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SQUEEZING IN COLLECTIVE RESONANCE FLUORESCENCE

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Squeezing - a new nonclassical effect in radiation theory - has recently become the subject of extensive theoretical /1-16/ and experimental works /21/. Squeezing is characterized by a field state in which the variance of two noncommuting observables is less than one half of the absolute value of their commutator. The reduced quantum fluctuations in resonance fluorescence have been investigated in /9-15/. In the work /15/ the collective effects on squeezing in resonance fluorescence have been discussed. In these works the received squeezing was small.

In this paper we discuss the reduced quantum fluctuation of the mixture of two spectrum sidebands in collective resonance fluorescence. A large squeezing of the mixture of sidebands is presented while it is absent for the whole field of resonance fluorescence. The N two-level atoms, concentrated in a region small compared to the wavelength of all the relevant radiation modes, interact with a monochromatic driving field with requency ω and with an emitted field (Fig.1). In treating the external field classically and using the Born and Markov approximation with respect to the coupling of the system with the vacuum field, one can obtain a master equation for the reduced density matrix ρ for the system alone in the form /17/

$$\frac{\partial \rho}{\partial t} = -i \left[\frac{\delta}{2} (J_{22} - J_{11}) + G (J_{21} + J_{12}), \rho \right] - \gamma_{21} (J_{21} J_{12} \rho - J_{12} \rho J_{21} + H.c.) = L\rho, \tag{1}$$

where $2\gamma_{21}$ is the radiative spontaneous transition probability per unit time for a single atom to change from the level $|2\rangle$ to $|1\rangle$; $\delta = \omega_{21} - \omega$ is the frequnecy detuning of a resonance; $G = d_{21} E_0$ is the matrix element of the driving field and atom interaction. The operators $J_{ij} = \sum_{k=1}^{N} |i\rangle_k \langle j|_k$; $(i,j = 1,2)$ are the collective angular momenta of the atoms. They satisfy the commutation relation

$$[J_{ij}, J_{i'j'}] = J_{ij'} \delta_{ji'} - J_{i'j} \delta_{ij'}$$

The atomic coherence phenomena an be illustrated with greater lucidity by introducing the Schwinger representation for angular momentum /22/

$$J_{ij} = C_i^+ C_j \qquad (i, j = 1, 2).$$

where C_i obey the boson commutation relation

$$[C_i, C_j^+] = \delta_{ij}.$$

Further, we consider only the case of intense driving field or of large detuning δ so that

$$\Omega = (\tfrac{1}{4}\delta^2 + G^2)^{1/4} >> N\gamma_{21}. \qquad (2)$$

After performing the canonical transformation

$$C_1 = \cos\varphi\, Q_1 + \sin\varphi\, Q_2,$$
$$C_2 = -\sin\varphi\, Q_1 + \cos\varphi\, Q_2,$$

where $$tg\, 2\varphi = \frac{2G}{\delta}, \qquad (3)$$

one can find that the Liouville operator L appearing in equation (1) splits into two components L_0 and L_1. The component L_0 is slowly varying in time whereas L_1 contains rapidly oscillating terms at frequencies 2Ω and 4Ω. For the case of in tense driving field or large detuning δ so that the condition (2) is satisfied, it is reasonable to make the secular approximation, i.e., to retain only the slowly varying part /17,18/. A correction to the results obtained in this fashion will be of an order of $(\gamma_{21} N/\Omega)^2$.

Making the secular approximation, one can find the stationary solution of the master equation

$$\tilde{\rho} = U\rho U^+ = Z^{-1} \sum_{N_1=0}^{N} X^{N_1} | N_1 \rangle\langle N_1 |. \qquad (4)$$

where U is a unitary operator representing the canonical transformation (3)

$$X = ctg^4\varphi \quad ; \quad Z = \frac{X^{N+1} - 1}{X - 1},$$

$|N_1\rangle$ is an eigenstate of the operators R_{11}, $\hat{N} = R_{11} + R_{22}$ here

$$R_{ij} = Q_i^+ Q_j \qquad (i, j = 1, 2).$$

The operators Q_i satisfy the boson commutation relation

$$[Q_i, Q_j^+] = \delta_{ij}.$$

so that

$$[R_{ij}, R_{i'j'}] = R_{ij'}\delta_{i'j} - R_{i'j}\delta_{ij'}. \qquad (5)$$

In the case of resonance $X = 1$, the solution (4) reduces to the solution of Agarwal et al./17/.

By using solution (4) one can calculate the statistical moments $\langle R^n \rangle_s$ where $\langle B \rangle_s$ indicates the expectation value of an operator B in steady-state (4). In particular, we find

$$\langle R_{11} \rangle_s = \frac{N X^{N+2} - (N+1) X^{N+1} + X}{(X-1)(X^{N+1} - 1)}, \qquad (6)$$

$$\langle R_{11}^2 \rangle_s = \frac{N^2 X^{N+3} - (2N^2 + 2N - 1) X^{N+2} + (N+1)^2 X^{N+1} - X^2 - X}{(X-1)^2 (X^{N+1} - 1)}. \qquad (7)$$

The variance of the fluctuations in the fluorescent field may be derived by using the following relation between the radiation field and the atomic operator in the far-field limit /12-15/:

$$E^{(+)}(\vec{x}, t) = E^{(+)}_{free}(\vec{x}, t) + \psi(\vec{x})\, J_{12}(t - \tfrac{r}{c})\, e^{-i\omega(t - r/c)}, \qquad (8)$$

where

$$\psi(\vec{x}) = \frac{\omega_{21}}{2\pi\varepsilon_0 c^2} \cdot \frac{\vec{x} \times (\vec{d} \times \vec{x})}{r^3},$$

$\vec{d}$ and $\vec{x}$ are the transition dipole moment and the observation point vector, respectively; $r = |\vec{x}|$; $E^{(+)}$ is the positive-frequency part of the radiation field.

We shall consider the variance of fluctuations in the in-phase (E_1) and out-of-phase components (E_2) of the scattered field amplitude

$$E_1 = \frac{1}{2}\left(E^{(+)} + E^{(-)}\right) \quad \text{and} \quad E_2 = \frac{-i}{2}\left(E^{(+)} - E^{(-)}\right). \tag{9}$$

We speak of squeezing in the radiation field if the normally ordered variance of the electric components E_1 or E_2 is less than zero /12-15/.

$$\langle : (\Delta E_1)^2 : \rangle = \langle : E_1^2 : \rangle - \langle E_1 \rangle^2 < 0$$

or

$$\langle : (\Delta E_2)^2 : \rangle = \langle : E_2^2 : \rangle - \langle E_2 \rangle^2 < 0. \tag{10}$$

From the canonical transformation (3), one can find

$$J_{21} = R_{21}\cos^2\varphi - R_{12}\sin^2\varphi + (R_{22} - R_{11})\sin\varphi\cos\varphi$$

$$J_{12} = R_{12}\cos^2\varphi - R_{21}\sin^2\varphi + (R_{22} - R_{11})\sin\varphi\cos\varphi. \tag{11}$$

Due to Apanasevich and Kilin /19/ one can consider the operators $R_{21}(t)$, $R_{12}(t)$ and $R_{22}(t) - R_{11}(t)$ as the sources of spectrum components at frequencies $\omega + 2\Omega$, $\omega - 2\Omega$ and ω. The reduced quantum fluctuations for the whole scattered field are investigated in the work /9-14/. In this paper we consider only the reductions of quantum fluctuations of the mixture of sidebands. After substituting the operator J_{12} in the relation (8) by the operator

$$\tilde{J}_{12} = R_{12}\cos^2\varphi - R_{21}\sin^2\varphi \tag{12}$$

and using the steady-state solution (4), one can find

$$\langle : (\Delta E_1)^2 : \rangle = \frac{1}{2}\langle R_{12}R_{21}\rangle_s (\sin^4\varphi - \sin^2\varphi\cos^2\varphi)$$

$$+ \frac{1}{2}\langle R_{21}R_{12}\rangle_s (\cos^4\varphi - \sin^2\varphi\cos^2\varphi), \tag{13}$$

$$\langle : (\Delta E_2)^2 : \rangle = \frac{1}{2}\langle R_{12}R_{21}\rangle_s \sin^2\varphi$$

$$+ \frac{1}{2}\langle R_{21}R_{12}\rangle_s \cos^2\varphi, \tag{14}$$

where

$$\langle R_{21}R_{12}\rangle_s = -\langle R_{11}^2\rangle_s + (N-1)\langle R_{11}\rangle_s + N \tag{15}$$

$$\langle R_{12}R_{21}\rangle_s = -\langle R_{11}^2\rangle_s + (N+1)\langle R_{11}\rangle_s. \tag{16}$$

The values $\langle R_{11}\rangle_s$ and $\langle R_{11}^2\rangle_s$ can be found in relations (6-7).

From relations (13-14) one can see that the squeezing occurs in the in-phase component E_1. The detailed behaviour of the normally ordered variance of the electric component E_1 against the parameter X is plotted in Fig. 2. As is shown in Fig. 2, the substantial squeezing occurs for the mixture of two sidebands while the optimum normally ordered variance of the electric components of the whole scattered field can reach the value $-\frac{1}{32}|\psi|^2$ /10/ and for our case of intense driving field or large detuning δ the squeezing of the whole scattered field is absent. In conclusion we note that the factor of squeezing for atomic operators

$$\tilde{J}_x = \frac{1}{2}(\tilde{J}_{12} + \tilde{J}_{21})$$

$$\tilde{J}_y = \frac{i}{2}(\tilde{J}_{12} - \tilde{J}_{21})$$

can be defined as /15/

$$F_x = \frac{2\langle (\Delta \tilde{J}_x)^2\rangle_s}{|\langle [\tilde{J}_x, \tilde{J}_y]\rangle_s|}. \tag{17}$$

$$F_y = \frac{2\langle (\Delta \tilde{J}_y)^2\rangle_s}{|\langle [\tilde{J}_x, \tilde{J}_y]\rangle_s|}. \tag{18}$$

By using the relation (12) one can show that the factor of squeezing can reach the value $F_x = 0.5$ for any number of atoms N.

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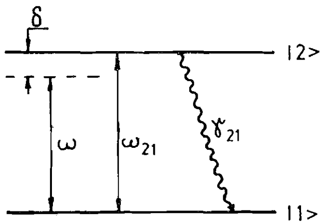

Fig.I. Schematic representation of two-level system interacting with monochromatic incident field.

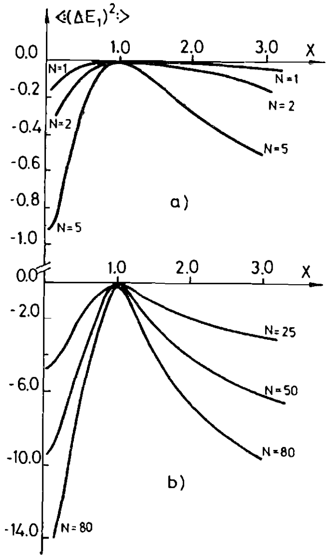

Fig. 2. Normally ordered variance $<:(\Delta E_1)^2:>$ as function of X.

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Боголюбов Н.Н./мл/, Шумовский А.С., Чан Куанг   E4-86-387
Сжатие света в коллективной резонансной флуоресценции

Обнаружено значительное понижение квантовых флуктуаций или сжатие света в смеси крайних спектров коллективной резонансной флуоресценции в случае сильного внешнего поля или большой частотной расстройки, когда сжатие света не существует для полного поля флуоресценции.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Bogolubov N.N.,Jr., Shumovsky A.S., Tran Guang   E4-86-387
Squeezing in Collective Resonance Fluorescence

The substantial reduced quantum fluctuations or squeezing of the mixture of two spectrum sidebands of collective resonance fluorescence are observed for the case of intense driving field or of large frequency detuning while they are absent in the whole field of resonance fluorescence.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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