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INTERACTING MULTI-BOSON MODEL

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1. The problem

The interacting boson model (IBM) [1-4] has got a great development and has been used in many applications [5]. As pointed out by its authors [1], in its initial version it is equivalent to the earlier proposed quadrupole phonon model (QPM) [6-8]. Its complete group structure has been described in ref. [9]. The relation between the coefficients of both models yielding their equivalence has been discussed by several authors, but exactly derived in ref. [10]. In spite of some drawbacks [11,10,12], it is the only model yielding such a simple and clear transition between nuclei of different shape and such a connection between vibrational and rotational motion.

The problem of its microscopic foundation [5,13,14] will not be considered here. Several well known extentions have been proposed: the proton-neutron IBM [15], the interacting boson fermion model including dynamical supersymmetries [16-19], the configuration mixing IBM [20,10,21], the higher order IBM [5,22]. The necessity of several other degrees of freedom, besides the monopole-quadrupole (s , d) one, has been realized immediately [1-3]. Additional bosons, such as pairing monopole-quadrupole (s', d'), dipole (p), octupole (f), hexadecapole (g) have been included [5]. However until very recently this has been done in a more or less empirical way.

One year ago it was realized that the inclusion of new bosons, either the g one [23] or the combination of p f [24], does not change at least the rotational limit drastically since the U(3) subgroup remains there. Therefore the introduction of a "unified IBM" [25] of a somewhat different type than here has been preliminarily suggested.

So our problem is to show that some extentions of the model with the inclusion of new bosons, aimed at better describing level energies and transition rates, if suitably chosen, are conserving its basic feature of a simple and clear description of transitional nuclei. Thus we stress that this advantage is not related to the monopole-quadrupole degree of freedom only. Special attention has been paid to the group-theoretical foundation of this statement.

2. Coordinates, bosons and groups

The collective coordinates we are going to use are of the type introduced in ref. /26/. However we consider $n-1$ power forms instead of 2 as follows:

$$q_{\varkappa_1 \varkappa_2 \dots \varkappa_{n-1}} = \sum_{k=1}^{A} \varkappa_1^k \varkappa_2^k \dots \varkappa_{n-1}^k . \qquad (1)$$

Here $\varkappa_i^k = x_i^k, y_i^k, z_i^k$ is the usual coordinate of the k-th nucleon. Thus we do not introduce spin and isospin yet, but this can be done later on in an extended version of the model. The coordinates q are symmetric with respect to any permutation of $\varkappa_1 \varkappa_2 \dots \varkappa_{n-1}$. We denote by n_x, n_y, n_z the numbers of x, y, z indices, $n_x + n_y + n_z = n - 1$. So we have $(n-1)!/(n_x!\, n_y!\, n_z!)$ different index sequences giving the same coordinate, and $n(n+1)/2$ different coordinates.

We notice that if we decompose (1) into irreducible tensors with respect to the usual rotational O(3) group, we get one tensor of each rank $j = n-1, n-3, \dots 1$ or 0, which we denote by q_m^j. We denote also by $p_m^j = i^{-1} \partial/\partial q_m^j$ their momenta. Now we can define our bosons in the usual way:

$$b_m^{j+} = \frac{1}{\sqrt{2}}\left(q_m^j - i(-)^m p_{-m}^j\right), \quad b_m^j = \left(b_m^{j+}\right)^+ = \frac{1}{\sqrt{2}}\left((-)^m q_{-m}^j + i p_m^j\right). \quad (2)$$

We notice that $b_{m_1}^{j_1+} \cdot b_{m_2}^{j_2}$ are generators of a $U(n(n+1)/2)$ group. If in (1) we change n to $n' = n-1$, we obtain tensors of rank $j = n-2, n-4, \dots 0$ or 1, of which we construct the generators of a $U(n(n-1)/2)$ group in the same way. Altogether both cases give n bosons of rank $j = n-1, n-2, \dots 0$, and the generators of a $U(n^2)$ group. Let us notice also that all our bosons b_m^{j+}, $\bar{b}_m^j = (-)^m b_{-m}^j$ have + time parity, and the bosons of rank j have $(-)^j$ space parity.

To find all subgroups of $U(n^2)$ of physical significance, containing the usual rotational O(3) group at the end, is of course a nontrivial group-theoretical problem. However in addition to the usual group-theoretical methods /27/ for finding the branching rules of e.g. $U(r) \supset O(r)$, there is a powerful method related to the so-called outer plethysm, introduced in ref. /28/, described in ref. /29/ and used for tabulation of $O(r) \supset O(3)$, tabulated in refs. /30,31/ in a form convenient for $U(r) \supset U(s)$. We have used this method to find the $U^{0,2}(3)$ subgroup embedded in the usual s d boson $U^{0,2}(6)$ group/9/.

We denote the momenta with respect to (1) by $p_{\varkappa_1 \varkappa_2 \dots \varkappa_{n-1}} = i^{-1} \partial/\partial q_{\varkappa_1 \varkappa_2 \dots \varkappa_{n-1}}$. We define their bosons similarly to (2) but in a Cartesian basis. Then we use combinations of the $U(n(n+1)/2)$ generators by making s contractions:

$$\sum_{\varkappa_1 \dots \varkappa_s} b^+_{\varkappa'_{n-1} \dots \varkappa'_{s+1} \varkappa_s \dots \varkappa_1} \; b_{\varkappa_1 \dots \varkappa_s \varkappa''_{s+1} \dots \varkappa''_{n-1}} .$$

If we make suitable additional combinations, we may hope to get the generators of a subgroup. One example is the usual $n = 3$ s d $U^{0,2}(6)$ group where one contraction gives the generators of its $U^{0,2}(3)$ subgroup /9/. Another example is the $n = 4$ p f $U^{1,3}(10)$ group where two contractions give its $U^{1,3}(3)$ subgroup suggested in ref. /24/. Further examples will be given in section 3.

3. s p d f boson model

For $n' = n - 1 = 3$ and $n = 4$ we obtain $b^j = s, p, d, f$ bosons of multipolarity $j = 0, 1, 2, 3$ and their $U^{0,1,2,3}(16) \supset U^{0,2}(6) \times U^{1,3}(10)$ group. Thus U(16) contains the direct product of the usual s d $U^{0,2}(6)$ group and a p f $U^{1,3}(10)$ group. For the subgroups $U^{0,2}(3)$ and $O^{0,2}(6)$ of the usual s d $U^{0,2}(6)$ group there are phase ambiguities, used in a different way in refs. /1-4/ and in ref. /32/, and discussed in refs. /33,34/.

If for the moment we do not pay attention to time reversal invariance and normalize the $U^{0,2}(6)$ generators similarly to ref. /32/, we get the known table 1 ($L^1 = \frac{1}{\sqrt{2 \cdot 5}} \hat{I}$, where $\hat{I}$ is the angular momentum operator). The important for the subgroups existence /9,4/:

$$\begin{array}{llll} U^{0,2}(6) \supset U^2(5) & \supset O^2(5) & \supset O^2(3) & \\ U^{0,2}(6) \supset O^{0,2}(6) & \supset O^2(5) & \supset O^2(3) & \qquad (3) \\ U^{0,2}(6) & \supset U^{0,2}(3) & \supset O^2(3) & \end{array}$$

commutators are listed in table 2.

Now we apply the method mentioned at the end of the preceding section 2, and construct generators of $U^{1,3}(10)$ as listed in table 3. One can check directly that they obey the same commutation relations of table 2. Thus one sees that in $U^{1,3}(10)$, besides the new but trivial first and partly second row, the same subgroups as in the

Table 1

Generators of the sd $U^{0,2}(6)$ group, defining its $U(5)$ (N^0, L^1, N^2, L^3, N^4), $O(6)$ (L^1, L^2, L^3) and $U(3)$ (N^0, L^1, Q^2) subgroups; $\mu_+\mu_+^* = \nu_+\nu_+^* = 1$

$$N^0 = s^+s + \sqrt{5}\,[d^+d]^0$$

$$L^1 = [d^+d]^1$$

$$N^2 = [d^+d]^2$$

$$L^2 = [\mu_+ d^+s + \mu_+^* s^+d]^2$$

$$Q^2 = \frac{2}{\sqrt{3}}[\nu_+ d^+s + \nu_+^* s^+d]^2 + \sqrt{\frac{7}{3}}[d^+d]^2$$

$$L^3 = [d^+d]^3$$

$$N^4 = [d^+d]^4$$

Table 2

Commutators of the $U^{0,2}(6)$ and $U^{1,3}(10)$ group generators, proving the existence of their $U(5)$, $O(6)$ and $U(3)$ subgroups; T^k is any generator

$$[N^0, T^k]^j = 0$$

$$[L^1, T^k]^j = -\sqrt{\frac{k(k+1)}{2\cdot 5}}\, T^k \delta_{jk}$$

$$[Q^2, Q^2]^j = 5L^1\delta_{j1}$$

$$[L^3, L^3]^j = -\sqrt{\frac{2\cdot 7}{5}}\, L^1\delta_{j1} + \sqrt{\frac{3}{5}}\, L^3\delta_{j3}$$

$$[L^2, L^2]^j = 2L^1\delta_{j1} + 2L^3\delta_{j3}$$

$$[L^2, L^3]^j = -\sqrt{\frac{7}{5}}\, L^2\delta_{j2}$$

$$[N^2, N^2]^j = L^1\delta_{j1} - \frac{2^3}{7} L^3\delta_{j3}$$

$$[N^4, N^4]^j = \sqrt{2\cdot 3}\, L^1\delta_{j1} + \frac{3}{7}\sqrt{11}\, L^3\delta_{j3}$$

$$[N^2, N^4]^j = \frac{3}{7}\sqrt{2\cdot 5}\, L^3\delta_{j3}$$

$$[N^2, L^3]^j = \frac{2^3}{\sqrt{5\cdot 7}} N^2\delta_{j2} - \sqrt{\frac{2\cdot 5}{7}}\, N^4\delta_{j4}$$

$$[N^4, L^3]^j = -3\sqrt{\frac{2}{7}}\, N^2\delta_{j2} - \sqrt{\frac{11}{7}}\, N^4\delta_{j4}$$

Table 3

Generators of the pf $U^{1,3}(10)$ group, defining its $U(5)$ (N^0, L^1, N^2, L^3 with λ, N^4), $O(6)$ (L^1, L^2, L^3 with μ_-) and $U(3)$ (N^0, L^1, Q^2) subgroups; $\lambda\lambda^* = \mu_-\mu_-^* = \nu_-\nu_-^* = 1$

$$N^0 = -\sqrt{3}\,[p^+p]^0 - \sqrt{7}\,[f^+f]^0$$

$$L^1 = -\frac{1}{\sqrt{5}}[p^+p]^1 - \sqrt{\frac{2\cdot 7}{5}}\,[f^+f]^1$$

$$N^2 = \frac{1}{5}\sqrt{3\cdot 7}\,[p^+p]^2 + \frac{2}{5}\sqrt{2\cdot 3}\,[\lambda f^+p + \lambda^* p^+f]^2 - \frac{1}{5}\sqrt{2\cdot 3}\,[f^+f]^2$$

$$L^2 = -\frac{2}{5}\sqrt{2\cdot 3}\,[p^+p]^2 + \frac{1}{5}\sqrt{3\cdot 7}\,[\mu_- f^+p + \mu_-^* p^+f]^2 + \frac{2}{5}\sqrt{3\cdot 7}\,[f^+f]^2$$

$$Q^2 = \frac{3^2}{5}[p^+p]^2 + \frac{2}{5}\sqrt{3\cdot 7}\,[\nu_- f^+p + \nu_-^* p^+f]^2 + \frac{3}{5}\sqrt{2\cdot 7}\,[f^+f]^2$$

$$L^3 = \sqrt{\frac{2\cdot 3}{5}}\left[\binom{\lambda}{\mu_-} f^+p + \binom{\lambda^*}{\mu_-^*} p^+f\right]^3 + \sqrt{\frac{3}{5}}\,[f^+f]^3$$

$$N^4 = -\sqrt{\frac{2}{5}}\,[\lambda f^+p + \lambda^* p^+f]^4 + \sqrt{\frac{11}{5}}\,[f^+f]^4$$

$U^{0,2}(6)$ case are embedded:

$$\begin{array}{lllll}
U^{1,3}(10) \supset U^1(3)\times U^3(7) & \supset O^1(3)\times O^3(7) & \supset O^1(3)\times O^3(3) & \supset O^{1,3}(3) & \\
U^{1,3}(10) & \supset O^{1,3}(10) & \supset O^{1,3}(5) & \supset O^{1,3}(3) & \\
U^{1,3}(10) & \supset U^{1,3}(5) & \supset O^{1,3}(5) & \supset O^{1,3}(3) & (4) \\
U^{1,3}(10) & \supset O^{1,3}(6) & \supset O^{1,3}(5) & \supset O^{1,3}(3) & \\
U^{1,3}(10) & & \supset U^{1,3}(3) & \supset O^{1,3}(3) &
\end{array}$$

We are going to call these five rows separate vibrator, intermediate vibrator, mixed vibrator, unstable rotator and stable rotator.

The generators of table 3, at variance with those of table 1, do not lead to a $U^{1,3}(6)$ subgroup. The direct products of the (3) and (4) subgroups of the same name, with upper indices 0, 1, 2, 3, will be embedded in the $U^{0,1,2,3}(16) \supset U^{0,2}(6)\times U^{1,3}(10)$ group. One can see moreover that the phase ambiguities in both the $U^{0,2}(6)$ and $U^{1,3}(10)$ cases represent angles of rotation in the coordinate-momentum space of d bosons with respect to s for table 1, and f with respect to p for table 3.

Thus we can conclude that in spite of extending the boson space, the physical consequence of a good description of the transition between nuclei of different shape will be preserved. However, differences due to different possible representations of the subgroups will appear: see section 5.

4. Hamiltonian and transition operators

We limit the hamiltonian by the usual conditions: up to 4 boson operator terms, total boson number conservation, hermiticity, invariance with respect to rotation, time and space parity reversal. It is well known that in this way in the usual sd hamiltonian there remain 9 terms which can be written alternatively /32/ as the independent first and second order Casimir operators of all the groups in $U^{0,2}(6)$ (3) with $\mu_+ = 1, i$, $\nu_+ = \pm 1$ in table 1.

Let us for the moment try to solve this problem only for the pf case. Then we can show that one is left with 16 terms. Time reversal invariance is achieved by using real coefficients as in the sd case. Although one can write them explicitly, we are going to try to write the hamiltonian directly in terms of the Casimir operators of all groups in $U^{1,3}(10)$ (4). The first order Casimir operator of any group $U^s(r)$ will be denoted by $\hat{n}^s_r$, the second order one of $U^s(r)$ by $\hat{\nu}^s_r$ and the second order one of $O^s(r)$ by $\hat{\omega}^s_r$, normalized as in ref. /32/. Thus we obtain:

$$H = \sum_{j=1,3} \varepsilon^j \hat{n}^j_{2j+1} + \sum_{j \le k=1,3} \alpha^{j,k} \hat{n}^j_{2j+1} \hat{n}^k_{2k+1} + \sum_{j=1,3} \alpha^{1,3}_{j+2} \hat{\nu}^{1,3}_{j+2}$$

$$+ \sum_{j \le k = 1,3} \beta^k_{2j+1} \hat{\omega}^k_{2j+1} + \sum_{j,k=1,3} \beta^{1,3}_{(j+2)(k+1)/2} \hat{\omega}^{1,3}_{(j+2)(k+1)/2} . \qquad (5)$$

The hamiltonian in the spdf case can be written in a similar manner.

To obtain time reversal invariance, we have to insert into table 3 λ, μ_-, $\nu_- = \pm 1$. The terms in (5) are 14 instead of 16. The lack of two terms might be related to: 1) missing some subgroup of $U^{1,3}(10)$ in (4); 2) having an example where the Casimir operators do not exhaust all terms of the hamiltonian /35/. However, we might try to remedy this difficiency by adding the Casimir operators,

e.g. $\hat{\nu}$ of $U^{1,3}_{\lambda}(5)$ and $U^{1,3}_{\nu_-}(3)$ with $\lambda = \nu_- = -1$ to those with $\lambda = \nu_- = +1$, if they turn out to be independent.

We are going to show that something like this is e.g. the case with the $U^{0,2}(6)$ group. Instead of the $O^{0,2}(6)$ Casimir operator one might use the $U^{0,2}_{\nu_+}(3)$ Casimir operator with a phase factor $\nu_+ = -1$ added to that with $\nu_+ = +1$, or instead of the $O^2(5)$ Casimir operator the $O^{0,2}_{\mu_+}(6)$ Casimir operator with $\mu_+ = i$ added to that with $\mu_+ = 1$. In fact, they are related as follows (where $\hat{\nu}^{0,2}_{3,\nu_+} = (\hat{Q}_{\nu_+})^2 + \frac{1}{2}(\hat{I})^2 + \frac{4}{3}(\hat{N})^2$, $\hat{N} = \hat{n}^2_5 + \hat{n}^0_1$, $\hat{n} = \hat{n}^2_5$, $\hat{Q} = Q^2$, $\hat{I} = \sqrt{2\cdot 5}\, L^1$):

$$(\hat{Q}_{+1})^2 + (\hat{Q}_{-1})^2 = \frac{1}{3}\left[8\hat{\omega}_{6,1} - 12\hat{\omega}_5 + (\hat{I})^2 + 4\hat{n}(\hat{n}+5)\right]$$

$$\hat{\omega}_{6,1} + \hat{\omega}_{6,i} = 2\hat{\omega}_5 + 4(\hat{N} - \hat{n})(\hat{n} + 5/2) + 2\hat{n} \cdot \qquad (6)$$

These results mean that the $O(6)$ unstable rotator limit can be interpreted as appearing from the transition between $U(5)$ vibrator and two $U_{+1}(3)$ and $U_{-1}(3)$ stable rotators with different $\hat{Q}\cdot\hat{Q}$ interaction signs. By analogy, the $Q_i(6)$ unstable rotator is appearing from the transition between $U(5)$ vibrator and $O_1(6)$ unstable rotator.

Table 4

Electromagnetic transition operators

$$T^{E0} = \sum_{j=0}^{3} e^0_j [b^{j+} b^j]^0$$

$$T^{M1} = \sum_{j=1}^{3} m^1_j [b^{j+} b^j]^1$$

$$T^{E1} = \sum_{j=1}^{3} e^1_j [b^{j+} b^{j-1} + b^{j-1+} b^j]^1$$

$$T^{M2} = \sum_{j=2}^{3} m^2_j [b^{j+} b^{j-1} + b^{j-1+} b^j]^2$$

$$T^{E2} = \sum_{j=1}^{3} e^2_j [b^{j+} b^j]^2 + \sum_{j=2}^{3} e^2_{j,j-2} [b^{j+} b^{j-2} + b^{j-2+} b^j]^2$$

$$T^{M3} = \sum_{j=2}^{3} m^3_j [b^{j+} b^j]^3 + m^3_{3,1} [b^{3+} b^1 + b^{1+} b^3]^3$$

$$T^{E3} = \sum_{j=2}^{3} e^3_{j,j-1} [b^{j+} b^{j-1} + b^{j-1+} b^j]^3 + e^3_{3,0} [b^{3+} b^0 + b^{0+} b^3]^3$$

$$T^{M4} = m^4_{3,2} [b^{3+} b^2 + b^{2+} b^3]^4$$

$$T^{E4} = \sum_{j=2}^{3} e^4_j [b^{j+} b^j]^4 + e^4_{3,1} [b^{3+} b^1 + b^{1+} b^3]^4$$

The electromagnetic transition operators T^{XL} with their parities: time parity + for EL and − for ML, space parity + for EL = even, ML = odd, and − for EL = odd, ML = even, can be written in the lowest order directly for the spdf case as in table 4. One observes that at variance with the sd model /5/, here T^{E1}, T^{M2}, T^{E3}, T^{M4} are nonzero and will give corresponding transitions. For T^{E1} they must be described better than with f bosons only. Also T^{M1} can be different from the angular momentum in lowest order, and thus might give not only $I \to I$ g factors, but also $I \to I \pm 1$ transitions.

5. Spectra

To diagonalize algebraically the p f H (5) or the corresponding s p d f H, in some p f limit case (4) or in a corresponding s p d f limit, one has to obtain the eigenvalues n_r, ν_r, ω_r of the Casimir operators $\hat{n}_r$, $\hat{\nu}_r$, $\hat{\omega}_r$ of that case group chain. They are known /36/:

$$n_r = \sum_{k=1}^{r} h_k$$

$$\nu_r = \sum_{k=1}^{r} h_k(h_k + r + 1 - 2k) \qquad (7)$$

$$\omega_r = \sum_{k=1}^{[r/2]} \bar{h}_k(\bar{h}_k + r - 2k)$$

if the eigenstate quantum numbers, i.e. the irreducible representations (IR): $[h_1 \geqslant h_2 \geqslant \ldots h_r \geqslant 0]$ of all U(r) and $(\bar{h}_1 \geqslant \bar{h}_2 \geqslant \ldots \bar{h}_{[r/2]} \geqslant 0)$ of all O(r) in the group chain, are known ([r/2] is the integer part of r/2). Pay attention that at variance with the s d case where only completely symmetric representations, except for U(3), play role, here this might be not the case for other groups too. It will make the spectrum richer.

However group-theoretical methods /27-31/ will be necessary for finding the IR. As an example we mention the IR $[h_1\, h_2\, h_3]$ or $(\lambda = h_1 - h_2,\ \mu = h_2 - h_3)$ of $U^{1,3}(3)$ embedded in the IR [h] of $U^{1,3}(10)$, the last one in our case being completely symmetric. Several cases have been tabulated on p. 137 of ref. /37/, but can also be reproduced via the plethysm $[3] \otimes [h]$ tables, e.g. on p. 287-91 of ref. /30/. One can see that at variance with the s d case, in the p f case odd K bands and negative parity levels will appear.

To describe the transition between the limit cases, one has to diagonalize the hamiltonian numerically, possibly in a group chain basis, as in the s d case. Let us notice that for our purpose a concise version of the hamiltonian with a small number of parameters might be sufficient. E.g.:

$$H = \sum_j \varepsilon^j \hat{n}^j_{2j+1} + \delta(X^2 \cdot X^2), \quad X^2 = \alpha N^2 + \beta L^2 + \gamma Q^2 \qquad (8)$$
$$(\alpha + \beta + \gamma = 1)$$

(where $(X^j)^2 = (X^j \cdot X^j) = (-)^j \sqrt{2j+1}\,[X^j\, X^j]^0$) might cover all the transitions between the limit cases. For the s d case a similar

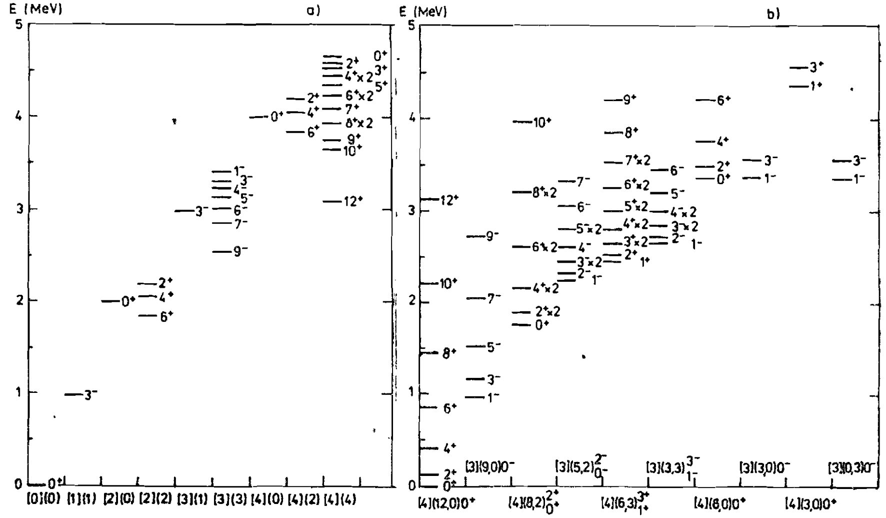

Figure 1. Schematic spectra in the p f boson model (5) after addition of the harmonic s d term $\sum_{j=0,2} \varepsilon^j \hat{n}^j_{2j+1}$ (ε^0 arbitrary, $\varepsilon^2 = \infty$) with $N = \sum_{j=0}^{3} n^j_{2j+1} = 4$ of: a) anharmonic octupole vibrator, hamiltonian (5) with $\varepsilon^1 = \infty$, $\varepsilon^3 = \varepsilon^0 + 1$ MeV, $\beta^3_7 = 18$ keV, $\beta^{1,3}_3 = -10$ keV, all other parameters equal to 0; below each multiplet: [h] of $U^{1,3}(10)$ and $U^3(7)$, $(\bar{h})$ of $O^3(7)$; b) axial stable rotator, hamiltonian (5) with $\varepsilon^1 = \varepsilon^3 = \varepsilon^0 + 1$ MeV, $\alpha^{1,3}_3 = -40$ keV multiplied by $\bar{\nu}^{1,3}_{s3}$ the SU(3) instead of $\hat{\nu}^{1,3}_3$ the U(3) Casimir operator, $\beta^{1,3}_3 = -\frac{1}{2}\alpha^{1,3}_3 = 20$ keV, corresponding to (8) with the same $\varepsilon^1 = \varepsilon^3$, $\gamma = 1$, $\delta = \alpha^{1,3}_3 = -40$ keV, all other parameters equal to 0; below each band: [h] of $U^{1,3}(10)$, $(\lambda = h_1 - h_2,\ \mu = h_2 - h_3)$ of $U^{1,3}(3)$, K^π; limited to (λ, μ) giving pairs of opposite parity bands, and to [h] = [4], [3].

hamiltonian, with the N^2, L^2, Q^2 operator expressions from table 1 and $\alpha = 0$, has been shown to work well /38/; for the p f case one has to take for (8) the operator expressions from table 3; for the complete s p d f case, the sum of their expressions from table 1 and from table 3.

In conclusion we show in figure 1 the schematic spectra of the p f model (5) for the separate vibrator: the first row of (4), and for the stable rotator: the last row of (4) limit cases. The s p d f model with nonschematic parameters will give of course modified and richer spectra due to the interactions between the s, d and p, f degrees of freedom, both for vibrations and for rotations. One can see that in principle one can reproduce the 0^+, 1^-, 2^+, 3^-, . . . level bands of the Ra - Th doubly even isotopes, discussed from the point of view of octupole deformation in many recent publications, e.g. /39,40/. The inclusion of sd - pf interaction may lead e.g. to Coriolis type terms (in fact, such a term appears also from the p - f interaction only due to (4): its first row) and thus, in particular, to a more realistic high spin behaviour. $U^{0,1,2,3}(16)$ configuration mixing for a similar purpose may also be performed in the way suggested in ref. /10/.

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Наджаков Е.Г., Михайлов И.Н. E4-86-510

Модель многих взаимодействующих бозонов

Введены обобщенные ядерные коллективные координаты. Через них построены бозоны разных мультиполярностей. Указана порождаемая ими U(r) группа. Предложен общий метод поиска ее подгрупп. Он проиллюстрирован на примере s p d f ММВБ с мультиполярностями бозонов 0, 1, 2, 3. Изложена процедура написания бозонного гамильтониана и операторов переходов через генераторы группы. Дана простая версия гамильтониана с несколькими членами. Обсуждены возможности модели улучшить обычную МВБ, описывая отсутствующие состояния и плохо воспроизводимые вероятности переходов. Отмечена возможность ее применения к недавно наблюденным низколежащим состояниям отрицательной четности в четно-четных изотопах Ra области с вероятной октупольной деформацией.

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Nadjakov E.G., Mikhailov I.N. E4-86-510

Interacting Multi-Boson Model

Generalized nuclear collective coordinates are introduced. Bosons of different multipolarities are constructed by using them. The created by them U(r) group is indicated. A general method of searching its subgroups is proposed. It is illustrated by the example of an s p d f IMBM of boson multipolarities 0, 1, 2, 3. The procedure to write the boson hamiltonian and transition operators in terms of the group generators is sketched. A concise version of a several term hamiltonian is given. The model possibilities to improve the usual IBM by describing lacking states and poorly reproduced transition rates are discussed. Its possible application to the recently observed low lying negative parity states in doubly even isotopes of the Ra region with probable octupole deformation is pointed out.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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