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PHOTON STATISTICS IN COLLECTIVE DOUBLE OPTICAL RESONANT PROCESS

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Recently there has been much interest in studies of cooperative effects in superradiance[1-4], resonance fluorescence[5-9], Raman scattering[10-12] double optical resonance[13], etc.

In the present paper we discuss the photon statistics of spectrum components of scattered light in a collective double optical resonant process. The correlation and anticorrelation between spectrum components are investigated.

The three-level atoms (Fig. 1), concentrated in a region small compared to the wavelength of all the relevant radiation modes, interact with two resonant driving fields and an emitted field[13]. In the boson representation of atoms[13,14] when each atomic level is compared with a boson variable, the master equation in the rotating frame with Markovian electric dipole and rotating wave approximation is[15]

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} = &- i \Omega \left[\left(\cos\alpha\, J_{12} + \sin\alpha\, J_{23} + H.c. \right), \rho \right] \\
&- \gamma_{21} \left(J_{21} J_{12} \rho - 2 J_{12} \rho J_{21} + \rho J_{21} J_{12} \right) \\
&- \gamma_{32} \left(J_{32} J_{23} \rho - 2 J_{23} \rho J_{32} + \rho J_{32} J_{23} \right),
\end{aligned} \tag{1}
$$

where ρ is the atomic density matrix, $2\gamma_{21}$ and $2\gamma_{32}$ are radiative spontaneous transition probabilities per unit time for a single atom to change from the level $|2\rangle$ to $|1\rangle$ and from $|3\rangle$ to $|2\rangle$ respectively; $\Omega = (\Omega_1^2 + \Omega_2^2)^{1/2}$ and $tg\alpha = \frac{\Omega_2}{\Omega_1}$, where Ω_1 and Ω_2 are the Rabi frequencies for the atomic transitions from the level $|2\rangle$ to $|1\rangle$ and from $|3\rangle$ to $|2\rangle$ respectively; $J_{ij} = C_i^+ C_j$ ($i, j = 1, 2, 3$), where C_i and C_i^+ are the annihilation and creation boson operators for the atoms populated on the level $|i\rangle$.

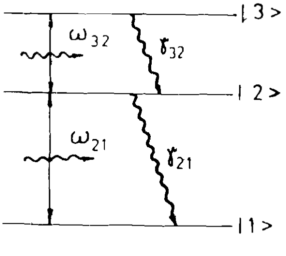

Fig. 1. Three-level system of atoms interacting with the two monochromatic applied fields.

After performing the canonical transformation

$$
\begin{aligned}
C_3 &= -\frac{1}{\sqrt{2}} \sin\alpha \, Q_1 + \cos\alpha \, Q_2 + \frac{1}{\sqrt{2}} \sin\alpha \, Q_3 \\
C_2 &= \frac{1}{\sqrt{2}} Q_1 + \frac{1}{\sqrt{2}} Q_3 \qquad (2) \\
C_1 &= -\frac{1}{\sqrt{2}} \cos\alpha \, Q_1 - \sin\alpha \, Q_2 + \frac{1}{\sqrt{2}} \cos\alpha \, Q_3
\end{aligned}
$$

and using the secular approximation[8,13] for the case of intense external field $\Omega \gg N\gamma_{21}, N\gamma_{32}$, one can find the stationary solution of the master equation (1) in the form

$$\tilde{\rho} = Z^{-1} \sum_{R=0}^{N} X^R \sum_{M=0}^{R} |M,R\rangle\langle R,M|, \qquad (3)$$

where $\tilde{\rho} = U \rho U^+$, here U is the unitary operator representing the canonical transformation (2),

$$X = \frac{\gamma_{32}\cos^2\alpha}{\gamma_{21}\sin^2\alpha} \;; \quad Z = \frac{(N+1)X^{N+2} - (N+2)X^{N+1} + 1}{(X-1)^2}.$$

The state $|M,R\rangle$ is an eigenstate of the operators R_{11}, $R = R_{11} + R_{33}$ and $\hat{N} = R_{11} + R_{22} + R_{33}$ here

$$R_{ij} = Q_i^+ Q_j \qquad (i,j = 1,2,3).$$

By using solution (3) the characteristic function can be defined similarly to Louisell[16]

$$\chi_R(\xi) = \langle e^{i\xi R}\rangle_s = Z^{-1} \frac{(N+1)Y^{N+2} - (N+2)Y^{N+1} + 1}{(Y-1)^2}, \qquad (4)$$

where

$$Y = X \cdot e^{i\xi}.$$

Here $\langle B\rangle_s$ indicates the expectation value of an operator B in the steady state (3).

Once the characteristic function is known, it is easy to calculate the statistical moments

$$\langle R^n\rangle_s = \frac{\partial^n}{\partial(i\xi)^n} \chi_R(\xi) \Big|_{i\xi=0}. \qquad (5)$$

Now we investigate the statistical properties of scattered light corresponding to the atomic transition $|2\rangle$ to $|1\rangle$.

It is easy to find from the canonical transformation (2) that

$$
\begin{aligned}
J_{21}(t) &= \frac{1}{2}\cos\alpha \left(R_{33}(t) - R_{11}(t)\right) + \frac{1}{\sqrt{2}}\sin\alpha \left(R_{32}(t) + R_{12}(t)\right) + \frac{\cos\alpha}{2}\left(R_{13}(t) - R_{31}(t)\right) \qquad (6) \\
J_{12}(t) &= \frac{\cos\alpha}{2}\left(R_{33}(t) - R_{11}(t)\right) + \frac{1}{\sqrt{2}}\sin\alpha \left(R_{23}(t) + R_{21}(t)\right) + \frac{\cos\alpha}{2}\left(R_{31}(t) - R_{13}(t)\right). \qquad (7)
\end{aligned}
$$

By using the results of our previous paper[13], the operators $R_{33}(t) - R_{11}(t)$, $R_{32}(t)$, $R_{12}(t)$, $R_{31}(t)$ and $R_{13}(t)$ can be considered as the sources of spectrum components at frequencies ω_{21}, $\omega_{21} + \Omega$, $\omega_{21} - \Omega$, $\omega_{21} + 2\Omega$ and $\omega_{21} - 2\Omega$ respectively. For simplicity we call the spectrum component at frequency $\omega_{21} + m\Omega$ ($m = 0, \pm 1, \pm 2$) by S_m.

By using the solution (3) the steady-state normalized intensity correlation functions of the spectrum components can be defined

$$
\begin{aligned}
g^{(2)}_{0,0} &= \langle (R_{33} - R_{11})^4\rangle_s / \langle (R_{33} - R_{11})^2\rangle_s^2 = \\
&= \frac{3}{5}\,\frac{3\langle R^4\rangle_s + 12\langle R^3\rangle_s + 8\langle R^2\rangle_s - 8\langle R\rangle_s}{(\langle R^2\rangle_s + 2\langle R\rangle_s)^2} \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
g^{(2)}_{-1,-1} &= \langle R_{12}R_{12}R_{21}R_{21}\rangle_s / \langle R_{12}R_{21}\rangle^2 = g^{(2)}_{1,1} = \\
&= \frac{4}{3}\,\frac{\langle R^4\rangle_s - 2(N+2)\langle R^3\rangle_s + (N^2+5N+5)\langle R^2\rangle_s - (N^2+3N+2)\langle R\rangle_s}{((N+1)\langle R\rangle_s - \langle R^2\rangle_s)^2} \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
g^{(2)}_{2,2} &= \langle R_{31}R_{31}R_{12}R_{12}\rangle_s / \langle R_{31}R_{13}\rangle_s^2 = g^{(2)}_{-2,-2} \\
&= \frac{6}{5}\,\frac{\langle R^4\rangle_s + 4\langle R^3\rangle_s + \langle R^2\rangle_s - 6\langle R\rangle_s}{(\langle R^2\rangle_s + 2\langle R\rangle_s)^2}. \qquad (10)
\end{aligned}
$$

where $g^{(2)}_{m,m}$ ($m = 0, \pm 1, \pm 2$) is the normalized intensity correlation function for the spectrum component S_m. Here the values $\langle R^n\rangle_s$ can be found in (5). The behaviour of the functions $g^{(2)}_{m,m}$ against the parameter X is shown in figs. 2. For the cases of one and several atoms the correlation functions $g^{(2)}_{\pm1,\pm1}$ and $g^{(2)}_{\pm2,\pm2}$ are less than unity for a suitable region of the parameter X; thus, the sidebands of the spectrum $S_{\pm1}$, $S_{\pm2}$ have subpoissonian photon statistics. The central component of the spectrum S_0 has the superpoissonian photon statistics for any value of

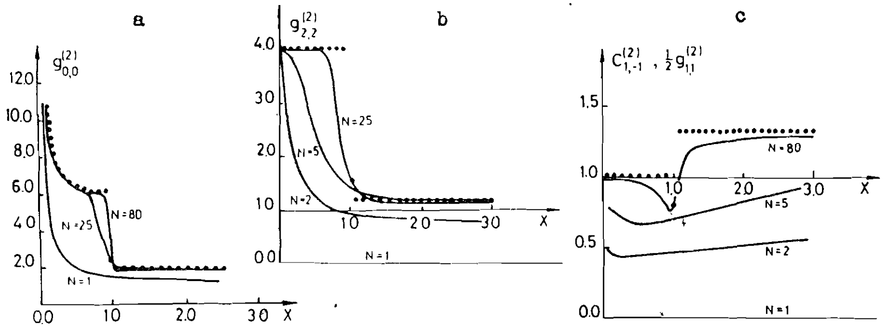

Fig. 2a-c. Normalized intensity correlation functions $g^{(2)}_{m,m}$ graphed against the parameter X. The dotted curves indicate the behaviour as $N \to \infty$.

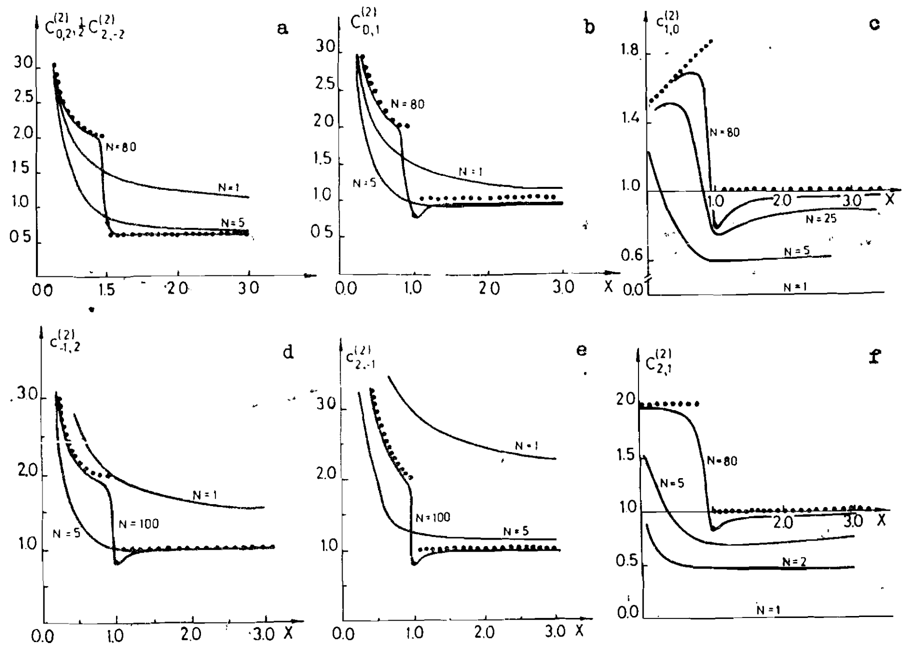

Fig. 3a-f. Cross correlation function $C^{(2)}_{m,n}$ graphed against the parameter X . The dotted curves indicate the behaviour as $N \to \infty$.

the number of atoms N and finite value of the parameter X. In the cooperative limit $N \to \infty$ the normalized intensity correlation function $g^{(2)}_{m,m}$ has a discontinuous transition, reminiscent of a typical nonequilibrium first-order phase transition at the critical point X = 1.

Now we discuss the cross correlation between the spectrum components. The magnitude of the cross correlation between the spectrum components S_m and S_n (m , n = 0, ± 1, ± 2) can be characterized by the steady-state cross-correlation functions $C^{(2)}_{m,n}$ using the stationary solution (3), one can find the cross-correlation functions between components of steady-state spectrum.

$$C^{(2)}_{1,-1} = \langle R_{32} R_{12} R_{21} R_{23} \rangle_s / \langle R_{32} R_{23} \rangle_s \langle R_{12} R_{21} \rangle_s = C^{(2)}_{-1,1} = \frac{1}{2} g^{(2)}_{1,1} \tag{11}$$

$$C^{(2)}_{0,2} = \langle \Delta_3 R_{31} R_{13} \Delta_3 \rangle_s / \langle \Delta_3^2 \rangle_s \langle R_{31} R_{13} \rangle_s = C^{(2)}_{2,0} = C^{(2)}_{0,-2} = C^{(2)}_{-2,0} \tag{12}$$

$$C^{(2)}_{2,-2} = \langle R_{31} R_{13} R_{31} R_{13} \rangle_s / \langle R_{31} R_{13} \rangle_s^2 = C^{(2)}_{-2,2} = 2 C^{(2)}_{0,2} \tag{13}$$

$$C^{(2)}_{2,1} = \langle R_{31} R_{32} R_{23} R_{13} \rangle_s / \langle R_{31} R_{13} \rangle_s \langle R_{32} R_{23} \rangle_s = C^{(2)}_{1,2} = C^{(2)}_{-1,-2} = C^{(2)}_{-2,-1} \tag{14}$$

$$C^{(2)}_{0,1} = \langle \Delta_3 R_{32} R_{23} \Delta_3 \rangle_s / \langle \Delta_3^2 \rangle_s \langle R_{32} R_{23} \rangle_s = C^{(2)}_{0,-1} \tag{15}$$

$$C^{(2)}_{1,0} = \langle R_{32} \Delta_3^2 R_{23} \rangle_s / \langle \Delta_3^2 \rangle_s \langle R_{32} R_{23} \rangle_s = C^{(2)}_{-1,0} \tag{16}$$

$$C^{(2)}_{2,-1} = \langle R_{31} R_{12} R_{21} R_{13} \rangle_s / \langle R_{31} R_{13} \rangle_s \langle R_{12} R_{21} \rangle_s = C^{(2)}_{-2,1} \tag{17}$$

$$C^{(2)}_{-1,2} = \langle R_{12} R_{31} R_{13} R_{21} \rangle_s / \langle R_{31} R_{13} \rangle_s \langle R_{12} R_{21} \rangle_s = C^{(2)}_{1,-2} \tag{18}$$

where $\Delta_3 = R_{33} - R_{11}$.

By using the solution (3) and boson's commutation relation for operator Q_i (i = 1,2,3), the cross-correlation functions $C^{(2)}_{m,n}$ in equations (11-18) can be expressed via the statistical moments $\langle R^n \rangle_s$ as done for the normalized intensity correlation functions $g^{(2)}_{m,m}$ in (8-10). We speak about anticorrelation (or correlation) between the spectrum components S_m and S_n when the cross-correlation function $C^{(2)}_{m,n}$ is less (or more) than unity [17,20]. Behaviour of the function $C^{(2)}_{m,n}$ (m , n = 0, ± 1, ± 2) against the parameter X is shown in figs. 3a-f and in fig. 2c. From these figures one can see that:

(i) for all values of the parameter X and number of atoms N the correlation between the extreme side bands S_{+2} and S_{-2} occurs, i.e., the photons of the extreme side band have a tendency to be emitted in pairs (fig. 3a).

(ii) The anticorrelation between all the spectrum components (beside between extreme side bands) occurs for suitable values of the parameter X and number of atoms N . Moreover, the anticorrelation between the central spectrum component S_0 and the extreme sidebands $S_{\pm 2}$, between S_2 and S_{-1} and between S_1 and S_2 **occurs only in the collective case $N \neq 1$. Thus, the measurable cross-correlation functions $C^{(2)}_{0,\pm 2}$, $C^{(2)}_{2,-1}$ and $C^{(2)}_{1,2}$ provide a new toole for the study of cooperative effects.**

(iii) In cooperative limit $N \to \infty$, the cross correlation functions $C^{(2)}_{m,n}$ have a discontinuous transition reminiscent of a typical nonequilibrium first-order phase transition at the critical point X = 1.

In conclusion, we have shown that in the case of one two-level atom, i.e., N = 1, $X \to \infty$ our results reduce to those of the works [17-19].

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Статистика фотонов в процессе коллективного двойного оптического резонанса

В статье рассмотрена статистика фотонов спектральных компонент рассеянного света в процессе коллективного двойного оптического резонанса. Обнаружены корреляция и антикорреляция между спектральными компонентами.

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Photon Statistics in Collective Double Optical Resonant Process

The photon statistics of spectrum components of scattered light in a collective double optical resonant process [13] is discussed. The correlation and anticorrelation between spectrum components are observed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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