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**STATISTICAL PROPERTIES
OF PHOTONS IN COLLECTIVE RESONANCE
FLUORESCENCE**

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I. Introduction

In the last few years the interest in the statistical properties of resonance fluorescence has been increased. The photon antibunching effect, characterized by a nonclassical state of the field in which the variance of the number of photons is less than the mean number of photons has been observed in experiments by Kimble et al. (1977), Dagenais and Mandel (1978), Short and Mandel (1983). The recent publications deal with collective effects in resonance fluorescence (Marducci et al. 1978, Compagno and Persico 1982, Ficek et al. 1981, 1984, London 1980, Swain 1980, Steudel 1978, Wiegand 1983).

In the theoretical works by Cohen-Tannoudji and Reynaud (1979), Apanasevich and Kilin (1979), and the experimental work by Aspect et al. (1980) the statistical properties of spectrum components of resonance fluorescence for the one-atom case had been investigated. The correlation between sidebands and no correlation between the central component and the sidebands are observed.

In the present paper the photon statistics of spectrum components and the cross-correlation between them are investigated for the collective case of N atoms. Besides the correlation between sidebands, in the collective case the anticorrelation between a side components and the central one is observed.

II. Master Equation

The N two-level atoms (Fig.1) concentrated in a region small compared to the wavelength of all the relevant radiation modes (model Dicke 1954) interact with a monochromatic driving field with a frequency ω and with an emitted field.

In treating the external fields classically and using the Born and Markov approximation with respect to the coupling of the system with the vacuum field and atomic reservoir, one can obtain a master

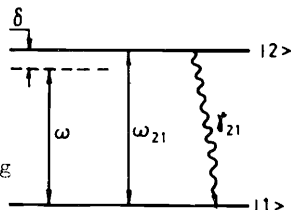


Fig. 1. Two-level system of atoms interacting with the monochromatic applied fields.

equation for the reduced density matrix for the system alone in the form

$$\frac{\partial \rho}{\partial t} = -i \left[\frac{d}{2} (\mathcal{J}_{22} - \mathcal{J}_{11}) + G (\mathcal{J}_{21} + \mathcal{J}_{12}) + H.C., \rho \right] - \gamma_{21} (\mathcal{J}_{21} \rho \mathcal{J}_{12} - \mathcal{J}_{12} \rho \mathcal{J}_{21} + H.C.) = \mathcal{L} \rho \quad (1)$$

where $2\gamma_{21}$ is radiative spontaneous transition probabilities per unit time for a single atom to change from excited level $|2\rangle$ to ground state $|1\rangle$; $\delta = \omega_{21} - \omega$ is the frequency detuning of resonance (the system of $\hbar=1$) and $\mathcal{J}_{ij} = \sum_{k,k'} |i\rangle_k \langle j|_{k'}$, ($i, j = 1, 2$) are the collective angular momenta of the atoms. They obey the commutation relation

$$[\mathcal{J}_{ij}, \mathcal{J}_{i'j'}] = \mathcal{J}_{ij} \delta_{j'i} - \mathcal{J}_{i'j'} \delta_{ij'}$$

The atomic coherence phenomena can be illustrated with greater lucidity by introducing the Schwinger representation for angular momentum (Schwinger 1965, Wall 1971, Bogolubov et al. 1985).

$$\mathcal{J}_{ij} = c_i^+ c_j, \quad (i, j = 1, 2),$$

where c_i obey boson commutation relation

$$[c_i, c_j^+] = \delta_{ij}$$

Further, we investigate only the case of an intense external field or much detuning δ so that

$$\Omega = \left(\frac{1}{4} \delta^2 + G^2 \right)^{1/2} \gg N \gamma_{21} \quad (2)$$

After performing the canonical transformation

$$\begin{aligned} c_1 &= Q_1 \cos \varphi + Q_2 \sin \varphi, \\ c_2 &= -Q_1 \sin \varphi + Q_2 \cos \varphi, \end{aligned} \quad (3)$$

where

$$\tan 2\varphi = \frac{2G}{\delta}$$

one can find that the Liouville operator \mathcal{L} appearing in equation (1) splits into two components \mathcal{L}_0 and \mathcal{L}_2 . The component \mathcal{L}_0 is slowly varying in the time whereas \mathcal{L}_2 contains rapidly oscillating terms at frequencies 2Ω and 4Ω . For the case, when relation (2) is fulfilled, one can make the secular approximation, i.e., to retain only the slowly varying part (Agarwal 1978, Bogolubov et al. 1985). Correction to the results obtained in this fashion will be of an order of $(\gamma_{21} N / \Omega)^2$.

Making the secular approximation, one can find the stationary solution of the master equation (1) in the form

$$\tilde{\rho} = \mathcal{Z}^{-1} \sum_{N_i=0}^N X^{N_i} |N_i\rangle \langle N_i| \quad (4)$$

where $\tilde{\rho} = U \rho U^{-1}$, here U is the unitary operator representing the canonical transformation (3)

$$X = \cot^2 \varphi, \quad \mathcal{Z} = \frac{X^{N+1} - 1}{X - 1}$$

The state $|N_i\rangle$ is an eigenstate of the operators R_{ii} and $\hat{N} = R_{11} + R_{22}$ here $R_{ij} = Q_i^+ Q_j$ ($i, j = 1, 2$). The operators Q_i satisfy the boson commutation relation

$$[Q_i, Q_j^+] = \delta_{ij} \quad (5)$$

so

$$[R_{ij}, R_{i'j'}] = R_{ij} \delta_{j'i} - R_{i'j'} \delta_{ij'} \quad (6)$$

In the case of resonance, i.e. when $X=1$, the solution (4) reduces to the solution by Agarwal (1978).

By using equation (4), the characteristic function can be defined as done by Louisell (1964)

$$\chi_{R_{ii}}(\xi) = \langle e^{i\xi R_{ii}} \rangle_s = \mathcal{Z}^{-1} \frac{Y^{N+1} - 1}{Y - 1},$$

where

$$Y = \mathcal{Z} e^{i\xi}$$

Here $\langle A \rangle_S$ indicates the expectation value of an operator A in the steady state (4).

Once the characteristic function is known, it is easy to calculate the statistical moments

$$\langle R_{11}^n \rangle_S = \frac{\partial^n}{\partial (i\mathcal{F})^n} \chi_{R_{11}}(\mathcal{F}) \Big|_{i\mathcal{F}=0} \quad (7)$$

III. Photon statistics of spectrum components

In this section we discuss the influence of collective effects and detuning δ^n on the photon statistics of the components of the steady-state spectrum. One can find from canonical transformation (3) that

$$\begin{aligned} \hat{M}_1 &= (R_{22} - R_{11}) \sin\varphi \cos\varphi + R_{21} \cos^2\varphi - R_{12} \sin^2\varphi, \\ \hat{M}_2 &= (R_{22} - R_{11}) \sin\varphi \cos\varphi + R_{12} \cos^2\varphi - R_{21} \sin^2\varphi. \end{aligned} \quad (8)$$

Similarly to Apanasevich and Kilin (1979), we can consider the operators $R_{22}(t) - R_{11}(t)$, $R_{21}(t)$ and $R_{12}(t)$ as the sources of spectrum components at frequencies ω , $\omega + 2\Omega$ and $\omega - 2\Omega$. For simplicity we call the spectrum component at frequency $\omega + m\Omega$ ($m=0, \pm 1$) S_m and the steady-state normalized intensity correlation function of spectrum component S_m , $g_{m,m}^{(2)}$. By using the stationary solution (4) and commutation relations (5)-(6), one can find correlation functions $g_{m,m}^{(2)}$ in the form

$$g_{0,0}^{(2)} = \langle (R_{22} - R_{11})^4 \rangle_S / \langle (R_{22} - R_{11})^2 \rangle_S^2, \quad (9)$$

$$g_{-1,-1}^{(2)} = \langle R_{12} R_{12} R_{21} R_{21} \rangle_S / \langle R_{12} R_{21} \rangle_S^2 = g_{+1,+1}^{(2)}, \quad (10)$$

where

$$\langle (R_{22} - R_{11})^2 \rangle_S = 4 \langle R_{11}^2 \rangle_S - 4N \langle R_{11} \rangle_S + N^2, \quad (11)$$

$$\langle R_{12} R_{21} \rangle_S = -\langle R_{11}^2 \rangle_S + (N+1) \langle R_{11} \rangle_S, \quad (12)$$

$$\begin{aligned} \langle (R_{22} - R_{11})^4 \rangle_S &= 16 \langle R_{11}^4 \rangle_S - 32N \langle R_{11}^3 \rangle_S + \\ &+ 24N^2 \langle R_{11}^2 \rangle_S - 8N^3 \langle R_{11} \rangle_S + N^4, \end{aligned} \quad (13)$$

$$\begin{aligned} \langle R_{12} R_{12} R_{21} R_{21} \rangle_S &= \langle R_{11}^4 \rangle_S - 2(N+2) \langle R_{11}^3 \rangle_S + \\ &+ (N^2 + 5N + 5) \langle R_{11}^2 \rangle_S - (N^2 + 3N + 2) \langle R_{11} \rangle_S. \end{aligned} \quad (14)$$

Here the values $\langle R_{11}^n \rangle_S$ can be found in equation (7). The behaviour of the functions $g_{m,m}^{(2)}$ against the parameter X is plotted in figs. 2-a and 2-b. For the one-atom case the central spectrum component S_0 has poissonian statistics ($g_{0,0}^{(2)} = 1$) while the sidebands have subpoissonian statistics ($g_{\pm 1, \pm 1}^{(2)} < 1$). These results coincide with the one by Aspect et al. (1980), Cohen-Tannoudji et al. (1979), Apanasevich and Kilin (1979).

For the collective case the central component S_0 has superpoissonian statistics ($g_{0,0}^{(2)} > 1$). The collective effects reduce the antibunching of sidebands (Fig. 2b). For the case of several atoms, the sidebands have subpoissonian statistics only for a suitable region of parameter X and for the case of large N ($N \gg 5$) they have superpoissonian statistics for all values of parameter X .

IV. Cross-Correlation between Spectrum Components

More interesting is the question of cross-correlation between spectrum components. The magnitude of the cross correlation between the spectrum components S_m and S_n ($m, n=0, \pm 1$) can be characterized by the Steady-state cross-correlation function $C_{m,n}^{(2)}$. By using solution (4) and commutation relation (5)-(6), one can find

$$C_{0,-1}^{(2)} = \langle \Delta_3 R_{12} R_{21} \Delta_3 \rangle_S / \langle \Delta_3^2 \rangle_S \langle R_{12} R_{21} \rangle_S = C_{-1,0}^{(2)}, \quad (15)$$

$$C_{0,+1}^{(2)} = \langle \Delta_3 R_{21} R_{12} \Delta_3 \rangle_S / \langle \Delta_3^2 \rangle_S \langle R_{21} R_{12} \rangle_S = C_{-1,0}^{(2)}, \quad (16)$$

$$C_{-1,-1}^{(2)} = \langle R_{21} R_{12} R_{21} R_{12} \rangle_S / \langle R_{12} R_{21} \rangle_S \langle R_{21} R_{12} \rangle_S, \quad (17)$$

$$C_{-1,+1}^{(2)} = \langle R_{12} R_{21} R_{12} R_{21} \rangle_S / \langle R_{12} R_{21} \rangle_S \langle R_{21} R_{12} \rangle_S, \quad (18)$$

where $\Delta_3 = R_{22} - R_{11}$,

$$\langle \Delta_3 R_{12} R_{21} \Delta_3 \rangle_S = -4 \langle R_{11}^4 \rangle_S + (8N+4) \langle R_{11}^3 \rangle_S - (5N^2+4N) \langle R_{11}^2 \rangle_S + (N^3+N^2) \langle R_{11} \rangle_S,$$

$$\langle \Delta_3 R_{21} R_{12} \Delta_3 \rangle_S = -4 \langle R_{11}^4 \rangle_S + (8N-4) \langle R_{11}^3 \rangle_S - (5N^2-8N) \langle R_{11}^2 \rangle_S + (N^3-5N^2) \langle R_{11} \rangle_S + N^3,$$

$$\langle R_{21} R_{12} R_{21} R_{12} \rangle_S = \langle R_{11}^4 \rangle_S - (2N-2) \langle R_{11}^3 \rangle_S + (N^2-4N+1) \langle R_{11}^2 \rangle_S + (2N^2-2N) \langle R_{11} \rangle_S + N^2,$$

$$\langle R_{12} R_{21} R_{12} R_{21} \rangle_S = \langle R_{11}^4 \rangle_S - 2(N+1) \langle R_{11}^3 \rangle_S + (N+1)^2 \langle R_{11}^2 \rangle_S,$$

$$\langle R_{21} R_{12} \rangle_S = - \langle R_{11}^2 \rangle_S + (N-1) \langle R_{11} \rangle_S + N.$$

The values $\langle R_{12} R_{21} \rangle_S$ and $\langle \Delta_3^2 \rangle_S$ can be found in eqs. (11)-(12).

We speak about anticorrelation (or correlation) between the spectrum components S_m and S_n when $C_{m,n}^{(2)}$ is less (or more) than unity. The behaviour of the functions $C_{m,n}^{(2)}$ against the parameter X is shown in figs. 3-a,b,c,d). From these figures one can see that:

(i) For the one-atom case the correlation between the sidebands comes into existence while there are no correlations between side components and the central ones (Apanasevich et al. 1979, Aspect et al. 1980).

(ii) For the collective case the correlation between sidebands reduces (Figs. 3-a,b) but as in the case of one atom the sidebands of the resonance scattered light have a tendency to be emitted in pairs. For the case of $X < 1$ (i.e. the detuning $\delta < 0$) we have $C_{1,-1}^{(2)} > C_{-1,1}^{(2)}$; it means that the photon with frequency $\omega + 2\Omega$ has a tendency to be emitted before the photon with frequency $\omega - 2\Omega$. (Aspect et al. 1980). For the case of $X > 1$ the photons of sidebands have a tendency to be emitted in a reverse time order.

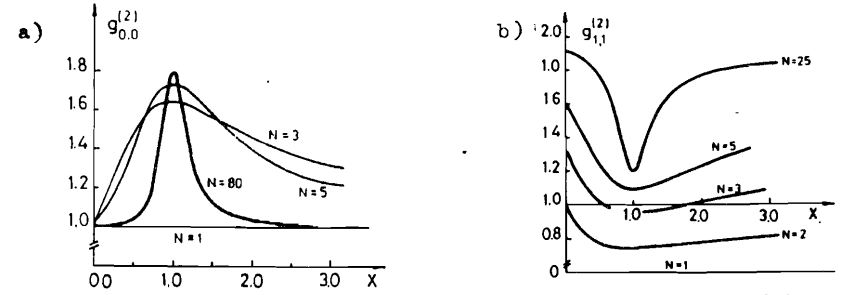


Fig. 2. Normalized intensity correlation functions $g_{m,m}^{(2)}$ graphed against the parameter X .

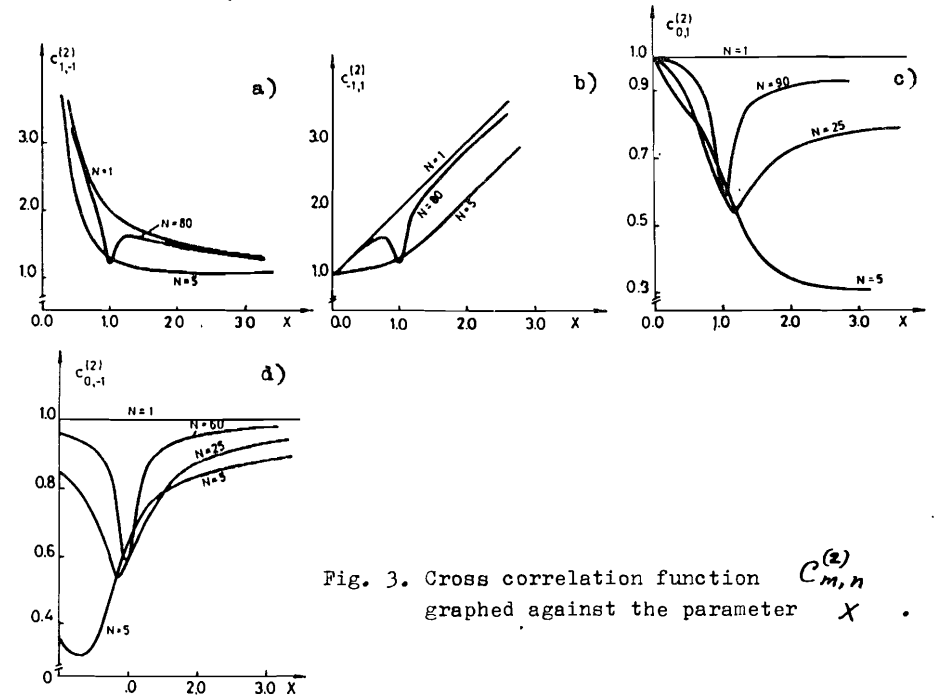


Fig. 3. Cross correlation function $C_{m,n}^{(2)}$ graphed against the parameter X .

(iii) For the collective case the anticorrelation between the central component S_0 and sidebands S_{\pm} occurs (Fig.3-c,d). Physically it means that in the collective case and especially in the region of resonance the atoms have a tendency to emit simultaneously only the photons of the central component or the pairs of sidebands.

In conclusion, we note that since the anticorrelation between spectrum components occurs only for the cooperative case, the measurable cross-correlation functions $C_{0,\pm 1}^{(2)}$, $C_{\pm 1,0}^{(2)}$ provide a new tool for the study of collective effects.

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Статистика фотонов в коллективной резонансной флуоресценции

Обсуждены статистика фотонов спектральных компонент и кросс-корреляция между ними. Для коллективного случая обнаружена антикорреляция между центральными и крайними спектральными компонентами.

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Statistical Properties of Photons in Collective Resonance Fluorescence

The photon statistics of spectrum components and the cross-correlation between spectrum components are discussed. For the collective case the anticorrelation between the central component and sidebands is observed.

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