сообщения
объединенного
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ядерных
исследований
дубна

E4-85-934

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ALPHA-LIKE FOUR NUCLEON CORRELATIONS IN THE SUPERFLUID PHASE OF ATOMIC NUCLEI

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1985

1. Introduction

Recently the Fermi liquid model for alpha clusterization and alpha decay has been proposed /1/ and tested /2-4/ on different alpha transitions. This model has been born as a result of a comprehensive analysis /5/ of the current alpha decay models and introduces a four particle interaction for the irreducible reaction amplitude of the alpha cluster formation in the four particle channel, based on the prescriptions resulted from the Landau-Migdal /6/ theory of quantum liquids.

A rather good description of different alpha transitions (both favoured and unfavoured) of light and heavy nuclei in the framework of this model and the fact that the alpha clusterization process is enhanced for nuclei existing in the superfluid phase as compared to those in the normal phase lead us to the idea that may be the four-nucleon correlations could have an important contribution to the structure of superfluid nuclei.

The Fermi liquid model four-nucleon interaction is perhaps that interaction leading to the condensation of two protons and two neutrons into the α-clusters. It is important to recognize that an α-particle cluster in the nucleus is not the same as a free α-particle. It is naturally distorted by the fields of the surrounding nucleons and may be violently changed by close interactions between them. By α-clusters one should probably understand an agregate formed from two protons and two neutrons with strong spatial correlations between them, such that the spatial localization of this agregate to be (much) smaller than the spatial localization of the nucleus itself.

Another type of α-agregate determined by the Fermi liquid model four-nucleon interaction may be formed from two correlated Cooper pairs (one proton pair and one neutron pair). Let us christen this type of agregate- the α-superfluid agregate. This agregate surely has weaker spatial correlations as compared to α-clusters, but stronger correlations in the angular momentum space. This α-superfluid agregate in coordinate space is spread throughout the nucleus in the same way as the Cooper pairs of a superconductor. We shall show that the α-superfluid agregates are more probable present in a larger class of atomic nuclei as compared to the α-clusters and they play an important role in the α-clusterization process and also in the clusterization process of heavier clusters.

Similar correlations leading to an α-like "condensate" have been recently[7] proposed in the framework of the interacting boson model (IBM) with the aim of describing the "α-clusters" in nuclei as a bound state of two bosons, each boson corresponding to a pair of nucleons. Although very interesting and of broader relevance (inspired from recent investigations[8,9] on the physics of biexcitons in semiconductors) the IBM greatly cancels the mechanism of formation of the α-agregates by approximating the condensed pairs of nucleons by bosons. It is known[8,9] that such an approximation is only valid for weakly interacting fermion systems in low-density regime, a condition which is far to be met in the nuclear systems. The underlying fermionic structure of the paired bosons certainly plays an important role in the formation of α-agregates in nuclei, a role which the IBM fails to take care of. On the other hand the odd-even staggering of the experimental (average) one pair binding energies brought up[7] as evidence for the α-like condensate in nuclei includes to a large extent the effect of the nuclear symmetry energy also[10] and it is difficult to assess[11] at this level how much of the staggering amplitude comes exclusively from the α-like correlations. In this respect it would be desirable to have a more nonambiguous quantitative measure of these α-like correlations in nuclei.

The underlying fermionic structure of the bosons ignored in the IBM-approach[7] seems to be of great importance as far as genuine four-nucleon correlations are looked for. Such an investigation raises the question of whether a condensed state could directly be obtained by starting with purely fermionic Hamiltonian and incorporating the four-nucleon correlations just at the out set. Doing so the ensuing condensed state might be viewed as corresponding to genuine four fermion correlations rather to a bound state of two already condensed pairs of fermions.

Early attempts[12-18] to account for α-superfluid agregates in nuclei as arising from four-nucleon correlations use a trial wave function with such correlations included, thereby simulating the four-fer-

mion condensate, the interaction remaining of the two-fermion type.

A different point of view is assumed in the present work by using the well-known BCS-like pairing wave function and accounting for four-fermion correlations by two pair (proton and neutron) correlations. It is shown that, within this simple pairing approximation, the four fermion correlations lead to a condensed state of the Fermi gas model which consists of correlated fermion pairs. Such point of view may get a support by using the Green's function method[5,6]. Some preliminary results using the mean field approach[19] and BCS approach[20,21] have been reported.

The paper is organized as follows. In section II we consider the general formulation of the problem. The discussion of the gap equations is presented in section III. In section IV is presented the procedure for extracting the pairing- and α-like-correlations strengths from the experiment. In section V is presented the enhancement factor of the two-nucleon-, α-transfer reactions and α-decay. In the last section we present the conclusions.

II. Outline of the Model

We consider a system of nucleons (protons and neutrons) which are moving in a certain axially symmetric self-consistent well as, e.g., a deformed Saxon-Woods one[22]. As basic functions of the second quantization representation we choose the wave functions of a nucleon in this well. States which differ only in the sign of the projections of angular momentum along the symmetry axis, are degenerate and conjugate with respect to the time reversal operation[23,24].

The Hamiltonian for the system of interacting nucleons is

$$H = H_{op} + H_{on} + H^{(p)}_{pair} + H^{(n)}_{pair} + H_4, \tag{1}$$

where

$$H_{op} = \sum_{\nu\tau} E_\nu a^+_{\nu\tau} a_{\nu\tau} \tag{2}$$

$$H_{on} = \sum_{\omega\sigma} E_\omega a^+_{\omega\sigma} a_{\omega\sigma} \tag{3}$$

$$H^{(p)}_{pair} = -G_Z \sum_{\nu\nu'} b^+_\nu b_{\nu'} \tag{4}$$

$$H^{(n)}_{pair} = -G_N \sum_{\omega\omega'} b^+_\omega b_{\omega'} \tag{5}$$

$$H_4 = -G_4 \sum_{\nu\nu'} \sum_{\omega\omega'} b^+_\nu b^+_\omega b_{\omega'} b_{\nu'}. \tag{6}$$

Here $a^+_{\nu\tau}$ ($a_{\nu\tau}$) and $a^+_{\omega\sigma}$ ($a_{\omega\sigma}$) are the Fermi operators which create (destroy) the nucleon in (from) the single particle state ($|\nu\tau\rangle$ and $|\omega\sigma\rangle$), τ and σ are the signs of the projections of the nucleon (proton and neutron) angular momenta onto the nuclear symmetry axis, ν and ω being the residual quantum numbers that label the one-particle (proton and neutron) energy levels. The proton and neutron pair operators are here denoted by b_ν and b_ω, respectively, where

$$b_s = a_{s-}a_{s+}. \qquad (7)$$

The last term (6) in the eq.(1) is an effective, coherent, two-pair (four nucleon) interaction term, which is expected to induce the α-like four nucleon correlations in the superfluid nucleus. The other terms in the eq.(1) describe[24] the usual BCS-superfluidity. The G_Z, G_N, G_4 - quantities are positive-valued coupling strengths, nonvanishing within a certain energetical range - the cut-off energy range. The summation in the eqs.(2-6) is taken over distinct nucleon levels belonging to the cut-off energy range.

According to our above-mentioned particular outlook at the problem at hand we use the BCS trial wave function:

$$|BCS\rangle = \prod_\nu (u_\nu + v_\nu b_\nu^+) \prod_\omega (u_\omega + v_\omega b_\omega^+) |0\rangle . \qquad (8)$$

where $u^2_{\nu(\omega)} + v^2_{\nu(\omega)} = 1$, and $|0\rangle$ denotes the absolute vacuum, to get the BCS-energy functional in the first approximation (dropping out, for example, as usually, the self-consistent field corrections):

$$\begin{aligned} W &= \langle BCS | H - \lambda_p \hat{Z} - \lambda_n \hat{N} | BCS \rangle \\ &= \sum_\nu 2(E_\nu - \lambda_p) v_\nu^2 + \sum_\omega 2(E_\omega - \lambda_n) v_\omega^2 \\ &\quad - G_Z \chi_p^2 - G_N \chi_n^2 - G_4 \chi_p^2 \chi_n^2 , \end{aligned} \qquad (9)$$

where $\lambda_{p(n)}$ denotes the proton (neutron) chemical potential, $\hat{N}$ is the proton (neutron) number operator and

$$\chi_{p(n)} = \langle BCS | \sum_{\nu(\omega)} b^+_{\nu(\omega)} | BCS \rangle = \sum_{\nu(\omega)} u_{\nu(\omega)} v_{\nu(\omega)} \qquad (10)$$

are the pairing correlation functions.

The minimization of W from eq.(9) leads to the following gap equations

$$\begin{aligned} &\tfrac{1}{2}(G_Z + G_4 \chi_n^2) f_p(\Delta_p, \lambda_p) = 1 \\ &\sum_\nu \left(1 - \frac{E_\nu - \lambda_p}{\mathcal{E}_\nu}\right) = Z \\ &\tfrac{1}{2}(G_N + G_4 \chi_p^2) f_n(\Delta_n, \lambda_n) = 1 \\ &\sum_\omega \left(1 - \frac{E_\omega - \lambda_n}{\mathcal{E}_\omega}\right) = N \end{aligned} \qquad (11)$$

for doubly even nuclei and

$$\begin{aligned} &\tfrac{1}{2}(G_Z + G_4 \chi_n^2) \tilde{f}_p(\Delta_p, \lambda_p) = 1 \\ &1 + \sum_{\nu \neq \nu_0} \left(1 - \frac{E_\nu - \lambda_p}{\mathcal{E}_\nu}\right) = Z \\ &\tfrac{1}{2}(G_N + G_4 \tilde{\chi}_p^2) f_n(\Delta_n, \lambda_n) = 1 \\ &\sum_\omega \left(1 - \frac{E_\omega - \lambda_n}{\mathcal{E}_\omega}\right) = N \end{aligned} \qquad (12)$$

for odd proton and even neutron nuclei.

Here

$$f_{p(n)}(\Delta_{p(n)}, \lambda_{p(n)}) = \sum_{\nu(\omega)} \frac{1}{\mathcal{E}_{\nu(\omega)}} \; ; \; \tilde{f}_{p(n)}(\Delta_{p(n)}, \lambda_{p(n)}) = \sum_{\nu(\omega) \neq \nu_0(\omega_0)} \frac{1}{\mathcal{E}_{\nu(\omega)}} \qquad (13)$$

$$\mathcal{E}_{\nu(\omega)} = \left[(E_{\nu(\omega)} - \lambda_{p(n)})^2 + \Delta_{p(n)} \right]^{1/2} \qquad (14)$$

$$u^2_{\nu(\omega)} = \frac{1}{2}\left(1 + \frac{E_{\nu(\omega)} - \lambda_{p(n)}}{\mathcal{E}_{\nu(\omega)}}\right) \; ; \; v^2_{\nu(\omega)} = \frac{1}{2}\left(1 - \frac{E_{\nu(\omega)} - \lambda_{p(n)}}{\mathcal{E}_{\nu(\omega)}}\right) \qquad (15)$$

and

$$\tilde{\chi}_{p(n)} = \sum_{\nu(\omega) \neq \nu_0(\omega_0)} u_{\nu(\omega)} v_{\nu(\omega)} . \qquad (16)$$

For odd neutron and even proton nuclei the gap equations are analogous. $Z(N)$ is the number of protons (neutrons) belonging to the cut-off energy range. The gap equations for an odd system have been obtained by minimizing the quantity

$$W = E_{\nu_0} - \lambda_p + \sum_{\nu \neq \nu_0} 2(E_\nu - \lambda_p) V_\nu^2 + \sum_{\omega} 2(E_\omega - \lambda_n) V_\omega^2$$
$$- G_Z \tilde{X}_p^2 - G_N X_n^2 - G_4 \tilde{X}_p^2 X_n^2 \qquad (17)$$

similar to that from the eq.(9), where we have taken care of the blocking effect.

With these solutions we may describe as, e.g., in ref. /24/ the low-lying excited states of superfluid deformed nuclei also.

III. Discussion of the Solutions of the Gap Equations

To understand which are the conditions that the coupling strengths whould fulfil we analyse an instructive schematic two-level model with two nucleons. We assume to deal with one sort of nucleons, i.e.,

$$G_Z = G_N = G_2 \qquad (18)$$
$$\Delta_p = \Delta_n = \Delta$$
$$\lambda_p = \lambda_n = \lambda$$

than we use the following notations

$$\frac{2}{\rho} = E_2 - E_1 = 2|E_2 - \lambda| = 2|E_1 - \lambda|$$
$$x = \rho\Delta \qquad (19)$$
$$g_2 = \rho G_2$$
$$g_4 = \rho G_4 .$$

The gap equation becomes:

$$F(x) \equiv \left(g_2 + g_4 \frac{x}{1+x}\right) \frac{1}{\sqrt{1+x}} = 1 \qquad (20)$$

With $F(0) = g_2$ and $F(\infty) = 0$ and the correlation energy has the form

$$E_{corr} = W(x) - W(0) =$$
$$= \frac{1}{\rho}\left[4\left(1 - \frac{1}{\sqrt{1+x}}\right) - 2g_2 \frac{x}{1+x} - g_4 \left(\frac{x}{1+x}\right)^2\right]. \qquad (21)$$

where the energy W has been defined in the eq.(9).

The function $F(x)$ from (20) has a maximum at

$$x_m = \frac{2g_4 - g_2}{g_4 + g_2} \qquad (22)$$

and

$$F(x_m) = \frac{2}{3\sqrt{3g_4}} (g_2 + g_4)^{3/2} . \qquad (23)$$

If

$$g_4 < \frac{1}{2} g_2 \qquad (24)$$

$\frac{dF}{dx} < 0$ and the eq.(20) has a unique solution if $g_2 > 1$ and no solutions for $g_2 < 1$ and if

$$g_4 > \frac{1}{2} g_2 \qquad (25)$$

$\frac{dF}{dx} > 0$ for $x < x_m$ and $\frac{dF}{dx} < 0$ for $x > x_m$. In this case we distinguish three cases:

$$g_2 > 1 \qquad (26)$$

the eq.(20) has an unique solution,

$$g_2 < 1 < F(x_m) \qquad (27)$$

the eq.(20) has two solutions and

$$F(x_m) < 1 \qquad (28)$$

the eq.(20) has no solutions.

Let us analyse the phase diagram g_2 versus g_4 (figure). The critique curve is

$$F(x_m) = 1 \qquad (29)$$

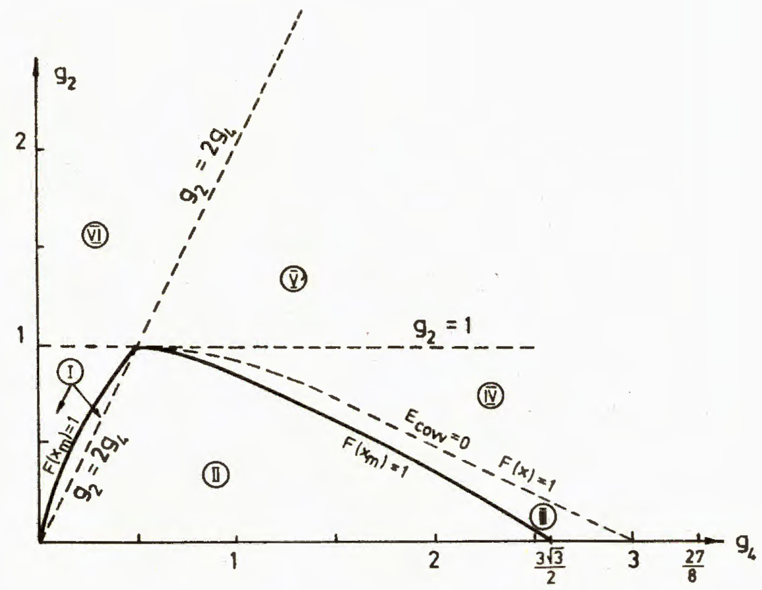
or

$$g_2 = 3 \cdot 2^{-\frac{2}{3}} g_4^{\frac{1}{3}} - g_4 . \qquad (30)$$

The space between this curve and $g_2 = 0$ coincides to the case (28), i.e., we do not have the nontrivial solutions, i.e. $\Delta = 0$.

The space determined by the intersection of the following regions

$$g_2 < 1 \quad ; \quad g_4 > 0 \quad ; \quad g_2 > 2 g_4 .$$



The g_2 versus g_4 phase diagram in the framework of the two level with two-nucleons model. The regions I and II correspond to the normal fluid nuclear matter, the region IV, V and VI correspond to the most stable superfluid nuclear matter and the region III correspond to the metastable superfluid nuclear matter. The transitions I → VI correspond to the second order phase transition and the other ones - to the first order phase transition.

correspond to the case (24) with no nontrivial solutions, i.e. $\Delta = 0$ too.

Analysing the second derivative of the correlation energy (21) we find that for $g_2 > 1$ (the case (26)) the unique solution correspond to the minimum of this energy. In the remaining space determined by the intersection of the following regions

$$g_2 > 3.2^{-\frac{2}{3}} g_4^{\frac{1}{3}} - g_4 \; ; \quad g_2 < 1 \; ; \quad g_4 > 0$$

that correspond to the case (27) we have two solutions, the greatest in the right of the dashed line (defined by $E_{corr}(x) = 0$ (see eq.(21)) and the gap eq.(20), corresponds to the minimum of the correlation energy (21) and the smallest (in the left of the dashed line) - to the maximum of the correlation energy (21).

It should be mentioned here that for the more realistic case studied in section II with many levels and nucleons, we may find one, two, three and no nontrivial solutions and these solutions should be analyzed in a three-dimensional phase diagram, which determine at least three phases of nuclear matter

1) normal fluid phase $\Delta = 0$ (the regions I and II in the figure),
2) superfluid phase, in which we have an unique $\Delta \neq 0$ solution corresponding to the deepest minimum (the regions IV, V, VI in the figure),
3) a superfluid metastable phase, where $\Delta_i \neq 0$ correspond to other minima or maxima of the correlation energy (the region III in the figure).

One should say that the phase transition I → VI is of the second order because the gap and the correlation energy have a smooth transition from the normal fluid matter to the superfluid matter, while the phase transition II → III, IV, V is of the first order, i.e., Δ and E_{corr} suffer a jump.

IV. Determination of the Coupling Strengths from the Experiment

To fix the coupling strengths G_Z, G_N and G_4 from the experiments we use the well-known odd-even mass defference

$$P_Z = \frac{1}{2}\left\{ 2\mathcal{E}(Z-1,N) - \mathcal{E}(Z,N) - \mathcal{E}(Z-2,N)\right\} \quad (31)$$

$$P_N = \frac{1}{2}\left\{ 2\mathcal{E}(Z,N-1) - \mathcal{E}(Z,N) - \mathcal{E}(Z,N-2)\right\} \quad (32)$$

for G_Z and G_N and

$$P_4 = \mathcal{E}(Z,N) - \mathcal{E}(Z-2,N-2) - \mathcal{E}(Z+1,N) + \mathcal{E}(Z-1,N) - \mathcal{E}(Z,N+1) + \mathcal{E}(Z,N-1) \quad (33)$$

for G_4.

Here

$$\mathcal{E}(Z,N) = \sum_{\nu} 2E_\nu V_\nu^2 + \sum_{\omega} 2E_\omega V_\omega^2 - G_Z \chi_p^2 - G_N \chi_n^2 - G_4 \chi_p^2 \chi_n^2 \quad (34)$$

for a doubly even nucleus and

$$\mathcal{E}(Z+1,N) = E_{\nu_0} + \sum_{\nu\neq\nu_0} 2E_\nu V_\nu^2 + \sum_\omega 2E_\omega V_\omega^2 - $$
$$- G_Z \tilde{\chi}_p^2 - G_N \chi_n^2 - G_4 \tilde{\chi}_p^2 \chi_n^2 \qquad (35)$$

for an odd-mass one.

The experimental P_Z, P_N and P_4 quantities are obtained from (31-33) by replacing $\mathcal{E}$ by $-B$, B being the binding energy.

The P_4 -quantity has been chosen in the same spirit as the analogous quantity for pairing vibrations has been chosen in ref. /25/, i.e.:

$$P_4 = \mathcal{E}(Z,N) - \mathcal{E}(Z-2,N-2) - 2\lambda_p - 2\lambda_n . \qquad (36)$$

In the first approximation this quantity is determined by

$$P_4 \cong \Delta_p(G_4\neq 0) - \Delta_p(G_4=0) + \Delta_n(G_4\neq 0) - \Delta_n(G_4=0) \qquad (37)$$

analogous /23,24/ to $P_Z \cong \Delta_p$ and $P_N \cong \Delta_n$.

The quantities P_Z, P_N and P_4 involve 8 nuclei. For each nucleus of this set we have to solve the gap eqs.(11) or (12). Thus we have in all a nonlinear system of 35 equations with 35 unknowns for each nucleus (Z,N). Using the $A = Z+N$ dependence of the coupling strengths G_Z and G_N as usual

$$G_Z = \frac{C_Z}{A}\ \mathrm{MeV} \quad ; \quad G_N = \frac{C_N}{A}\ \mathrm{MeV} \qquad (38)$$

and for G_4

$$G_4 = \frac{C_4}{A^2}\ \mathrm{MeV} \qquad (39)$$

we have calculated for some rare-earth and actinide nuclei the C_Z C_N and C_4 constants. The results are given in table 1.

The expression (39) is obtained using the assumption that for G_4 it may be expected an approximate factorization of the two-pair vertex interaction into two one-pair vertex interaction whose coupling strengths (38) are known /24/. In the performed calculations the cut off energy range contains approximately 40 nucleon energy levels of the deformed Saxon-Woods potential /22/. For the solutions we have taken care of that they should correspond to the deepest negative minimum of the correlation energy.

Table 1. The results of the numerical determination of the pairing and α-type correlations coupling strengths (38,39), the gap parameters (Δ_p, Δ_n) and the experimental and theoretical P_Z, P_N and P_4 (31,33)-quantities.

Nucleus	C_Z	C_N	C_4	Δ_p (MeV)	Δ_n (MeV)	$P_{Z,exp}$ (MeV)	$P_{Z,th}$ (MeV)	$P_{N,exp}$ (MeV)	$P_{N,th}$ (MeV)	$P_{4,exp}$ (MeV)	$P_{4,th}$ (MeV)
$^{152}_{60}Nd_{92}$	19.340 26.490	17.168 23.644	27.720 0	1.17 0.92	1.16 1.11	0.675	0.676 0.6745	0.9715	0.9735 0.9714	0.163	0.1043 -0.736
$^{156}_{62}Sm_{94}$	18.310 24.430	18.090 21.870	29.200 0	0.777 0.685	0.889 0.850	0.474	0.474 0.473	0.721	0.7213 0.7207	-0.177	-0.181 -0.617
$^{160}_{64}Gd_{96}$	21.043 26.740	21.652 24.935	27.060 0	0.832 0.752	0.991 0.925	0.5175	0.5172 0.5175	0.7545	0.7525 0.7545	-0.216	-0.2185 -0.766
$^{164}_{66}Dy_{98}$	24.200 27.781	21.820 24.693	22.788 0	0.818 0.739	0.833 0.800	0.4285	0.4293 0.4286	0.6845	0.6845 0.6847	-0.582	-0.576 -0.825
$^{168}_{68}Er_{100}$	25.538 29.447	22.183 26.100	27.688 0	0.949 0.832	0.887 0.839	0.5045	0.5045 0.5046	0.6675	0.6676 0.6677	-0.401	-0.401 -0.883
$^{176}_{72}Hf_{104}$	23.280 27.866	19.911 22.328	19.196 0	0.755 0.737	0.843 0.840	0.594	0.5938 0.5939	0.728	0.7273 0.7283	-0.348	-0.340 -0.576
$^{180}_{74}W_{106}$	26.839 31.329	19.884 23.040	22.280 0	0.908 0.878	0.803 0.786	0.681	0.6813 0.6807	0.7415	0.7420 0.7419	0.076	0.070 -0.508
$^{184}_{76}Os_{108}$	24.164 31.640	25.293 26.390	22.184 0	0.688 0.670	1.175 1.080	0.4395	0.4405 0.4393	0.9105	0.9125 0.9107	-0.823	-0.830 -1.222
$^{240}_{94}Pu_{146}$	34.074	21.590	15.229	0.900	0.685	0.591	0.603	0.443	0.419	-0.313	-0.326
$^{246}_{98}Cf_{148}$	31.708	19.944	14.555	0.609	0.355	0.538	0.5426	0.546	0.543	-0.266	-0.253

A straightforward conclusion from table 1 we have concerning the P_4 quantity. For $G_4 = 0$ this quantity cannot be reproduced. The C_4 constant has almost the same variation as C_Z and C_N. The Δ_p and Δ_n gap parameters suffer an increase when G_4 is switched on. The P_Z and P_N quantities practically do not depend on G_4.

V. Superfluid Enhancement Factor for the Alpha Clusterization Probabilities and Two-Nucleon Transfer Reactions Probabilities

The partial alpha-decay width for an alpha transition between the state $|I_i K_i \pi_i\rangle$ and the state $|I_f K_f \pi_f\rangle$ of deformed nuclei has the following expression[5]:

$$\Gamma_\ell (I_i\pi_i K_i, I_f\pi_f K_f) = \left| \sum_{\substack{\nu\nu'\omega\omega' \\ \tau\tau'\sigma\sigma'}} \left[\Gamma_\ell^{1/2} (I_i\pi_i K_i, I_f\pi_f K_f)\right]^{(\tau\tau'\sigma\sigma')}_{\nu\nu'\omega\omega'} \times \right.$$

$$\left. \times \langle BCS | a^+_{\nu\tau} a^+_{\nu'\tau'} a^+_{\omega\sigma} a^+_{\omega'\sigma'} | BCS\rangle \right|^2 . \qquad (40)$$

For the favoured ground-ground α-transitions between doubly even mass deformed nuclei the expression (40) becomes

$$\Gamma_0 (0^+0, 0^+0) = \left| \sum_{\nu\omega} \left[\Gamma_0^{1/2} (0^+0, 0^+0)\right]^{(+-+-)}_{\nu\nu\omega\omega} \times \right.$$

$$\left. \times\; U_\nu (Z-2, N-2) V_\nu (Z,N)\, U_\omega (Z-2,N-2) V_\omega (Z,N) \right|^2 . \qquad (41)$$

Assuming that the single particle amplitudes $\left[\Gamma_0^{1/2}(0^+0,0^+0)\right]^{(+-+-)}_{\nu\nu\omega\omega}$ do not depend on ν and ω indices[24], i.e.,

$$\left[\Gamma_0^{1/2}(0^+0, 0^+0)\right]^{(+-,+-)}_{\nu\nu\,\omega\omega} \cong \langle \Gamma^{1/2}_{s.p} \rangle \qquad (42)$$

the partial α-decay width (41) becomes:

$$\Gamma_0 (0^+0, 0^+0) \cong \langle \Gamma^{1/2}_{s.p.} \rangle^2 R_p^2 R_n^2 , \qquad (43)$$

where $R_p^2 R_n^2$ is the superfluid enhancement factor in which

$$R_{p(n)} = \sum_{\nu(\omega)} U_{\nu(\omega)} (Z-2, N-2)\, V_{\nu(\omega)} (Z,N) \cong \chi_{p(n)} \qquad (44)$$

in which the quantities $\chi_{p(n)}$ are defined in eq.(10).

Analogously the spectroscopic factors entering the cross sections for the α-transfer reactions are proportional to $R_p^2 R_n^2$, while the spectroscopic factors entering the cross sections for two-nucleon transfer reactions are proportional to R_p^2 or R_n^2 depending on the transferred pair (proton or neutron, respectively).

In table 2 we have calculated the superfluid enhancement factors with and without α-type correlations included. The over all conclusion is that the superfluid enhancement factor increases with G_4 with 5-20% for two-nucleon transfer reactions and 10-50% for alpha clusterization processes in the superfluid region of atomic nuclei.

More interesting are the cases when one subsystem (protons or neutrons) has a magic (or about) number of particles and the other one has the number of particles corresponding to the middle of the major shell. In such cases the usual BCS pairing neutron or proton superfluidity can mutually be induced by one another via the α-type correlations.

This is the case experimentally observed[26] in the Z=82 region. The experimental α-reduced widths of $^{184-192}Pb$ isotopes have values about 0.1 MeV, the same value as for doubly superfluid actinide $U-Pu-Cm$-nuclei. The conclusion that Z=82 is not a magic number seems to be wrong.

Table 2. Superfluid enhancement factors for favoured two- and four-nucleon transitions

Nucleus	C_4	χ_p^2	χ_n^2
$^{152}_{60}Nd_{92}$	27.72	37.725	53.675
	0	27.975	51.500
$^{156}_{62}Sm_{94}$	29.200	22.475	38.750
	0	19.125	37.500
$^{160}_{64}Gd_{96}$	27.060	23.410	38.345
	0	20.260	35.275
$^{164}_{66}Dy_{98}$	22.788	22.080	29.775
	0	19.045	28.250
$^{168}_{68}Er_{100}$	27.688	26.970	31.250
	0	22.225	29.225
$^{176}_{72}Hf_{104}$	19.196	22.370	44.025
	0	21.925	43.860
$^{180}_{74}W_{106}$	22.28	26.650	38.900
	0	25.450	37.730
$^{184}_{71}Os_{108}$	22.184	15.910	63.120
	0	15.200	56.975
$^{240}_{94}Pu_{146}$	15.229	33.74	47.61

VI. Summary and Conclusions

A new type of superfluid phase induced by α-like four nucleon correlations seems to be predicted in the heavy deformed nuclei. The model includes in addition to the usual BCS-pairing correlations a two-pair α-like four nucleon correlation which is assumed to be responsible for the formation of the α-superfluid agregates. Further on a BCS-like trial wave function is used, which contains correlated pairs of protons and neutrons, and not the correlated products of four nucleon operators which seems to have rather small contribution to the ground state wave function. The main advantage of such an approach consist in that it is able to take into account the underlying fermionic structure of the α-superfluid agregates, which are not longer approximate /7/ by two interacting bosons in a bound state. For sufficiently large coupling constants the condensed, symmetry-broken ground-state mode of such correlated α-superfluid agregates is energetically favoured with respect to the normal fluid one. Consequently, under this circumstances, a normal fluid superfluid transition(of first and second order) is predicted.

Experimental evidences are found for the existence of such α-superfluid agregates by estimating the P_4-quantities (see table 1). Following the standard procedure a set of 35 nonlinear equations are solved for each nucleus (table 1). It is found that the P_4-quantity cannot be reproduced without including the α-like four nucleon correlations in addition to the BCS pairing correlations ones.

The proton and neutron gap parameters are slightly increased by the presence of the α-type correlations, which induces a further enhancement of the probabilities of two-nucleon and α-transfer reactions and favoured α-decay. In addition, the proton and neutron BCS pairing superfluidity can mutually be induced by one another via the α-type correlations. This point might be of relevance in explaining the "anomalous" α-decay rates of light Pb isotopes near Z=82 magic number, rather than to assume the nonmagicity of this proton number. All these questions require further investigations.

In conclusion, one may say that the α-type correlations enlarges the region of superfluid nuclei in ground and low lying states.

Acknowledgements

One of the authors (O.D.) would like to acknowledge Prof. V.G.Soloviev for his continued interest and encouragement concerning this work and Dr. W.Rybarska for helpful remarks and precious criticism.

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Received by Publishing Department

on December 23, 1985.

Апостол М., и др. E4-85-934
Четырехчастичные корреляции в сверхтекучей фазе атомных ядер

В рамках БКШ модели получено, что вклад четырехчастичных корреляций в структуру основных и низколежащих возбужденных состояний немаловажен. Показано, что эти корреляции очень важны в обьяснении процесса α-кластеризации (спектроскопических факторов α-распада и реакции с передачей α-частиц). Показано также, что область сверхтекучих ядер расширяется особенно благодаря тому, что обычная БКШ сверхтекучесть протонов (нейтронов) индуцирует сверхтекучесть в системе нейтронов (протонов) при помощи четырехчастичных корреляций.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Apostol M., et al E4-85-934
Alpha-Like Four Nucleon Correlations in the Superfluid Phase of Atomic Nuclei

An interplay between the pairing correlations and the α-like four-nucleon correlations (α-type correlations) in the ground and low-lying excited states is predicted within a BCS-like model. It is shown that these α-type correlations are especially important in describing the α-clusterization process (the spectroscopic factors entering the α-decay and α-transfer reaction probabilities). It is also shown that the region of superfluid nuclei is enlarged due to the fact that the usual BCS neutron and proton superfluidity can mutually be induced by one another via the α-type correlations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985