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THE PERTURBATION APPROACH TO SPIN RESONANCE AND SPIN WAVES IN NONMAGNETIC METALS

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1. Introductory Remarks; the Long-Wavelength Limit

We are going to apply the perturbation methods to the equation describing spin waves in normal metals. This equation, in the collisionless limit, can be written as follows [1,2] (cf. also [3])

$$-\omega\, P\tilde{M} + Q_{\vec{k}}\tilde{M} = \frac{1}{2}\beta\omega H' g(\vec{p}) . \qquad (1)$$

Let us list the appearing symbols: $-\omega, \vec{k}$ denote the frequency and the wave vector respectively, $\tilde{M}$ is the deviation of the instantaneous magnetization vector from its local equilibrium value, $\tilde{M}=\tilde{M}_x + i\tilde{M}_y$ and the dc external magnetic field H is oriented along the z-axis, $H'_\perp = H'_x + iH'_y$, where H' denotes ac magnetic field, $g(\vec{p})$ denotes the quasimomentum dependent electron g-factor, β the Bohr magneton and the operators $Q_{\vec{k}}$ and P are defined as follows

$$Q_{\vec{k}} = \beta H g(\vec{p}) + \frac{ie}{c}(\vec{H}\times\vec{V}_{\vec{p}})\vec{V}_{\vec{p}} + (\vec{k}\vec{V}_{\vec{p}}), \qquad (2)$$

$$P\tilde{M}(\vec{p}) = \tilde{M}(\vec{p}) - \int \frac{g S_F}{4\pi^3 V_{\vec{p}'}} G(\vec{p},\vec{p}')\tilde{M}(\vec{p}'), \quad h=1. \qquad (3)$$

Here $G(\vec{p},\vec{p}')$ denotes the exchange part of the scattering amplitude of quasiparticles [4,5], $dS_{\vec{p}'}$ the area element of the Fermi surface (FS) near the quasimomentum $\vec{p}'$, $V_{\vec{p}}$ the velocity of quasiparticles. The $\vec{p}$-dependence of g was detected experimentally for Al, Cu and Ag [6,7].

For irrelevant spin-orbit coupling the action of the operator P on the function $\tilde{M}=2$ gives $g(\vec{p})$, i.e., $g(\vec{p})=P\cdot 2$ [3], If P^{-1} g varies on FS then the homogeneous $(\vec{k}=0)$ spin resonance has, in the collisionless limit, more than one line, with a possible exception of suitably directed $\vec{H}$ [8]. Let us introduce, according to [8], the scalar product

$$(\phi,\psi)=\int \frac{dS_F}{4\pi^3 v_p \nu}\phi^*(\vec{p})\psi(\vec{p}), \tag{4}$$

where ν denotes the density of states per unit volume. The operators P and $Q_{\vec{k}}$ are hermitean operators. The dynamic magnetic susceptibility χ^+ can be expressed in terms of (4) as

$$\chi^+=\frac{1}{2}\nu\beta(g,P\tilde{M})/H_+' . \tag{5}$$

Following [8] define the system of eigenfunctions $\tilde{M}_{\alpha k}$ as

$$\omega_{\alpha\vec{k}} P\tilde{M}_{\alpha\vec{k}}=Q_{\vec{k}}\tilde{M}_{\alpha\vec{k}}, \tag{6}$$

where the variable α numerates different eigenvalues $\omega_{\alpha\vec{k}}$, $\vec{k}$ is treated as a parameter. For closed FS one can always assume that, [8],

$$(\tilde{M}_{\alpha\vec{k}},P\tilde{M}_{\beta\vec{k}})=\delta_{\alpha\beta} . \tag{7}$$

Similarly as in [6] one then obtains

$$\tilde{M}=\frac{1}{2}\beta\omega H_+' \sum_\alpha \frac{(\tilde{M}_{\alpha\vec{k}},g)\tilde{M}_{\alpha\vec{k}}}{\omega_{\alpha\vec{k}}-\omega}, \tag{8}$$

and

$$\chi^+=\frac{1}{4}\nu\beta^2\omega\sum_\alpha \frac{(\tilde{M}_{\alpha\vec{k}},g)(g,P\tilde{M}_{\alpha\vec{k}})}{\omega_{\alpha\vec{k}}-\omega}. \tag{9}$$

Our formal solution (9), (10) is valid only in the collisionless limit. In order to introduce the damping $Q_{\vec{k}}$ have to be changed by $Q_{\vec{k}}+i/\tau$, where τ^{-1} is some her-

mitean and positive definite operator. The assumption that τ is some constant does not lead to any important simplification of the results. Now equation (1) with $Q_{\vec{k}}$ replaced by $Q_{\vec{k}}+i/\tau$ cannot be simply expressed in terms of the functions $\tilde{M}_{\alpha\vec{k}}$. Assuming that i/τ could be treated as a perturbation one finds, with the linear accuracy, that

$$\tilde{M}=\frac{1}{2}\beta\omega H_{+}^{\prime}\sum_{\alpha}[\tilde{\omega}_{\alpha\vec{k}}-\omega]^{-1}\{(\tilde{M}_{\alpha\vec{k}},g)\tilde{M}_{\alpha\vec{k}}+\sum_{\beta(\beta\neq\alpha)}[\omega_{\alpha\vec{k}}-\omega_{\beta\vec{k}}]^{-1}\times$$

$$\times[(i/\tau)_{\alpha\beta}(\tilde{M}_{\beta\vec{k}},g)\tilde{M}_{\alpha\vec{k}}+(i/\tau)_{\beta\alpha}(\tilde{M}_{\alpha\vec{k}},g)\tilde{M}_{\beta\vec{k}}]\}, \qquad (10)$$

where

$$(i/\tau)_{\alpha\beta}=(\tilde{M}_{\alpha\vec{k}},(i/\tau)\tilde{M}_{\beta\vec{k}}),\ \tilde{\omega}_{\alpha\vec{k}}=\omega_{\alpha\vec{k}}+(i/\tau)_{\alpha\alpha}.$$

The matrix elements of the operator (i/τ) are $\vec{k}$-dependent. Substituting (10) into (5) we find χ^{+}. The poles and residues of the response function χ^{+} give the excitation energies and intensities respectively. Hence, it is necessary to apply the perturbation method in both these quantities separately; for example, perturbational substitution

$$(\tilde{\omega}_{\alpha\vec{k}}-\omega)^{-1}\to(\omega_{\alpha\vec{k}}-\omega)^{-1}-(i/\tau)_{\alpha\alpha}(\omega_{\alpha\vec{k}}-\omega)^{-2}$$

into (10) is not a good procedure. The immediate application of the second step of the perturbation approach results in the second-order poles. These poles have no meaningful physical interpretation. They can be eliminated if one introduces the second-order correction to $\tilde{\omega}_{\alpha\vec{k}}$ and one treats the second order poles as a result of series development of $(\tilde{\omega}_{\alpha\vec{k}}-\omega)^{-1}$. The collision integral used in [1] and [9] such that the spin density is less damped than, e.g., the spin current, can also be treated according to our scheme.

Let us treat the last term in (12) as a perturbation. The same method was used in (10) for isotropic systems. Applying the usual perturbation procedure to equation (6) one finds $(\omega_{\alpha\vec{k}}=\omega_{\alpha 0}+\omega_{\alpha 1}+\omega_{\alpha 2}+\ldots)$

$$\omega_{\alpha 1} = (\tilde{M}_{\alpha 0}, (\vec{k}\vec{V})\tilde{M}_{\alpha 0}). \qquad (11)$$

Since the operators P and Q_0 commute with the parity operator, as a result of time-reversal invariance, $\tilde{M}_{\alpha 0}$ has definite parity and hence $\omega_{\alpha 1} = 0$. Note that the assumption about the strength of spin-orbit coupling is not important for this statement. The second-order correction to the frequency can be written as follows

$$\omega_{\alpha 2} = \sum_{\gamma(\gamma \neq \alpha)} \frac{|(\tilde{M}_{\alpha 0}, (\vec{k}\vec{V})\tilde{M}_{\alpha 0})|^2}{\omega_{\alpha 0} - \omega_{\gamma 0}}. \qquad (12)$$

Note that the spectrum of equation (6) for $\vec{k} = 0$ is unbounded and it is impossible to deduce simply the sign of $\omega_{\alpha 2}$ for any α. The first order correction to the function $\tilde{M}_{\alpha k}$ can be written as

$$\tilde{M}_{\alpha 1} = \sum_{\gamma(\gamma \neq \alpha)} \frac{(\tilde{M}_{\gamma 0}, (\vec{k}\vec{V})\tilde{M}_{\alpha 0})}{\omega_{\alpha 0} - \omega_{\gamma 0}} \tilde{M}_{\gamma 0}, \qquad (13)$$

whereas the second order correction has the form

$$\tilde{M}_{\alpha 2} = \sum_{\gamma(\gamma \neq \alpha)} \sum_{\delta(\delta \neq \alpha)} \frac{(\tilde{M}_{\delta 0}, (\vec{k}\vec{V})\tilde{M}_{\gamma 0})(\tilde{M}_{\gamma 0}, (\vec{k}\vec{V})\tilde{M}_{\alpha 0})}{(\omega_{\alpha 0} - \omega_{\delta 0})(\omega_{\alpha 0} - \omega_{\gamma 0})} \tilde{M}_{\delta 0} -$$

$$- \frac{1}{2} \tilde{M}_{\alpha 0} \sum_{\gamma(\gamma \neq \alpha)} \frac{|(\tilde{M}_{\gamma 0}, (\vec{k}\vec{V})\tilde{M}_{\alpha 0})|^2}{(\omega_{\alpha 0} - \omega_{\gamma 0})^2}. \qquad (14)$$

These functions and eigenvalues can be substituted into (8) and (9) or (10). Taking into account that for $\vec{k} = 0$ in (8-10) only even functions $\tilde{M}_{\alpha 0}$ appear, and that parities of $\tilde{M}_{\alpha 0}$ and $\tilde{M}_{\delta 0}$ are the same and reciprocal to that of $\tilde{M}_{\gamma 0}$ one can establish simple selection rules. The fact that the subspace of functions $\tilde{M}_{\alpha 0}$ with definite parity remains invariant

under the operator τ^{-1} has to be taken into account too. The formula (12) allows one to reproduce the well-known results for the dispersion of spin waves in the long-wavelength limit in a very simple manner. Note that for isotropic systems the operators P and Q_0 commute due to symmetry; this fact highly simplifies calculations in this case.

Our further interest will be restricted to eigenvalues $\omega_{\alpha 0}$ and $\omega_{\alpha 2}$; for convenience we will omit the index zero near $\omega_{\alpha 0}$ and $M_{\alpha 0}$; the index 2 near $\omega_{\alpha 2}$ will be preserved. We are going to develop other perturbation procedures, the perturbation order will be now denoted by upper indices.

2. The Perturbation Method for Spin-Orbit Coupling Contributing to the Anisotropy of the g-Factor

In this section we are going to develop a more general approach to the formulated problem than that used in papers /6/ and /9/. Let us define the operator G as P-E, where E is the identity operator. Let us define the function ϕ as the difference between the action of the operator G on the function identically equal to 1 and the constant (1, G1). This fact can be written symbolically as $\phi = G \cdot 1 - (1, G1)$ and hence $(1, \phi) = 0$. Define the kernel $\tilde{G}(\vec{p}, \vec{p}')$ such that

$$\tilde{G}(\vec{p}, \vec{p}') = G(\vec{p}, \vec{p}') - \phi(\vec{p}) - \phi(\vec{p}'). \qquad (15)$$

The integral operator $\tilde{G}$ will be determined by the kernel $\tilde{G}(\vec{p}, \vec{p}')$ in such a way as the operator G is determined by the kernel $G(\vec{p}, \vec{p}')$; $\tilde{P} = E - \tilde{G}$. If the system characterized by the operator P is stable then the system characterized by the operator $\tilde{P}$ is stable too. This means, that if $(M, PM) > 0$ for every function, nonzero in a domain of FS of nonzero measure then also $(M, \tilde{P}M) > 0$, (cf. /11/ and also /12/). Let us prove this statement.

According to basic ideas of Landau's Fermi-liquid approach, the operator G should be determined by comparison with the experiment. Hence, without important loss of generality one can assume that G has a degenerate kernel. It can be written as follows

$$G(\vec{p},\vec{p}\,') = G_{00} + \sum_{i>0} G_{0i}\,(M_i(\vec{p}) + M_i(\vec{p}\,')) + \sum_{i,j>0} G_{ij}\,M_i(\vec{p})M_j(\vec{p}\,'), \qquad (16)$$

with $M_0(\vec{p})=1$, $G_{ij}=G_{ji}$ and $(M_i, M_j)=\delta_{ij}$ for $i,j \geq 0$. The kernel $G(\vec{p},\vec{p}\,')$ can be written in the form of (16) if we put there $G_{0i}=0$ for $i>0$. The system characterized by the operator P will be stable if all principal minors of the matrix $\|\delta_{ij} - G_{ij}\|$, $i,j > 0$ will be positive. From the obtained form of the kernel $G(\vec{p},\vec{p}\,')$ one finds that the system characterized by the operator P will be stable if all principal minors of the matrix $\|\delta_{ij} - G_{ij}\|$, $i,j > 0$ will be positive and $1-G_{0(F}=1-(1,Gl) > 0$. It is clear that these conditions are the consequence of the previous ones and hence we complete our proof.

Now let us pass to the perturbation procedure if the anisotropy of the g-factor is small, analogously as the ϕ-terms in comparison to $\bar{G}$. Let us determine the operator $\bar{Q}$ in the form (2) for $\vec{k}=0$ but with $\overline{g(\vec{p})}$, instead of $g(\vec{p})$, where the bar over g denotes some linear functional. Define the operator Λ as $Q-\bar{Q}$ (the lower index zero near Q will be omitted for simplicity) and Ω as $P-\bar{P}$. Now the functions $\tilde{M}^{\circ}_{\alpha}$ will obey the orthogonality relations (7), but with $\bar{P}$ instead of P. One can verify that the function $\tilde{M}^{\circ}_{0}=[1-(1,Gl)]^{-1/2}$ is the eigenfunction of the zeroth order equation (6) for $\vec{k}=0$, i.e., $\omega^{\circ}_{\gamma}\bar{P}\tilde{M}^{\circ}_{\gamma} = Q\tilde{M}^{\circ}_{\gamma}$ and the eigenvalue

$$\omega^{\circ}_{0} = 2\beta H\bar{g}[1-(1,G1)]^{-1}. \qquad (17)$$

Applying the first order perturbation procedure, slightly more general that the quantum-mechanical ,one finds

$$\omega^{1}_{\alpha} = (\tilde{M}^{\circ}_{\alpha}, \Delta\tilde{M}^{\circ}_{\sigma}) - \omega^{\circ}_{\alpha}(\tilde{M}^{\circ}_{\alpha}, \Omega\tilde{M}^{\circ}_{\alpha}). \qquad (18)$$

For $\alpha=0$ the second term vanished and it is convenient

to choose the linear functional $\overline{g(\vec{p})}$ so that for $a=0$ the first term vanishes too. Hence

$$\overline{g(\vec{p})} = (1,g), \tag{19}$$

and formula (17) becomes identical with that used in [6,9]. Note that $\omega_0^\circ + \omega_0^1$ does not depend on the particular choice of the functional $\overline{g(\vec{p})}$. If the spin-orbit coupling is not important then formula (17) gives the well-known free electron result, i.e., we obtain a proper result in the first (or even zeroth) order of the perturbation method (cf. [13] and also [14]). The first order correction to eigenfunctions has the form

$$M_\gamma^1 = \sum_{\beta(\beta\neq\gamma)} \tilde{M}_\beta^\circ \frac{(\tilde{M}_\beta^\circ, (\Delta - \omega_\gamma^\circ \Omega)\tilde{M}_\gamma^\circ)}{\omega_\gamma^\circ - \omega_\beta^\circ} - \frac{1}{2}(\tilde{M}_\gamma^\circ, \Omega\tilde{M}_\gamma^\circ)\tilde{M}_\gamma^\circ, \tag{20}$$

with the last term vanishing for $\gamma=0$. Let us write also the second order correction to the eigenvalue ω_0°. It has the form

$$\omega_0^2 = \sum_{\beta\neq 0} \frac{|(\tilde{M}_\beta^\circ, [\Delta - \omega_0^\circ \Omega]\tilde{M}_0^\circ)|^2}{\omega_0^\circ - \omega_\beta^\circ}. \tag{21}$$

In proving this formula the relations $(\tilde{M}_0^\circ, \Delta\tilde{M}_0^\circ) = (\tilde{M}_0^\circ, \Omega\tilde{M}_0^\circ) = 0$ were taken into account. For the operator G used in papers [6,9] $\Omega = 0$, but even in this case $\omega_0^2 \neq 0$. Using formulae (18) and (20) one can find the first order correction to the spectrum of spin waves (SW).

3. SW for Nearly Spherical, Cubically Distorted FS

Here we are going to obtain the first order correction with respect to the cubic distortion of FS to the main SW frequency. It will also be assumed that the cubic terms of the effective quasiparticle interaction, as well as the forward scattering amplitude, are of the same order of

magnitude as the distortion. The alkali metals, with the exception of lithium, have to be well described by such a model. The nearly spherical isoenergetic surface can be described by the equation $p = p_0(1 + U(\theta, \phi))$, where p, θ, ϕ denote spherical coordinates in the momentum space, p_0 is the parameter of the surface and $|U(\theta,\phi)| \ll 1$ The function $U(\theta,\phi)$ can be p_0-dependent; without any loss of generality one can assume that the integral of this function over spherical angles vanishes. Hereafter only such functions U will be used. Since p_0 is the parameter of the isoenergetic surface thus, expressing $\epsilon(\vec{p})$ by θ, ϕ and $p = p_0[1 + U(\theta, \phi)]$ one obtains the function which is only p_0-dependent. Note that the functions U were determined for FS of all alkalis by applying the de Haas-van Alphen effect (see the review article by Shoenberg /15/ containing all references to this subject.

Simple application of the differential geometry shows that the element of area of the isoenegetic surface has the form

$$dS = p_0^2(1+2U)d\Omega, \quad d\Omega = \sin\theta d\theta d\phi, \tag{22}$$

if one restricts oneself to the linear terms in U. In the same approximation, the unit vector normal to the isoenergetic surface, is given by

$$\hat{\vec{n}} = [\hat{\vec{n}}_p - \frac{\partial U}{\partial \theta}\hat{\vec{n}}_\theta - \frac{1}{\sin\theta}\frac{\partial U}{\partial \phi}\hat{\vec{n}}_\phi]. \tag{23}$$

In the above formula $\hat{\vec{n}}_p$, $\hat{\vec{n}}_\theta$, $\hat{\vec{n}}_\phi$, denote unit vectors normal to the surface of constant p , θ , ϕ , respectively. In order to express the orientation of these vector let us write $\hat{\vec{n}}_p = \vec{p}/p$, $\hat{\vec{n}}_\phi = [-\sin\phi, \cos\phi, 0]$ and $\hat{\vec{n}}_\theta = \hat{\vec{n}}_\phi \times \hat{\vec{n}}_p$. Let us assume that $V_{\vec{p}} = V[1 + q(\theta,\phi)]$, where q is of the same order of magnitude as U and the integral of q over $d\Omega$ vanishes. We have with our accuracy

$$V_{\vec{p}} = V_{\vec{p}}\hat{\vec{n}} \approx V(1+q)\hat{\vec{n}}_p - V[\frac{\partial U}{\partial \theta}\hat{\vec{n}}_\theta + \frac{1}{\sin\theta}\frac{\partial U}{\partial \phi}\hat{\vec{n}}_\phi]. \tag{24}$$

Taking into account (22) one finds that the electron density $\rho = p_F^3/3\pi^2 (\hbar = 1)$, where p_F is the parameter of FS. With the same accuracy, using (22) one can find that the density of states per unit volume, ν, is given by $p_F^2/V\pi^2$. The function q can be determined by U and the p_0-derivatives of U and V. This expression is given in the Appendix, as well as the expression for the cyclotron mass, complementary to that obtained in [16]. Taking into account formula (24) one can write

$$\frac{ie}{c}(\vec{H}\times\vec{V}_{\vec{p}})\,\vec{V}_{\vec{p}} = -\frac{eV}{cp_0}(1+q-U)(\vec{HL}) + \tag{25}$$

$$+\frac{ieHV}{cp_0}\cdot[\sin\alpha\cos(\beta-\phi)+\operatorname{ctg}\theta\cos\alpha]\left[\frac{\partial U}{\partial\phi}\frac{\partial}{\partial\theta}-\frac{\partial U}{\partial\theta}\frac{\partial}{\partial\phi}\right],$$

where α,β are spherical angles determining the direction of H, with β being the azimuthal angle and $L=-i(p\times\nabla_p)$. It is convenient to apply (25) if α,β are spherical angles in the frame of reference of 4 – fold axes of a cube. Another simplifying choice $\alpha=0$, when ϕ becomes the azimuthal angle about the H-axis, leads to a very complicated form of cubic invariants appearing in the functions U and q. The term $-eV(\vec{HL})/cp_0$ in (25) should be treated as the unperturbed term, the remaining ones as the perturbation terms. Let us pass to the operator P. According to [17], let us write $g(\hat{\vec{p}},\hat{\vec{p}}') = \langle G(\hat{\vec{p}},\hat{\vec{p}}')\rangle$, in the following form

$$g(\hat{\vec{p}},\hat{\vec{p}}') = \sum_{\lambda}\sum_{\ell=0}^{\infty}\sum_{a=1}^{n_\lambda^{(\ell)}}\sum_{b=1}^{n_\lambda^{(\ell')}}[(g_\ell\delta_{\ell\ell'}\delta_{ab}+\Lambda g_{\ell\ell'}^{\lambda ab})\times \tag{26}$$

$$\times\sum_{k=1}^{d_\lambda}\lambda_{\ell k}^{a}(\hat{\vec{p}})\lambda_{\ell' k}^{b}(\hat{\vec{p}}')] = g(\hat{\vec{p}}\hat{\vec{p}}')+\Lambda g(\hat{\vec{p}},\hat{\vec{p}}').$$

Here $\lambda_{\ell k}^{a}(\hat{\vec{p}})$ is the function constructed out of $Y_{\ell m}(\hat{\vec{p}})$, $-\ell\le m\le\ell$ and transfroming as the k-th row of the λ-th irreducible representation of the group O_h, the

variables a,b describe the degeneration of the cubic harmonics (cf. [18]). The function λ are chosen so that

$$\int \frac{d\Omega}{4\pi} \lambda^{a}_{\ell k}(\hat{\vec{p}}) \mu^{b}_{\ell' k'}(\hat{\vec{p}}) = \delta_{\lambda\mu} \delta_{ab} \delta_{\ell\ell'} \delta_{kk'}. \qquad (27)$$

If all $\Delta g = 0$ then (26) reduces to the sum over ℓ of $(2\ell+1) g_\ell P_\ell(\hat{\vec{p}}\hat{\vec{p}}')$; [17] denoted by $g(\hat{\vec{p}}\hat{\vec{p}}')$. Hence, without any loss of generality, one can assume that the sum of $\Delta g^{\lambda}_{\ell\ell'}{}^{a}$ over a and λ venishes. Taking into account that

$$dS_F / 4\pi^3 v_{\vec{p}} \nu \simeq (1 + 2U - q)\, d\Omega / 4\pi \qquad (28)$$

one can rewrite the operator P(3), with the accuracy up to linear terms with respect to distortion, as follows

$$P\tilde{M}(\hat{p}) = \tilde{M}(\hat{p}) - \int \frac{d\Omega'}{4\pi} g(\hat{p}\hat{p}') \tilde{M}(\hat{p}') - \int \frac{d\Omega'}{4\pi} [\, g(\hat{p}\hat{p}')(2U(\hat{p}') - q(\hat{p}')) + \Lambda g(\hat{p}, \hat{p}')] \tilde{M}(\hat{p}'). \qquad (29)$$

In order to preserve an analogy with the previous section, let us denote the first two terms of operator (29) by $\bar{P}$, whereas the small third term by Ω. An analogous notation will be used for the operator Q, too. If one assumes that the spin-orbit souping is irrelevant for our system then, substituting $\tilde{M}(\vec{p}) = 2$ into (29), one has to obtain the elecron g-factor. Denoting the expansion coefficients of the function $2\mu(\hat{\vec{p}}) - q(\hat{\vec{p}})$ over the invariants of the cubic group, $a^{a}_{\ell}(\vec{p})$, by $C_{a\ell}$ one finds from (29) for $\tilde{M}(\vec{p}) = 2$

$$g(\hat{\vec{p}}) = 2\,[\,1 - g_0 - \sum_{\ell=4}^{\infty} \sum_{a=1}^{n_a(\ell)} (g_\ell C_{a\ell} + \Lambda g^{a\,a\,1}_{\ell 0}) a^{a}_{\ell}(\hat{\vec{p}})\,]. \qquad (30)$$

In proving formula (30), the addition theorem for crystal harmonics [17], formulae (26), (27) and the information about invariants with lowest ℓ were used. According to formula (30) g is anisotropic unless $g_\ell C_{a\ell} + \Lambda g^{a\,a\,1}_{\ell 0} = 0$ for all ℓ and a.

Let us turn now to the perturbation method for equation (6) for $\vec{k} = 0$. The perturbation terms will be i) just defined

operator Ω and, ii) the operator Λ being the sum of the perturbation terms of operator (25) and operator $\beta H(g(\hat{p})-2+2g_0)$ (30). One can see easily that formula (18) for the first order correction to ω_α° still remains valid, but with the scalar product defined as in the spherically symmetric case, i.e.,

$$(\Psi, \sigma) = \int \frac{d\Omega}{4\pi} \Psi^*(\hat{\vec{p}})\, \sigma(\hat{\vec{p}}) . \qquad (31)$$

Here $\alpha \to \ell m$ and, in the reference frame such that ϕ is the azimuthal angle about the H-axis, $\tilde{M}_\alpha^\circ \sim Y_{\ell m}$ and hence

$$\omega_{\ell m}^\circ = [2\beta H(1-g_0) - eHVm/cp_0](1-g_\ell)^{-1}, \quad -\ell \le m \le \ell, \qquad (32)$$

(cf. /4/). In order to express these functions in the reference frame of 4-fold axes of a cube one has to apply the transformation given by the matrices $D^\ell_{mm'}$. Denoting the spherical harmonics in this last reference frame by $N_{\ell m}$, $(N_{\ell m}, N_{\ell m'}) = 1$, one finds from the proper orthonormality relations, $(\tilde{M}_\alpha^\circ, \bar{P}\tilde{M}_\beta^\circ) = \delta_{\alpha\beta}$ that $\tilde{M}_{\ell m}^\circ(\hat{\vec{p}}) = (1-g_\ell)^{-1/2} N_{\ell m}(\hat{\vec{p}})$.

According to the results of papers /13/ and /14/ $\omega_\alpha = 2\beta H$ for $\tilde{M}_\alpha = \text{const}$, i.e., for $\ell = 0$, $m = 0$, and hence $\omega_{00}^1 = 0$. Applying the recurrence properties of spherical harmonics and taking into account that the first nontrivial cubic invariant appears for $\ell = 4$ one can show that all contributions to ω_{1m}^1 vanish with the possible exception of the last term appearing in (25), being the part of the operator Δ. Denoting the last term of (25) by R, one can write $\omega_{1m}^1 = (1-g_1)^{-1}(N_{1m}, RN_{1m})$. Putting here $\alpha = 0$, integrating by parts, so that the function U will be free of derivatives after the integration, one can show by means of the mentioned methods that $\omega_{1m}^1 = 0$. Let us investigate $\omega_{00,z}^1$ i.e., the first order correction to the spectrum of long-wavelength spin waves based on the usual spin resonance. After rather simple calculations, using the above results, one obtains

$$\omega^{1}_{00,2} = 2k\mathrm{V}\mathrm{Re}\sum_{m=-1}^{1}\{[(2\beta H - \omega^{\circ}_{1m})(1-g_0)(1-g_1)]^{-1}\ N^{*}_{1m}(\hat{k}) \times$$

$$\times[(\vec{k}\vec{V}_1, N_{1m}) + (1-g_0)^{1/2}(\tilde{M}^{1}_{00,}(\vec{k}\vec{V}_0)N_{1m}) + (1-g_1)^{1/2}(\vec{k}\vec{V}_0, \tilde{M}^{1}_{1m})]\}. \tag{33}$$

Here $\vec{V}_0(\hat{p}) = V\hat{p}$, $\vec{V}_1(\hat{p}) = \vec{V}(\hat{p}) - \vec{V}_0(\hat{p})$ and the functions $N_{1m}(\hat{k})$ can be written, in the invariant form, as follows: $N_{1m}(\hat{k}) = \sqrt{3}(\vec{b}_m\hat{k})$ with $\vec{b}_0 = \hat{H}$ and $\vec{b}_m = 2^{-1/2}[\hat{a} + im(\hat{H}\times\hat{a})]$ for $m \neq 0$, where $(\vec{a}\vec{H}) = 0$. The function $\tilde{M}^{1}_{\ell m}$ denotes the first order correction, with respect to the distortion of FS, to the function $\tilde{M}^{\circ}_{\ell m}$. It can be obtained easily that $\tilde{M}^{1}_{00} = 0$. On the other hand, $\tilde{M}^{1}_{1m}$ can be written in a form very similar to (20) where, in the last term, the operator Ω should be replaced by $\Omega + C\bar{P}$ with C being the operator of multiplication by the function $2U(\vec{p}) - q(\vec{p})$. One can simply deduce, using considerations identical with previous ones, that the analog of the last term in (20) vanishes for $\ell = 1$. Moreover, only the terms of $\tilde{M}^{1}_{1m}$ proportional to $\tilde{M}^{\circ}_{1m'}$, $-1 \leq m' \leq 1$, $m' \neq m$ can contribute to the scalar product $(\vec{k}\vec{V}_0, \tilde{M}^{1}_{1m})$ appearing in (33). One can simply show that $(\tilde{M}^{\circ}_{1m}, \Omega\tilde{M}^{\circ}_{1m'}) = 0$ and $(\tilde{M}^{\circ}_{1m}, \Lambda\tilde{M}^{\circ}_{1m'}) = (\tilde{M}^{\circ}_{1m}, R\tilde{M}^{\circ}_{1m'})$, where R denotes the last term of the operator (25). Hence, one can write

$$(1-g_1)^{\frac{1}{2}}(\vec{k}\vec{V}_0, \tilde{M}^{1}_{1m}) = \frac{cp_0k}{cH}\sum_{m'=-1}^{1}{}_{(m'\neq m)}\frac{N_{1m'}(\hat{k})(N_{1m'}, RN_{1m})}{m'-m}. \tag{34}$$

Applying to nondiagonal elements of the R-operator the same procedure as previously to its diagonal elements (i.e., for ω^{1}_{1m}) one can show that all elements of the operator R vanish in the subspace with $\ell = 1$ and that (34) vanishes too. Hence, the single term contributing to $\omega^{1}_{00,2}$ is connected with $(\vec{k}\vec{V}_1, N_{1m})$, i.e., with the

deviation of the velocity vector from the spherical symmetry. The result that the anisotropic part of the effective quasiparticle interaction does not contribute to $\omega^{1}_{00,2}$ in our approximation is not surprising (cf. with the estimation in the paper /19/). Using the formula (24), by means of the integration by parts and some application of the angular momentum methods, one can write

$$(\vec{k}\vec{V}_l, N_{lm}) = kV \int \frac{d\Omega}{4\pi} U \{ \frac{1}{\sin\theta} \frac{\partial}{\partial\phi} [N_{lm} \sin\gamma \sin(\delta-\phi)] + \tag{35}$$

$$+ N_{lm} \operatorname{ctg}\theta [\sin\gamma \cos\theta \cos(\delta-\phi) - \cos\gamma \sin\theta] \},$$

where γ, δ denote spherical angles of the $\vec{k}$ -vector in the reference frame of 4-fold aces of a cube with δ being the aximuthal angle. Note that $N_{lm}(\vec{p})$ in (35) should be expressed in the following form

$$N_{10}(\hat{p}) = \sqrt{3} [\sin\alpha \sin\theta \cos(\phi-\beta) + \cos\alpha \cos\theta] , \tag{36}$$

$$N_{l,\pm1}(\hat{p}) = \sqrt{\frac{3}{2}} [\cos\alpha \sin\theta \cos(\phi-\beta) - \sin\alpha \cos\theta \pm i \sin\theta \sin(\phi-\beta)],$$

whereas for $N_{lm}(\hat{k})$ appearing in (33), the angles θ, ϕ should be replaced by γ, δ . If one expresses the function U in its usual form, i.e., as the finite sum of cubic invariants, then integral (35) can be calculated without any important difficulty for a few first l. The integration result will be an elementary function which cannot be written in a compact form and this is a reason why we do not write down these integrals. It is interesting that in formula (35) the function q disappears. This means that de Haas-van Alphen measurements are sufficient for determination of the correction term $\omega^{1}_{00,2}$. Moreover, only the transversal deviation of the vector $\vec{V}_{\vec{p}}$, appearing in the combination $(\vec{k}\vec{V}_{\vec{p}})$ in formula (12) is effective for $\omega^{1}_{00,2}$.

Appendix

Let us start with the proof of the mentioned above identity for the function q. Applying the rotation taken in the spherical variables to vector (24) one gets

$$\frac{\partial q}{\partial \psi} = -\left(1 + 3\,\frac{\partial \ln V}{\partial \ln \rho_0}\right)\frac{\partial U}{\partial \psi} - 3\,\frac{\partial^2 U}{\partial \psi\, \partial \ln \rho_0}\,, \qquad (37)$$

where ψ denotes the variable θ or ϕ and $\rho_0 = p_0^3/3\pi^2$ is the density of electrons occupying a domain in the momentum space with the parameter of isoenergetic surface smaller than p_0. If the functions U and q, integrated over $d\Omega$ give zero, then from (37) one gets

$$q = -u\left(1 + 3\,\frac{\partial \ln V}{\partial \ln \rho_0}\right) - 3\,\frac{\partial U}{\partial \ln \rho_0}\,, \qquad (38)$$

and we have our identity.

Let us pass the formula for cyclotron mass m^* for nearly spherical isoenergetic surfaces. In contrast to the formula obtained in /16/ we will express m^* only by local quantities, i.e., by U and q for given p_0. We have (cf. /2/)

$$m^* = \frac{1}{2\pi}\int \frac{dp_\perp}{v_\perp(\vec{p})}\,, \qquad (39)$$

where the integral is taken over the length element $dp_\perp$ of the curve $[\epsilon(\vec{p}) = \epsilon, \vec{H}\vec{p} = p_H = \text{const}]$ and $v_\perp^2(\vec{p})\,V_p^2 = (V_p \times \hat{H})^2$. Simple application of the differential geometry shows that

$$dp_\perp = p_0\,(1+U)\,\sin\theta\,d\phi\,. \qquad (40)$$

Integral (39) is taken for $p_H = \text{const}$. If θ is the angle between the vectors $\vec{H}$ and $\vec{p}$ then this condition is equivalent to $\theta = \text{const}$ only the zeroth approximation

with respect to U. Taking into account that $p_0(1+U)\cos\theta = p_{\vec{H}}$ and putting $\theta = \theta_0 + \theta_1$ we find that

$$\cos\theta_0 = p_{\vec{H}} / p_0 , \quad \theta_1 = U \ \mathrm{ctg}\theta_0 , \tag{41}$$

where in U the variable is put equal to θ_0. Substituting θ of the above form into (40) we get

$$d p_{\perp} = p_0 \sin\theta_0 [\, 1 + U / \sin^2\theta_0 \,]\, d\phi . \tag{42}$$

Note that this approximation is valid unless $\sin\theta_0 \ll 1$. Using formula (24) we can find

$$V_{\perp}^{-1}(p) = \frac{1}{V \sin\theta_0} [1 - q - U \ \mathrm{ctg}^2\theta_0 + \mathrm{ctg}\,\theta \ \frac{\partial U}{\partial \theta_0} \ ] . \tag{43}$$

Substituting (42) and (43) into (39) we find

$$m^* = \frac{p_0}{V} [\, 1 + (1 + \mathrm{ctg}\,\theta \frac{\partial}{\partial\theta}) \frac{1}{2\pi} \int_0^{2\pi} U \ d\phi - \frac{1}{2\pi} \int_0^{2\pi} q \, d\phi], \tag{44}$$

where the subscript zero near θ was omitted. The integration in (44) goes over the azimuthal angle about $\vec{H}$-axis. The cyclotron mass depends on θ and on the direction cosines of the axis H. Formula (44) is valid unless $\sin\theta \ll 1$. If $\sin\theta = 0$ (i.e., at the lunetary point) then, according to /2/, one can express m^* by the Gauss curvature of FS, K, and $V_{\vec{p}}$. We have

$$m^* = K^{-1/2} V_{\vec{p}}^{-1} = \frac{p_0}{V} [\, 1 + U - q \,] . \tag{45}$$

Substituting U and q expanded in series of cubic invariants into (44) and applying our results for the azimuthal angle integrals of cubic invariance /12/ we get

$$m^* = \frac{p_0}{V} \{ 1 + \sum_{k=2} \sum_{a=1}^{n_\alpha(2k)} \alpha_{2k}^{a} (\alpha, \beta) \times \times [\, U_{2k}^{a} \mathrm{ctg}\,\theta \, P_{2k}^{1}(\cos\theta) + (U_{2k}^{a} - q_{2k}^{a}) P_{2k}(\cos\theta) \,] \} , \tag{46}$$

where U_{2k}^{a} and q_{2k}^{a} denote the expansion coefficients of

the functions U and q over α_{2k}^{a} and α, β denote the spherical angles determining the orientation of the vector $\vec{H}$ with respect to 4-fold axes of a cube. As one can verify, for $\theta = 0$ or π (46) turns into (45).

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