объединенный
институт
ядерных
исследований
дубна

E4-86-29

A.Adamczak, V.S.Melezhik, L.I.Menshikov*

ELECTRON SCREENING IN LOW ENERGY SCATTERING OF MUONIC HYDROGEN ON HYDROGEN ATOMS

Submitted to "Zeitschrift für Physik"

* I.V.Kurchatov Institute of Atomic Energy, Moscow, USSR

1986

1. Introduction

The knowledge of the cross sections for low energy scattering of muonic hydrogen on hydrogen nuclei, atoms or molecules is needed in many branches of muon physics, e.g., in the kinetics of muon catalyzed fusion /1/. The scattering on isolated nuclei requires the solution of a three-body problem with Coulomb interaction /2/, and it has been investigated in Refs./3,4/. The phase shifts and cross sections obtained there are used in this paper to calculate the cross sections for the scattering of muonic hydrogen on hydrogen atoms with inclusion of an effective potential describing the electron screening. The electron screening potential used in our calculations is discussed in Sec. 2. In Sec. 3 a method of deriving the screening corrections to the elastic, spin-flip and isotopic exchange cross sections is described. Some results obtained for different isotopes of hydrogen are presented in Sec. 4.

2. Electron Screening Potential

For the sake of brevity let us denote the muonic and electronic hydrogen atoms by $(a\mu)$ and (be), respectively, where the nuclei a and b can be any hydrogen isotopes. Let us consider the $(a\mu)+(be)$ scattering as shown in Fig. 1.

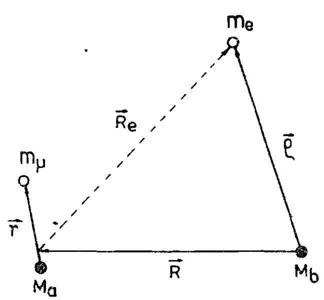

Fig. 1. Coordinates used for the description of scattering of muonic hydrogen on hydrogen atoms.

The electronic and muonic masses are denoted by m_e and m_μ and those of the respective nuclei by M_a and M_b $(M_a \geq M_b)$. Let the reduced mass of $(a\mu)$ be m_a. Upon separation of the overall center of mass motion the Schrödinger equation of this system is $(\hbar = e = m_a = 1)$

$$(H-E)\Psi(\vec{r},\vec{\varrho},\vec{R}) = 0 \ , \tag{1}$$

where

$$H = -\nabla_R^2/2\mathcal{M} + H_\mu + H_e + V \quad , \quad V = V_{nucl} + V_e \quad , \tag{2}$$

$$H_\mu = -\nabla_r^2/2 - 1/r \quad , \quad (H_\mu - E_{\mu l})\varphi_{\mu l} = 0 \quad , \tag{3}$$

$$H_e = -\nabla_\varrho^2/2m_e - 1/\varrho \quad , \quad (H_e - E_{ek})\varphi_{ek} = 0 \quad , \tag{4}$$

$$V_{nucl} = -|\vec{R} + M_a\vec{r}/(M_a+m_\mu)|^{-1} + |\vec{R} - m_\mu\vec{r}/(M_a+m_\mu)|^{-1} , \tag{5}$$

$$V_e = -|\vec{R} - \vec{\varrho} + m_\mu\vec{r}/(M_a+m_\mu)|^{-1} + |\vec{R} - \vec{\varrho} - M_a\vec{r}/(M_a+m_\mu)|^{-1} , \tag{6}$$

$$m_a^{-1} = M_a^{-1} + m_\mu^{-1} \quad , \quad \mathcal{M}^{-1} = (M_a+m_\mu)^{-1} + (M_b+m_e)^{-1} .$$

Vector $\vec{R}$ connects the nucleus b with the centre of mass of the muonic atom $(a\mu)$. The vectors $\vec{r}$ and $\vec{\varrho}$ represent, respectively, the internal coordinates of $(a\mu)$ and (be). The terms of the order of m_e/M_b are neglected. The eigenvalues and eigenfunctions of the Hamiltonians H_μ and H_e are denoted by $E_{\mu l}, \varphi_{\mu l}$ and E_{ek}, φ_{ek}, respectively (l, k are sets of quantum numbers).

Our aim is to investigate how slow collisions of the neutral systems under consideration are influenced by the electron- $(a\mu)$ interaction potential $V_e(\vec{r},\vec{\varrho},\vec{R})$. Since the nuclei are heavy compared to the negatively charged particles, their relative motion is very slow, and the Born-Oppenheimer approximation can be applied yielding an effective screening potential depending exclusively on R. To obtain this potential, the wave function $\Psi(\vec{r},\vec{\varrho},\vec{R})$ is expanded over the basis of eigenfunctions $\varphi_{\mu l}$ and φ_{ek} corresponding to the isolated atoms

$$\Psi(\vec{r},\vec{\varrho},\vec{R}) = \sum_n F_n(\vec{R})\varphi_n(\vec{r},\vec{\varrho}) , \tag{7}$$

where

$$\varphi_n(\vec{r},\vec{\varrho}) \equiv \varphi_{\mu l}(\vec{r})\varphi_{ek}(\vec{\varrho}) \equiv |n\rangle .$$

Inserting (7) into (1) one obtained the system of equations for F_n

$$(\nabla_R^2 + k_n^2)F_n = 2\mathcal{M}\sum_{n'} V_{nn'}F_{n'} , \tag{8}$$

$$k_n^2 = 2\mathcal{M}(\varepsilon + E_0 - E_n) \quad , \tag{9}$$

$$\varepsilon = E - E_0 \quad , \quad E_n = E_{\mu l} + E_{ek} \quad , \quad E_0 = E_{\mu 0} + E_{e0} \quad , \tag{10}$$

$$V_{nn'} \equiv \langle n|V(\vec{r},\vec{\varrho},\vec{R})|n'\rangle \quad , \tag{11}$$

where ε is the collision energy, $E_{\mu 0}$ and E_{e0} are the internal energies of the two atoms.

Let us consider the region $R > R_{min} \gg a_\mu \approx 1$ where a probability of $(a\mu)$ excitation by the nuclei b is small (a_μ is the Bohr radius of muonic hydrogen)*. One can expect that the contribution of V_e to the scattering amplitude from $R < R_{min}$ is negligible because the corresponding volume is small compared to that of the electron cloud characterized by Bohr radius $a_e (a_e \approx 200,\ m_e \approx a_e^{-1} \approx 1/200$ in muonic units. The first two orders of a perturbation expansion lead to the following equation for F_0 /5,6/

$$(-\nabla_R^2/2\mathcal{M} + \tilde{V}(R) - \varepsilon)F_0(\vec{R}) = 0 , \tag{12}$$

where

$$\tilde{V}(R) = V_{00} + \sum_n |V_{n0}|^2/(E_0 - E_n) . \tag{13}$$

When the initial states are the ground states of the isolated atoms, i.e.,

$$\varphi_{\mu 0} \equiv |0_\mu\rangle = \pi^{-\frac{1}{2}}\exp(-r) \quad , \quad \varphi_{e0} \equiv |0_e\rangle = (\pi a_e^3)^{-\frac{1}{2}}\exp(-\varrho/a_e) , \tag{14}$$

the potential $\tilde{V}(R)$, for $k_0 = (2\mathcal{M}\varepsilon)^{1/2} \ll E_0 \sim 1$, is given by /5,6/

$$\tilde{V}(R) = -9/4R^4 + U(R) \quad , \tag{15}$$

where the first term is the asymptotic form of (5), and $U(R)$ is the effective screening potential

$$U(R) = \langle 0_e|[U_0(R_e) + U_1(\vec{R}_e,\vec{R})]|0_e\rangle \quad , \quad \vec{R}_e = \vec{\varrho} - \vec{R} \quad , \tag{16}$$

$$U_0(R_e) = -\varkappa_\mu(1 + 1/R_e)\exp(-2R_e) \quad , \quad \varkappa_\mu = (M_a - m_\mu)/(M_a + m_\mu) , \tag{17}$$

$$U_1(\vec{R}_e,\vec{R}) = \frac{9}{2}\frac{\vec{R}_e\cdot\vec{R}}{R_e^3 R^3} - \frac{2}{3}\frac{m_e}{R_e^4}\sum_{l\neq 0}\frac{|\langle l|\vec{r}|0\rangle|^2 g_l(R_e)}{\gamma_l^2} \quad , \tag{18}$$

$$g_l(R_e) = 1 - (1 + \gamma_l R_e)\exp(-\gamma_l R_e) \quad , \quad \gamma_l = [2m_e(E_{\mu l} - E_{\mu 0})]^{\frac{1}{2}} .$$

R_e is the distance between the electron and the $(a\mu)$ centre of mass. The first order potential $U_0(R_e)$ represents the energy of interaction between the electron and the average electrostatic field of $(a\mu)$ which, at $R_e \gg 1$, falls rapidly with R_e. The second order term $U_1(\vec{R}_e,\vec{R})$ describes at $R_e \gg 1$ the polarisation interaction between $(a\mu)$ and the electron. By averaging (17) and (18) over φ_{e0} one obtains /6/

---

* The results of /3,4/ show that $R_{min} \approx 30$.

$$U(R)=-\left[2\varkappa_\mu+\frac{8}{3}m_e^{1/2}\sum_{l\neq 0}\frac{|\langle l|\vec{r}|0\rangle|^2}{(E_{\mu l}-E_{\mu 0})^{1/2}}\right]\frac{\exp(-2R/a_e)}{a_e^3}+\frac{9}{4}\,\frac{\Phi(R/a_e)}{a_e^4}\,, \qquad (19)$$

where

$$\Phi(y)=\frac{2}{y^4}\left[1-(1+2y+2y^2)\exp(-2y)\right]+\left[(1/y-2)\exp(-2y)Ei(2y)-\right.$$
$$\left.-(1/y+2)\exp(2y)Ei(-2y)\right], \qquad y=R/a_e\,,$$

$$Ei(-z)=-\int_z^\infty e^{-t}t^{-1}dt\,.$$

At $R\gg a_e$ the component of $U(R)$ which is proportional to a_e^{-4} $a_e^{-4}\approx m_e^4$ approaches $9/4R^4-\mathrm{const}/R^6$, and the total effective potential of two neutral atoms (15) follows a Van der Waals R^{-6} behaviour. However, the main contribution of $U(R)$ to the scattering amplitude comes from the region $R\sim a_e$, where the exponential terms of the order of a_e^{-3} and $a_e^{-3.5}$ are dominant *. Moreover, (19) does not include the polarization interaction at distances $R_e\sim 1$ which also gives a contribution to $U(R)$ of the order of a_e^{-4}. Therefore, in our calculations we have neglected the terms of an order greater than that of $a_e^{-3.5}$. Finally, after the numerical estimation of the sum over the (a_μ) eigenstates in (19), we have obtained the screening potential

$$U(R)=-C\,a_e^{-3}\exp(-2R/a_e)\,, \qquad 30\lesssim R\lesssim 800\,, \qquad (20)$$

$$C=2\varkappa_\mu+8.40\,m_e^{1/2}\approx 2.3\,. \qquad (21)$$

The magnitude of this potential for $R\sim a_e$ is of the order of 10^{-6} in muonic units. Thus, we are dealing with a three-body potential, relatively strong at $R\ll a_e$, and a weak effective screening potential $U(R)$ which dominates at $R\sim a_e$.

Let us consider a particle with momentum k and mass m scattering on the potential $U(R)$. A relative change of the wave function of particle at $R=0$, due to $U(R)$, is given by the first Born approximation and is of the order of $mC/a_e^2(1+k^2)^{\frac{1}{2}}\sim 10^{-4}$. That proves

* Practically, in the numerical calculations it is sufficient to consider only the region $R<R_{max}\approx 4a_e\approx 800$. The absolute value of the last term of (19) which decreases as $R_e^{-4}(R\gg a_e)$ is greater than the absolute value of the first exponential term at distances $R\gtrsim 5a_e$, and it gives the contribution to the screening corrections equal about 6% .

the applicability of Born approximation to the potential (20) for any collision energy.

3. Cross Sections

Below we estimate the corrections to the elastic spin-flip and muon isotopic transfer cross sections for the scattering of $(a\mu)$ on hydrogen nuclei which arise due to inclusion of the screening potential (20). The values for the phase shifts and cross sections for these processes reported in Refs /3,4/ were used as an input in our calculations.

The phase shifts of $(a\mu)$ scattering on nuclei have been obtained from the solution of the three-body problem in the framework of adiabatic representation /2,3/. The calculation of the initial phase shifts in this representation requires the solution of a multichannel scattering problem given by the system of N differential equations

$$\frac{d^2}{dR'^2}y_i^{(\alpha)}(R')+\left[k_i^2-\frac{L(L+1)}{R'^2}\right]y_i^{(\alpha)}(R')-\sum_{j\neq i}^{N}H_{ij}(R')y_j^{(\alpha)}(R')=0\,, \qquad (22)$$

where $i,j=1,\dots,N$, $\alpha=1,\dots,\nu$, R' is the internuclear distance, L is the orbital angular momentum of the system, k_i are the momenta in respective channels, and $y_i^{(\alpha)}(R')$ are the wave functions describing the relative motion of the nuclei. The effective potentials $H_{ij}(R')$ of the three-body problem are defined in /2/. If the number of open channels equals to ν, one has

$$k_i^2>0\,,\quad i\leqslant\nu$$
$$k_i^2<0\,,\quad i>\nu$$

and for $R'\to\infty$

$$y_i^{(\alpha)}(R')\to\begin{cases} j_L(k_iR')\delta_{i\alpha}-n_L(k_iR')\delta_{ij}T_{j\alpha}\,, & i\leqslant\nu\,,\\ \exp(-|k_i|R')\,, & i>\nu\,,\end{cases} \qquad (23)$$

$$\delta_{ij}=\begin{cases}1\,, & i=j\\ 0\,, & i\neq j\end{cases}\,,$$

where $j_L(k_iR')$, $n_L(k_iR')$ are the spherical Bessel functions, and T is a reaction matrix. Usually, at low collision energies it is necessary to consider a great number $(N-\nu\sim 300)$ of closed channels in order to determine T accurately /7,8/.

The influence of the electron screening on the phase shifts can be investigated by direct solution of the multichannel problem (22) with the potential $\bar{H}_{ij}\equiv H_{ij}+U_{ij}$, where U_{ij} denotes a matrix element of the screening potential. The main contribution to initial

T comes from the region $R' \ll a_e$ where the screening effects are negligible. On the other hand, the influence of the electron screening is dominant at $R' \sim a_e$ where the functions of adiabatic basis are approximated by the eigenfunctions $\varphi_{\mu\nu}$ of $(a\mu)$ [9]. Therefore, to obtain $V_{ij}(R')$ one can use the method presented in the previous section. Let us note that the potentials H_{ij} in (22) depend on the distance R' between the nuclei, while the screening potential (20) depends on the distance R between the centres of mass of the atoms. However, for $R \sim R' \sim a_e$ this difference is negligible [6].

We have solved system of equations (22) with the potentials $\bar{H}_{ij} = H_{ij} + V_{ij}$ using the method reported in [7,8] for $N = 2$ and 14 at low collision energies ($\varepsilon \lesssim 10$ eV). In this case only one or two channels are open ($\nu = 1$ or 2). The elastic scattering of the ground state $(a\mu)$ is a first channel ($M_a \gg M_b$), and the second one which corresponds to an excited state of $(a\mu)$ hyperfine structure ($M_a = M_b$) or to the ground state of the system $(b\mu)$ consisting of the muon and the lighter nucleus b. Numerical results have shown that the two channel approximation of the phase shift screening corrections is sufficient for any combination of hydrogen isotopes, although it is not always satisfactory for the initial phase shifts [8]. The error introduced by this approximation is less than 1 per cent.

The second and more simple way of determining the screening corrections is the variable phase method [10] applied earlier to the two-channel scattering problem [11]. In this case the reaction matrix $\bar{T}$ is written in the form

$$\bar{T} = T + \Delta T, \tag{24}$$

where T is the reaction matrix of the pure three-body problem, and ΔT describes the influence of the electron. $\bar{T}$ is related to the scattering matrix $\bar{S}$ by $\bar{S} = (1+i\bar{T})(1-i\bar{T})^{-1}$. Neglecting at $R \gtrsim R_{min}$ the three-body potential and taking T as the first approximation of a variable reaction matrix $T(R)$, we obtain

$$\Delta T = \Delta T(R_{max}), \quad \frac{d}{dR}\Delta T(R) \approx -2m\,U(R)(u+Tv)(u+vT), \quad \Delta T(R_{min}) = 0. \tag{25}$$

The diagonal matrices u and v consisting of the Riccati-Bessel functions are given in [11].

When the collision energy ε is greater than $\Delta E = E_1 - E_0$ (E_1 is a threshold energy of the second channel), the partial cross sections are [11]

$$\bar{\sigma}^L_{ij} = \frac{4\pi}{k_i^2}(2L+1)\left|\frac{D_T\delta_{ij} + i\bar{t}^L_{ij}}{(D_T-1)+i(\bar{t}^L_{11}+\bar{t}^L_{22})}\right|^2, \quad L = 0, 1, \ldots, \tag{26}$$

$$D_T = \det \bar{T}^L = \bar{t}^L_{11}\bar{t}^L_{22} - (\bar{t}^L_{12})^2,$$

where $i = 1,2$ denotes the channel number, and k_i are the momenta in the respective channels

$$k_1 = (2m\varepsilon)^{1/2}, \quad k_2 = [2m(\varepsilon - \Delta E)]^{1/2}. \tag{27}$$

On the other hand, for $\varepsilon < \Delta E$ it is convenient to express the elastic cross section in the open channel by the well-known formula

$$\bar{\sigma}^L_{11} = \frac{4\pi}{k^2}(2L+1)\sin^2\bar{\delta}_L, \tag{28}$$

where

$$\tan\bar{\delta}_L = \bar{t}^L_{11}, \quad k \equiv k_1. \tag{29}$$

If $L = 0$, Eq.(25) leads to

$$\Delta t^0_{ii} = -\frac{2m}{k_i}\int_0^\infty U(R)[(\sin k_iR + t^0_{ii}\cos k_iR)^2 + (t^0_{12})^2\cos^2 k_iR]\,dR \tag{30}$$

for two open channels and to

$$\Delta\delta_0 = -\frac{2m}{k}\int_0^\infty U(R)\sin^2(kR+\delta_0)\,dR \tag{31}$$

for $\varepsilon < \Delta E$. The formulae above include the regions $R < R_{min}$ and $R > R_{max}$ which give negligible contributions to the integrals. By virtue of (20) and (31) we have

$$\Delta\delta_0 = \frac{mc}{2a_e}\,\frac{1}{x}\left(1 + \frac{x\sin 2\delta_0 - \cos 2\delta_0}{1+x^2}\right), \quad x \equiv ka_e. \tag{32}$$

When $k \ll 1/a_e$ ($\varepsilon \ll 0.01$ eV)

$$\Delta\delta_0 = -\Delta\lambda_1 k = -(\lambda_e - 2\lambda_1/a_e)k, \quad \lambda_e \equiv -\frac{mc}{2} \sim -10, \tag{33}$$

where λ_1 and λ_e are the scattering lengths for the three-body potential and the screening potential, respectively.

For muonic hydrogen scattering on nuclei it is sufficient to consider only a few partial waves in the energy range $\varepsilon \lesssim 1$ eV [3,4]. On the other hand, an analogous energy range is very narrow ($ka_e \ll 1$ for $\varepsilon \ll 0.01$ eV) in the case of the screening potential (20), and it is necessary to take into account more partial waves in the total elastic cross sections. Thus, it is convenient to use the Born amplitude of scattering of $(a\mu)$ on the potential $U(R)$

$$f(q) = -\lambda_e \left(1 + a_e^2 q^2/4\right)^{-2}, \tag{34}$$

where q is the momentum transfer.

The corresponding total cross section is

$$\sigma_{el}(x) = \sigma_{el}(0)\,\frac{1+x^2+x^4/3}{(1+x^2)^3} \xrightarrow[x\gg 1]{} \sigma_{el}(0)\frac{1}{3x^2}, \quad \sigma_{el}(0) = 4\pi\lambda_e^2 \tag{35}$$

and the first two Born phase shifts are

$$\Delta\delta_0^B = -\frac{\lambda_e}{a_e}\,\frac{x}{1+x^2}, \tag{36}$$

$$\Delta\delta_1^B = -\frac{\lambda_e}{a_e}\,\frac{1}{x^3}\left[\frac{x^2(x^2+2)}{1+x^2} - 2\ln(1+x^2)\right] \longrightarrow -\frac{\lambda_e}{3a_e}x^3, \quad (x\to 0). \tag{37}$$

Finally, the total elastic cross section $\bar{\sigma}_{11}$ for scattering of $(a\mu)$ on (be) is

$$\bar{\sigma}_{11}(k) = \sigma_{11}(k) + \sigma_{el}(k) + \frac{4\pi}{k^2}\sum_{L=0}^{n}(2L+1)\Delta\delta_L \sin 2\delta_L. \tag{38}$$

In the formula above n partial waves contribute to the initial cross section σ_{11} for scattering on hydrogen nuclei. The terms containing $(\Delta\delta_L)^2$ are neglected.

For $\varepsilon > \Delta E$ the following formulae are valid for elastic scattering in the respective channels

$$\bar{\sigma}_{ii} = \sum_{L=0}^{n}\left[\bar{\sigma}_{ii}^L(k_i) - \sigma_{el}^L(k_i)\right] + \sigma_{el}(k_i), \tag{39}$$

where the partial electronic cross sections σ_{el}^L are

$$\sigma_{el}^L(k_i) = \frac{4\pi}{k_i^2}\sin^2\Delta\delta_L^B \tag{40}$$

and $\bar{\sigma}_{ii}^L$ is defined by (26).

Now, let us consider the influence of the presence of the electron on the inelastic cross sections σ_{ij}^L $(i\neq j)$. The transitions between the states are due to the short-range forces at $R \ll a_e$ where the screening influence is negligible. Therefore, one can expect that the screening corrections to the inelastic cross sections which comes from the region $R \sim a_e$ are relatively small. Since the approximation of the scattering length is good for $k \ll 1/a_e$ ($\varepsilon \ll 0.01$ eV), it must not be introduced while considering inelastic processes. For these processes ΔE is greater than the collision energy $\varepsilon \approx 0.04$ eV, corresponding to the room temperature, where one always has $k_1 a_e > 1$. The estimates of the spin-flip cross sections given in [12] concern the three-body problem where both relations $k_1 R_0 \ll 1$ and

$k_2 R_0 \ll 1$ can be satisfied (R_0 is the characteristic radius of three-body potentials). They have led to the formulae expressed in terms of the scattering lengths. However, such formulae cannot be extended automatically to the case of scattering on atoms [13] by simply replacing the initial scattering lengths λ_i in the respective channels by the ones $\bar{\lambda}_i$ obtained with screening taken into account, where

$$\bar{\lambda}_i = \lambda_i + \Delta\lambda_i. \tag{41}$$

The variable phase method gives the screening corrections for any collision energy. From (25) one has

$$\bar{t}_{12}^{\,0} = t_{12}^0 \exp\left[-2m\sum_{i=1}^{2}\frac{1}{k_i}\int_0^\infty U(R)\left(\sin k_i R + t_{ii}^0 \cos k_i R\right)dR\right] = \tag{42}$$

$$= t_{12}^0 \exp\left\{-\frac{\lambda_e}{a_e}\sum_{i=1}^{2}\frac{1}{1+x_i^2}\left[1 + t_{ii}^0\,\frac{2+x_i^2}{x_i}\right]\right\}, \quad x_i = k_i a_e, \; i = 1,2.$$

Thus, neglecting higher-order terms corresponding to greater momenta $k_1 \gg (2m\Delta E)^{1/2}$ and to the quantity $\frac{t_{22}^0}{k_2 a_e} \sim \frac{\lambda_e}{a_e} \ll 1$, for $k_2 \ll k_1$ one obtains the leading term of (41)

$$\bar{t}_{12}^{\,0} = t_{12}^0\left[1 - \frac{\lambda_e}{a_e}\,\frac{1}{1+(k_2 a_e)^2}\right]. \tag{43}$$

It is evident that screening is considerable in the range $k_2 \lesssim 1/a_e$ where usually only the s-wave contributes to the initial inelastic cross section. Since according to (25), $\bar{t}_{12}^L$ is proportional in this case to t_{12}^L higher partial waves due to the pure $U(R)$ do not contribute to the cross section.

If the relations

$$|t_{11}^0| \ll 1, \quad |t_{22}^0| \ll 1, \quad |t_{12}^0| \ll 1 \tag{44}$$

are satisfied, then by virtue of (26) and (44), the ratio $\bar{\sigma}_{12}/\sigma_{12}$ is equal to

$$\frac{\bar{\sigma}_{12}}{\sigma_{12}} = \frac{\bar{\sigma}_{21}}{\sigma_{21}} \approx \left|\frac{\bar{t}_{12}^{\,0}}{t_{12}^0}\right|^2 \approx 1 - 2\,\frac{\lambda_e}{a_e}\,\frac{1}{1+(k_2 a_e)^2}. \tag{45}$$

4. Results

Some results of our calculations for different combinations of hydrogen isotopes are shown in Tables 1, 2 and Figs. 2,3,4,5. It can be seen from (35), (38) and (39) that the influence of the electron on elastic scattering is extremely important when the collision energy ε approaches zero ($\varepsilon \lesssim 10^{-3}$ eV). The value of the electronic scattering length λ_e defined by (33) is much greater than the

Table 1

Phase shifts and cross sections of elastic scattering $d\mu(\uparrow\downarrow)+D$, (S = 1/2)*

ε[eV]	$-\delta_0$	$^{**}\Delta\delta_0^{num}$	$\Delta\delta_0$	$\Delta\delta_0^B$	σ_{11}^0	$\bar\sigma_{11}^0$	σ_{11}	$\bar\sigma_{11}$
0	3.4 k	-	11.7 k	12.2 k	9.7	55.8	9.7	55.8
0.004	0.0127	0.0265	0.0270	0.0281	10.2	13.0	10.2	16.3
0.01	0.0211	0.0271	0.0278	0.0290	11.3	1.1	11.3	6.8
0.02	0.0307	0.0245	0.0248	0.0259	11.9	0.4	11.9	6.2
0.04	0.0448	0,0199	0.0202	0.0211	12.7	3.8	12.7	8.2
0.08	0.0661	0.0154	0.0154	0.0162	13.8	8.1	13.8	10.9
0.1	0.0751	0.0145	0.0140	0.0147	14.3	9.5	14.4	11.3
0.15	0.0949	0.0122	0.0117	0.0123	15.2	11.7	15.5	12.9
0.25	0.1274	0.0100	0.0092	0.0097	16.4	14.1	17.4	15.3

* Cross sections are given in 10^{-20} cm^2 units, scattering lengths in muonic units $a_\mu = \hbar^2/m_\mu e^2 = 2.56\cdot10^{-11}$ cm.

** Screening corrections obtained by numerical solution of (22) with additional effective screening potential.

three-body scattering lengths λ_i . Therefore, in this energy region the cross sections are mainly determined by the screening potential. For higher collision energies ($ka_e \gg 1$) σ_{el} decreases, according to (35) as k_i^{-2} , and the cross sections $\bar\sigma_{ii}$ of scattering on hydrogen atoms approach the respective cross sections σ_{ii} of scattering on nuclei at collision energies about 1 eV. If (44) is satisfied, the screening corrections Δt_{ii} , expressed by (30) are always positive and weakly depend on the initial phase shifts. In particular, (32) is approximated by (36). The variations of values of Δt_{ii} for different hydrogen isotopes are mainly due to the factor m .

The screening corrections $\Delta\delta_0$ in the case of $d\mu$ scattering on deuterium in the state S =1/2 of the total spin S of the muon and two nuclei are shown in Table 1 and Fig.2. One can see that both the methods of calculating of $\Delta\delta_0$ mentioned in the previous section give practically the same results. Since (44) is fulfilled in the energy

Table 2. Total cross sections and scattering lengths of elastic scattering of muonic atoms*

Process	$p\mu(\uparrow\downarrow)+H$		$d\mu+H$		$d\mu(\uparrow\downarrow)+D$ **		$t\mu+D$		$t\mu(\uparrow\downarrow)+T$	
ε [eV]	σ_{11}	$\bar\sigma_{11}$	σ_{11}	$\bar\sigma_{11}$	σ_{11}	$\bar\sigma_{11}$	σ_{11}	$\bar\sigma_{11}$	σ_{11}	$\bar\sigma_{11}$
0	2.0	52.0	2.4	79.4	13.8	48.3	14.4	72.2	0.002	252.7
0.01	1.9	23,8	2.1	30.0	15.6	6.0	14.8	9.2	0.18	35.6
0.02	1.9	15.7	1.8	18.0	16.5	7.5	15.3	8.5	0.34	17.2
0.04	1.9	10.0	1.6	10.2	17.5	11.2	16.0	10.4	0.62	8.2
0.1	2.4	6.2	1.2	4.7	19.3	15.8	19.4	14.9	1.3	3.7
0.15	3.1	5.9	1.0	3.3	20.3	17.7	22.0	17.7	1.6	3.1
0.25	-	-	0.75	2.0	21.7	19.9	27.2	23.6	-	-
1.00	-	-	0.07	0.03	27.5	26.7	46.8	45.2	-	-
Scattering length*	λ_1	$\bar\lambda_1$	λ_1	$\bar\lambda_1$	λ_1	$\bar\lambda_1$	λ_1	$\bar\lambda_1$	λ_1	$\bar\lambda_1$
This paper	-1.6	-8.0	-1.7	-9.8	3.4 $(S=\frac{1}{2})$ 4.2 $(S=\frac{3}{2})$	-8.2 $(S=\frac{1}{2})$ -7.4 $(S=\frac{3}{2})$	4.2	-9.4	0.05	-17.5
Kravtsov et al.***	-4.6	-11.1	-	-	3.6 $(S=\frac{1}{2})$ 4.6 $(S=\frac{3}{2})$	-7.6 $(S=\frac{1}{2})$ -6.6 $(S=\frac{3}{2})$	-	-	-0.6	-17.3

* Cross sections are given in $10^{-20}cm^2$ units, scattering lengths in muonic units $a_\mu = \hbar^2/m_\mu e^2$ =2.56 10-11 cm.

** Cross section averaged over the states S =1/2 and S =3/2 of the total spin of muon and two nuclei.

*** Values obtained in "simple approach" approximation /13/.

range $\varepsilon \lesssim 1$ eV, we have $\Delta\delta_0 \approx \Delta\delta_0^B$. Figure 3 represents the cross sections $\bar{\sigma}_{11}^0$ and $\bar{\sigma}_{11}$ of this process. The partial cross section $\bar{\sigma}_{11}^0$ vanishes for $\varepsilon \approx 0.018$ eV because $\Delta\delta_0 = -\delta_0$. The contributions of higher than zero partial waves due to $U(R)$ increase considerably the total cross section $\bar{\sigma}_{11}$ compared to $\bar{\sigma}_{11}^0$.

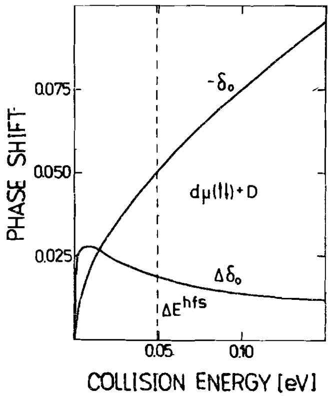

Fig. 2. The phase shift δ_0 and screening correction $\Delta\delta_0$ for elastic scattering $d\mu(\uparrow\downarrow)+D$ versus the collision energy ε.

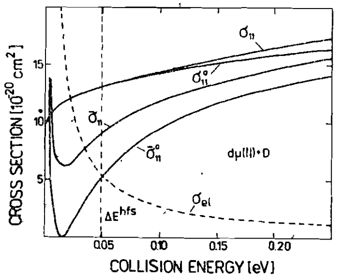

Fig. 3. Elastic cross sections $\bar{\sigma}_{11}^0$, $\bar{\sigma}_{11}$ of reaction $d\mu(\uparrow\downarrow)+D$ and σ_{11}^0, σ_{11} of reaction $d\mu(\uparrow\downarrow)+d$ versus the collision energy ε. The total cross section σ_{el} of scattering on the pure screening potential U(R) is represented by a dashed line.

The total cross section $\bar{\sigma}_{11}$ for scattering of $d\mu$ on a deuterium atom averaged over the states $S=1/2$ and $S=3/2$ (see Table 2 and Fig. 4) is less than the respective σ_{11} in the energy range $\varepsilon \gtrsim 0.005$ eV since for both the states δ_0 is negative, and its absolute value is greater than the positive $\Delta\delta_0$. Therefore, for $\varepsilon = 0.04$ eV our result $\bar{\sigma}_{11} = 11.2 \cdot 10^{-20}$ cm^2 is much closer to the experimental value $8 \pm 2 \cdot 10^{-20}$ cm^2 /14/ than the cross section $\sigma_{11} = 17.6 \cdot 10^{-20}$ cm^2 /4/.

On the other hand, in $p\mu$ scattering on proton δ_0 is positive (S=1/2). Thus, the positive $\Delta\delta_0$ must lead to an increase of $\bar{\sigma}_{11}$ com-

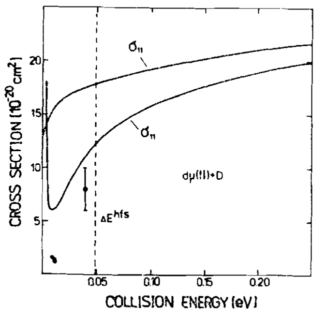

Fig. 4. Elastic cross sections $\bar{\sigma}_{11}$ and σ_{11} of scattering of $d\mu(\uparrow\downarrow)$ on deuterium atom and deuterium nuclei averaged over the states S=1/2 and S=3/2 of the total spin of muon and two nuclei. The experimental value 8 ± 2 10^{-20} cm^2 is given in Ref. /14/.

pared to σ_{11}. (Table 2). In particular, for $\varepsilon = 0.04$ eV we have obtained $\bar{\sigma}_{11} = 10.0 \cdot 10^{-20}$ cm^2, while $\sigma_{11} = 1.9 \cdot 10^{-20}$cm^2 /3/ and the experiment gives $1.7 \pm 3 \cdot 10^{-20}$cm^2 /15/. Let us note that to compare theory with experiment correctly, it is necessary to calculate the screening corrections for scattering on hydrogen molecules and to include the inelastic channel of the formation of muonic molecules.

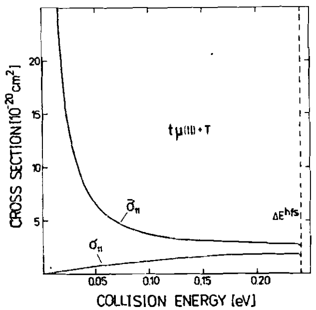

Fig. 5. Elastic cross sections $\bar{\sigma}_{11}$ of process $t\mu(\uparrow\downarrow)+T$ and σ_{11} of process $t\mu(\uparrow\downarrow)+t$ versus the collision energy ε.

Table 2 and Fig. 5 show also the results for the elastic processes $d\mu + H$, $d\mu + T$ and $t\mu(\uparrow\downarrow)+T$. The influence of screening is extremely strong for the reaction $t\mu(\uparrow\downarrow)+T$, where the reduced mass m is the greatest, and the initial scattering length λ_1 occurs to be very small.

Differences between the screening corrections to the spin-flip phase shifts obtained from formula (42) and from the numerical solution of the system of differential equations with the additional potential U(R) are less than 1 per cent. The ratio $(\bar{\sigma}_{21} - \sigma_{21})/\sigma_{21}$ given by (45) is equal to 8% for the reaction $d\mu(\uparrow\uparrow)+D \rightarrow d\mu(\uparrow\downarrow)+D$ at the collision energy corresponding to the temperature T=30K and is negligible at T=300K, which agrees well with the results of /4/ and experiment /16/. This effect is the strongest for the $t\mu$ scattering on tritium where the correction to the spin-flip cross section equals 18% at T=30 K leading to the additional weak dependence of the spin-flip cross section on temperature in the range $T \lesssim 300$K.

Analogous screening effects are also observed in scattering of muonic hydrogen on helium and lithium atoms /16,17/.

5. Conclusions

The electron screening in elastic scattering of muonic hydrogen on hydrogen atoms is important at collision energies lower than 1 eV. Therefore, this effect should be taken into account in the kinetics of muon catalysed fusion. The elastic scattering on atoms can be described in terms of a scattering length only at very low collision energies $\varepsilon \ll 0.01$ eV. To compare the results presented above with experiment, it would be of interest to have more experimental data, especially, for low temperatures of an order of 30K where the scattering of muonic hydrogen on electron is dominant.

The screening is relatively small for inelastic processes where the increase of cross sections does not exceed 20%. In the temperature range interesting for the muon catalysed fusion ($T \gtrsim 300K$) this effect is negligible.

The authors wish to thank Prof. L.I.Ponomarev for encouragement and interest. We are also indebted to Drs. M.P.Faifman, A.Guła, A.I. Mikhailov, N.P.Popov and J.Woźniak for helpful discussions.

References

1. Ponomarev L.I. Sixth International Conference on Atomic Physics, Mesic Atomic and Mesic Molecular Processes in the Hydrogen Isotope Mixtures. Zinatne, Riga, Plenum Press, N.Y., London, (1978).
   Ponomarev L.I. Atomkernenergie (Kerntechnik, 43, 175 (1983).
2. Vinitsky S.I., Ponomarev L.I. Fiz. Elem.Chastits At.Yadra, 13, 1336 (1982), (Sov. J.Part.Nucl. 13, 557 (1982)).
3. Melezhik V.S., Ponomarev L.I., Faifman M.P. Zh.Exp.Theor. Fiz. 85, 434 (1983), Sov. Phys. JETP, 58, 254 (1983).
4. Melezhik V.S., Ponomarev L.I., Woźniak J. Contribution to the X-th International Conference on Particles and Nuclei (PANIC), July 30- August 3, 1984, Book of Abstracts, Heidelberg, p. L40 (1984).
   Melezhik V.S., Woźniak J. JINR, E4-85-953, Dubna, (1985).
5. Kirzhnits D.A., Pen'kov F.M. Zh.Eksp.Theor.Fiz., 85, 80 (1983).
6. Kravtsov A.V., Mikhailov A.I., Popov N.P. Preprint LNPI n. 1037, Leningrad (1985).
7. Melezhik V.S., Puzynin I.V., Puzynina T.P. and Somov L.N. J.Comput. Phys., 54, 221 (1984).
8. Melezhik V.S. JINR, P4-84-643, Dubna, 1984, to be published in J.Comp.Phys.
9. Faifman M.P., Ponomarev L.I., Vinitsky S.I. J.Phys. B 9, 2255 (1976).
10. Calogero F. "Variable Phase Approach to Potential Scattering", Acad.Press, N.Y., London, 1967.
    Babikov V.V. "Variable Phase Method in Quantum Mechanics", Nauka, Moscow, 1976 (in Russian).
11. Matveenko A.V., Ponomarev L.I., Faifman M.P. Zh.Eksp.Theor.Fiz., 68, 437 (1975).
12. Gerstein S.S. Zh.Eksp.Theor.Fiz., 34, 463 (1958).
    Gerstein S.S. Zh.Eksp.Theor.Fiz., 40, 698 (1961).
13. Kravtsov A.V., Mikhailov A.I., Popov N.P. Preprint LNPI n. 1098, Leningrad (1985).

14. Bertin A. et al., Nuovo Cimento, 72A, 225 (1982).
15. Khomenko B.A., Bystritstsky V.H., Zh.Eksp.Theor.Fiz., 87, 384 (1984).
16. Kammel P. et al., Phys.Rev. A28, 2611 (1983).
17. Kravtsov A.V., Mikhailov A.I., Popov N.P. Preprint LNPI n. 1109, Leningrad, (1985).

Received by Publishing Department
on January 15, 1986.

WILL YOU FILL BLANK SPACES IN YOUR LIBRARY?

You can receive by post the books listed below. Prices - in US $, including the packing and registered postage

D1,2-82-27	Proceedings of the International Symposium on Polarization Phenomena in High Energy Physics. Dubna, 1981.	9.00
D2-82-568	Proceedings of the Meeting on Investigations in the Field of Relativistic Nuclear Physics. Dubna, 1982	7.50
D3,4-82-704	Proceedings of the IV International School on Neutron Physics. Dubna, 1982	12.00
D11-83-511	Proceedings of the Conference on Systems and Techniques of Analitical Computing and Their Applications in Theoretical Physics. Dubna, 1982.	9.50
D7-83-644	Proceedings of the International School-Seminar on Heavy Ion Physics. Alushta, 1983.	11.30
D2,13-83-689	Proceedings of the Workshop on Radiation Problems and Gravitational Wave Detection. Dubna, 1983.	6.00
D13-84-63	Proceedings of the XI International Symposium on Nuclear Electronics. Bratislava, Czechoslovakia, 1983.	12.00
E1,2-84-160	Proceedings of the 1983 JINR-CERN School of Physics. Tabor, Czechoslovakia, 1983.	6.50
D2-84-366	Proceedings of the VII International Cónference on the Problems of Quantum Field Theory. Alushta, 1984.	11.00
D1,2-84-599	Proceedings of the VII International Seminar on High Energy Physics Problems. Dubna, 1984.	12.00
D17-84-850	Proceedings of the III International Symposium on Selected Topics in Statistical Mechanics. Dubna, 1984. /2 volumes/.	22.50
D10,11-84-818	Proceedings of the V International Meeting on Problems of Mathematical Simulation, Programming and Mathematical Methods for Solving the Physical Problems, Dubna, 1983	7.50
	Proceedings of the IX All-Union Conference on Charged Particle Accelerators. Dubna, 1984. 2 volumes.	25.00
D4-85-851	Proceedings on the International School on Nuclear Structure. Alushta, 1985.	11.00

Orders for the above-mentioned books can be sent at the address:
Publishing Department, JINR
Head Post Office, P.O.Box 79 101000 Moscow, USSR

Адамчак А., Мележик В.С., Меньшиков Л.И. E4-86-29
Электронное экранирование в процессах низкоэнергетического рассеяния мезоатомов на атомах водорода

Вычислены поправки на электронное экранирование к сечениям низкоэнергетического рассеяния мезоатомов на атомах водорода. Показано, что его учет приводит к существенному изменению сечений упругого рассеяния мезоатомов при энергиях столкновения $\varepsilon < 1$ эВ. Влияние электронного экранирования на процессы изотопного обмена и переворота спина мезоатома относительно мало.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Adamczak A., Melezhik V.S., Menshikov L.I. E4-86-29
Electron Screening in Low Energy Scattering of Muonic Hydrogen on Hydrogen Atoms

Electron screening corrections to the cross sections for low energy scattering of muonic hydrogen on hydrogen atoms are calculated. It is shown that the presence of the electron influences considerably the elastic cross sections at collision energies below 1 eV. This influence is relatively small for the spin-flip and isotopic exchange processes.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1986