is seen that  $\widetilde{\lambda}_{21}$  exceeds  $\lambda_{21}$  by 20% at low temperatures. It is seen from the Table that  $\lambda_{21}^{-34}$  coincide with experiment<sup>42</sup> better than  $\widetilde{\lambda}_{21}$ . The reason for this disagreement is still unclear.

# 5. RESONANT FORMATION OF dd $\mu$ MOLECULES IN THE COLLISIONS d $\mu$ + DT AND d $\mu$ + DH

The scheme of calculations<sup>13)</sup> can be used directly for calculation of  $\lambda_p$  rates of dd  $\mu$  molecules in reactions:

$$d\mu)_{r} + DT \rightarrow \left[ (dd\mu)_{s} tee \right]$$
(33)

$$d\mu$$
)<sub>F</sub> + DH  $\rightarrow [(dd\mu)_{S}pee]$  (34)



Fig. 11. Into  $\lambda_{FF}$ , and  $\widetilde{\lambda}_{FF}$ , (23) of spin flip in direct reaction (16) and with the back decay processes (4a) taken into account. The contribution from the latter process reaches ~ 20% at T ~ 34 K.

18	b	1	8	
18	b	1	8	

leferences	$\lambda_{3/2}$ / $\lambda_{1/2}$	$\tilde{\lambda}_{21}$ , 10 <sup>6</sup> s <sup>-1</sup>
Kannal at al. <sup>43)</sup>	79.5 <u>+</u> 8	37.4 ± 1.5
Piesent papor		
$\lambda_{\rm P} + 0.41 \cdot 10^9  {\rm s}^{-1}$	73.3	48.6
$0.51 \cdot 10^9 \text{ s}^{-1}$	87.7	48.1

Remonance defects for these reactions equal, respectively:

for reaction (33)  $\varepsilon_0 = -0.1789 \text{ eV} (\gamma_f = 7)$   $\varepsilon_0 = 0.0478 \text{ eV} (\gamma_f = 8)$ for reaction (34)  $\varepsilon_0 = -0.1078 \text{ eV} (\gamma_f = 5)$  $\varepsilon_0 = 0.2151 \text{ eV} (\gamma_f = 6).$ 

The temperature dependences of partial resonant rates  $\lambda_F(T)$  for reactions (1) and (34) are given in Fig. 13 and 14.

It is nocessary to keep in mind that presented rates give only the unmeral outline of the resonance processes (33) and (34), but for the demuription of some important peculiarities of them it is necessary to perform the carefull consideration of the  $\mu$ CF kinetics in the mixtured  $D_{\mu} + H_{2}$  and  $D_{2} + T_{2}$ .



Fig. 13. Partial rates  $\lambda_{dd\mu}(T)$  of complex [(dd $\mu$ )tee] formation in states  $\mathcal{Y}_{f} = 7$ and  $\mathcal{Y}_{f} = 8$  in reaction (33).

In particular, the deep minimum in the cross-section of elastic scattering  $d\mu + p$  at  $\varepsilon \approx 1.6$  eV prevents from the fast thermalization of  $d\mu$  atoms in the mixture  $H_2 + D_2$ . That in a turn will influence the processes of resonant formation of  $dd\mu$  molecules in reaction (34) and leads to the spin-flip of  $d\mu$  atoms in the chain of reactions

$$(d_{\mu})_{\mathbf{F}} + (DH)_{\mathbf{v}_{i}K_{i}} - \left[ (dd_{\mu})_{\mathbf{S}}^{\mathrm{pee}} \right]_{\mathbf{v}_{f}K_{f}} - (d_{\mu})_{\mathbf{F}} + (DH)_{\mathbf{v}_{f}K_{f}}$$
(35)

The detailed study of this depolarization mechanism of dµ atoms in H<sub>2</sub> + D<sub>2</sub> mixture may turn necessary for the consecutive description of the nuclear fusion in pdµ molecule studies experiments<sup>46)</sup>, as well as for understanding of the well known but surprising results of the experiment<sup>47)</sup> devoted to the measurement of µ-capture rate by the deuterium nuclei in the H<sub>2</sub> + D<sub>2</sub> mixture.

#### 6. CONCLUDING REMARKS

The presented calculations of the dd  $\mu$  mesic molecule resonance formation rates crowns the program of studies outlined in paper<sup>35)</sup>. The various characteristics of mesic molecules and mesic molecular complexes which are necessary for calculations, namely, nonrelativistic values of the dd  $\mu$  molecule energy levels, their relativistic shifts including the vacuum polarization, nuclear recoil and nuclei form factors, spin splitting and other relativistic offoots; energy levels of mesic molocular complexes, the influence of a  $dd\mu$  molocule finite size on the energy of the complexes rates of cascade transitions in  $dd\mu$  molecule and of their nonresonance formation, etc., are also calculated without any free parameters. Only when calculating the fusion rate in  $dd\mu$  mesic molecule the measured cross section of reaction 1 ( $dd_{\mu}$ ) has been used. With this exception the presented calculations are the calculations *ab initio*.





The achieved agreement between theoretical and experimental values  $uf = \frac{1}{1+1} (T)$  syldences both the correctness of all the calculational achieved as a whole ( the choice of wave functions, transition operator, method of averaging, etc.) and high accuarcy of theoretical calculations of the objectives and  $[(dd \mu)dee]$  complexes.

The performed investigation demonstrates a possibility and gives a limit basis for precise measurements of the binding energy of the molecule (J = 1, v = 1) state with an accuracy of  $10^{-3} - 10^{-4}$  eV, i.e. with a relative accuracy of  $10^{-6} - 10^{-7}$  44).

The analysis of experimental data in the framework of the developed calculation scheme (see Fig. 8) gives the following value for the binding energy of dd  $\mu$  molecule bound state (J = v = 1):

$$|\mathbf{r}_{11}| = 1.9643 \pm 0.0005 \text{ eV}$$
 (36)

that is in a very good agreement with the theoretical value (30a). However, the presented analysis could be refined because in it the fine structure of ddµ mesic molecule energy levels (splitting  $\approx 0.6$  meV) having been not taken into account in the present analysis and the theoretical value  $\lambda_{nr}$  having been used. It is very desirable in this line to perform the new measurements of  $\lambda_{dd\mu}$  in liquid deuterium.

The present calculation is a numerical realization of the theoretical scheme developed in papers<sup>11-13)</sup> for the general case of resonance formation of ddµ and dtµ molecules. The achieved agreement between theory and experiment for the case of ddµ molecules allows one to use it with a confidence for a more complicated case of dtµ molecules. Corresponding calculations are being completed.

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D4 05 054	Proceedings on the International School on Nuclear Structure. Alushta, 1985.	11.00
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D11-85-791	Proceedings of the International Conference on Computer Algebra and its Applications in Theoretical Physics. Dubna, 1985.	,12.00

Вычислены скорости  $\lambda_{ddu}(T)$  резонансного образования мезомолекул ddµ при столкновениях мезоатомов dµ с молекулами D2, HD и DT при температурах 10≲T≲1000 К. Для реакции dµ+D<sub>2</sub> →  $\rightarrow$ [(ddµ)dee], вычисленные скорости  $\lambda_{dd\mu}$ (T) хорошо согласуются с экспериментально измеренными на всем интервале изменения температуры дейтерия. Работа выполнена в Лаборатории ядерных проблем ОИЯИ. Препринт Объединенного института ядерных исследований. Дубна 1986 Faifman M.P. et al. E4-86-541 Resonant Formation of ddµ Mesic Molecules The rates  $\lambda_{dd\mu}(T)$  of dd\mu mesic molecule resonant formation in dµ mesic atom collision with molecules  $D_2$ , HD and DT at temperatures 10 ST \$1000 K are calculated. For reaction  $d\mu + D_2 \rightarrow [(dd\mu)dee]$  the calculated rates  $\lambda_{dd\mu}(T)$  are in good agreement with the experimentally measured ones for a wide range of deuterium temperature. The investigation has been performed at the Laboratory of Nuclear Problems, JINR. Preprint of the Joint Institute for Nuclear Research. Dubna 1986

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Резонансное образование мезомолекул ddµ

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