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**HEXADECAPOLE STATES  
IN DEFORMED NUCLEI**

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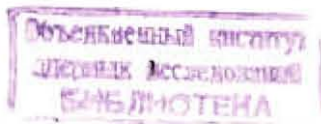
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## 1. Introduction

The quadrupole and octupole vibrational states are the most correctly described in the random phase approximation (RPA) and well studied experimentally among the low-lying states in doubly even nuclei<sup>/1-5/</sup>. There are numerous experimental data on the low-lying  $K^\pi = 3^+$  and  $4^+$  states ( $K$  is momentum projection onto the nuclear symmetry axis) in the rare-earth region<sup>/4,6-23/</sup>. The states with  $K^\pi = 4^+$  in Gd and Dy isotopes have large two-quasiparticle components<sup>/4,6-8/</sup>. The mixing of neutron and proton two-quasiparticle components in the  $3^+$  states has been observed in Yb isotopes in transfer reactions<sup>/12,13/</sup>. In some nuclei in the region  $A \approx 170$  the states  $I^\pi K = 4^+3$  are excited with large cross sections in the reaction  $(d, d')$ <sup>/12/</sup>. According to the experimental data<sup>/20/</sup> there are strong E4 transitions in the excitation of  $I^\pi K = 4^+4$  states in Os isotopes in  $(\alpha, \alpha')$  reaction. They indicate the hexadecapole structure of these states.

The available experimental data necessitate studies of hexadecapole states in deformed nuclei. The present paper is aimed at calculating the energies and  $B(E4)$ -values for the low-lying nonrotational  $K^\pi = 3^+$  and  $4^+$  states and elucidating to what extent these states are collective. We also consider the influence of hexadecapole forces on  $K^\pi = 2^+$  states, which is concluded from the experimental data of  $(\bar{p}, \bar{p}')$  reaction for  $^{168}\text{Er}$ <sup>/26/</sup>.

It is necessary to investigate hexadecapole states for solving the problem of existence of two-phonon collective states in deformed nuclei. A correct inclusion of the Pauli principle in two-phonon components of the wave function enabled one to make a conclusion about the absence of collective two-phonon states in deformed nuclei<sup>/27,28/</sup>. The study of  $K^\pi = 4^+$  and  $3^+$  states enables one to compare<sup>/29/</sup> the results of calculations within the quasiparticle-phonon nuclear model (QPNNM) and the interacting boson model (IBM) (particularly, in the case of introduction of g-boson into the IBM)<sup>/23-25/</sup>.



## 2. Details of calculations

The Hamiltonian of the quasiparticle-phonon nuclear model (QPNM) consists of an average field of the neutron and proton systems as the Saxon-Woods potential, superconducting pairing correlations and the multipole isoscalar and isovector forces. A phonon basis is constructed in the RPA. Using then the RPA secular equations the QPNM Hamiltonian is transformed to

$$H = \sum_{q\sigma} \epsilon_q a_{q\sigma}^\dagger a_{q\sigma} + H_2 + H_{1q} \quad (1)$$

containing free quasiparticles and phonons and the quasiparticle-phonon interaction  $H_{1q}$ . Here  $\epsilon_q$  is the quasiparticle energy,  $a_{q\sigma}^\dagger$  and  $a_{q\sigma}$  are the quasiparticle creation and absorption operators,  $q\sigma$  are quantum numbers of single-particle states,  $\sigma = \pm 1$ .

Hexadecapole nonrotational states with  $K^\pi = 3^+$  and  $4^+$  are generated by multipole isoscalar and isovector forces with  $\lambda = 4$ . However, as the calculations showed, the role of isovector forces is negligible and one can use formulae given in ref. /28/ for the isoscalar case.

The wave function of a nonrotational excited state of a doubly even nucleus is

$$\Psi_n(K^\pi) = \left\{ \sum_i R_i^n Q_{g_i}^+ + \sum_{\substack{g_1, g_2 \\ g_1, g_2}} \frac{\sqrt{1 + \delta_{g_1, g_2}}}{2} \delta_{\epsilon, \mu_1 + \epsilon, \mu_2, \epsilon K} P_{g_1, g_2}^n Q_{g_1, g_2}^+ Q_{g_2, g_1}^+ \right\} \Psi_0 \quad (2)$$

with the normalization condition

$$\sum_i (R_i^n)^2 + \sum_{g_1, g_2} \frac{1 + \delta_{g_1, g_2}}{2} (P_{g_1, g_2}^n)^2 \left\{ 1 + \frac{1}{1 + \delta_{g_1, g_2}} \mathcal{K}(g_1, g_1, g_1, g_2) \right\} = 1 \quad (3)$$

Here  $Q_{g_i}^+$  is the phonon creation operator,  $g = \lambda\mu_i$ ,  $i = 1, 2, \dots$  are numbers of the roots of secular equations in the RPA,  $\Psi_0$  is the ground state wave function of a doubly even nucleus,  $n = 1, 2, \dots$  is number of the state with given  $K$ . The notation and equations for the energies of excited states and the amplitudes  $R_i^n$  and  $P_{g_1, g_2}^n$  are given in ref. /28/. It should be noted that these equations were derived by taking strict account of the Pauli principle in the two-phonon components of the wave function (2).

The calculations have been performed with the parameters of the Saxon-Woods potential from refs. /30, 32/. The single-particle spectrum was taken from the bottom of the potential well up to +5 MeV. The pairing interaction constants were chosen according to pairing energies and the constants of residual multipole-multipole forces were adjusted so as to reproduce of experimental energies of the lowest nonro-

tational states using the wave function (2), for the isovector forces  $\chi_2^{(\lambda\mu)} = -1.2 \chi_2^{(\mu)}/30$ . The radial dependence of residual forces was taken in the form  $R(\tau) = \frac{\partial V(\tau)}{\partial \tau}$  where  $V(\tau)$  is the spherical Saxon-Woods potential. The multipolarities  $\lambda\mu = 20, 22, 30, 31, 32, 43, 44$  were taken into account, 10 RPA-phonons were used for each multipolarity. The reduced  $E\lambda$  transition probabilities were calculated in the adiabatic approximation with  $e_{\text{eff}} = 0.1$ . The two-quasiparticle state energies were calculated taking account of the blocking effect and the Gallagher-Moszkowski corrections. The latter was calculated by formula  $\Delta \epsilon_{q\sigma} = \chi_{\sigma\sigma} \langle q | \sigma_z | q \rangle \langle q' | \sigma_z | q' \rangle$  where  $\chi_{\sigma\sigma} = 0.26$  MeV is the constant of the isovector spin-spin interaction /31/ and  $\langle q | \sigma_z | q \rangle$  is the single-particle matrix element.

The energies and wave functions of  $K^\pi = 3^+$ ,  $3_2^+$ ,  $4_1^+$  and  $4_2^+$  states have been calculated with the constants  $\chi_2^{(32)} = 0.015 \frac{fm^2}{MeV}$  and  $\chi_2^{(44)} = 0.022 \frac{fm^2}{MeV}$ . The difference in values of the constants  $\chi_2^{(32)}$  and  $\chi_2^{(44)}$  is caused by the inclusion of only particle-hole excitations in the calculations of hexadecapole states and by that number of matrix elements of the particle-hole type at  $\lambda\mu = 44$  is less than at  $\lambda\mu = 43$  (for matrix elements of the particle-particle type we have an inverse case). The single-particle matrix elements of hexadecapole forces are close in value at different values of  $\mu$  but on the average the particle levels have larger values of  $K$  than the hole ones. The calculations have shown that the use of an incomplete single-particle basis ( $E_q < 5$  MeV) results in  $\chi_2^{(42)} < \chi_2^{(43)}$ . If the calculations are performed with the forces  $R(\tau) = \tau^\lambda$  the relevant value of  $\chi_2^{(44)}$  we used turns out to be larger than in ref. /30/ in describing the giant hexadecapole resonance.

## 3. Properties of $K^\pi = 3^+$ and $4^+$ states

Now we consider the low-lying states with  $K^\pi = 3^+$  and  $4^+$ . The results of calculations and experimental data /4, 6-23/ are shown in table 1. It includes the experimental energies, state structure and contribution of one-phonon components as well as the values calculated within the QPNM, namely reduced probabilities of  $E4$  transitions to  $4^+K$  states from the ground one, contributions of one-phonon configurations, the largest two-quasiparticle components. In the table phonons are denoted by  $\lambda\mu_i$ , their contribution to the wave function normalization is given in per cent. To denote the neutron (nn) and proton (pp) components the asymptotic quantum numbers  $N_{n_2} \Lambda$  (for  $K = \Lambda + \frac{1}{2}$  and for  $K = \Lambda - \frac{1}{2}$ ) were used.



In the Gd and Dy isotopes there are one or two  $K^\pi=4^+$  states with an energy less than 2 MeV<sup>4,6-8/</sup>. According to the calculations these states are slightly collectivized, for which  $B(E4)=0.1+0.4$  spu. The two-quasiparticle neutron configuration  $nn523\downarrow+521\uparrow$  appears in the excitation of  $K^\pi=4^+$  states in (d,p), (d,t) reactions and  $\beta$ -decay of the  $\alpha$  type. Of great importance is the proton configuration  $pp413\downarrow+411\uparrow$  in (t, $\alpha$ ) reactions. It should be noted that <sup>164</sup>Dy and <sup>166</sup>Er have almost four-quasiparticle  $K^\pi=3^+$  and  $4^+$  states (for instance the  $4_2^+$  state in <sup>164</sup>Dy shown in table 1)<sup>4/</sup>.

In <sup>166,168</sup>Er the calculations with  $\chi_0^{(4+)}=0.022$  fm<sup>2</sup>/MeV provide energies by 0.5 MeV higher than the experimental values and  $B(E4)<0.1$  spu. In <sup>168</sup>Er according to the calculations with  $\chi_0^{(4+)}=0.022$  fm<sup>2</sup>/MeV there are two states  $4_1^+$  and  $4_2^+$  with energies 2.57 and 2.59 MeV. For their wave functions the calculations give mixing of phonons 441 and 442 with structure 441:  $nn514\downarrow+521\downarrow$  - 99.8% and 442:  $nn512\uparrow+521\uparrow$  - 99.7%. In order the energy of the  $4^+$  state in <sup>168</sup>Er be equal to 2 MeV, one has to take  $\chi_0^{(4+)}=0.035$  fm<sup>2</sup>/MeV. In this case the collectivity of the  $4^+$  state will explicitly be overestimated (see table 1).

In the Sm, Gd and Dy isotopes there are no reliable data on  $K^\pi=3^+$  states with an energy less than 2 MeV. The analysis of (d,d') reaction with excitation of  $I^\pi K=4^+3$  states allowed one to make a conclusion on the existence of collective hexadecapole  $3_1^+$  states in nuclei with  $N=98-104$  and  $Z=68-72$ <sup>12/</sup>. Our calculations performed with fixed value of the constant  $\chi_0^{(4+)}=0.015$  fm<sup>2</sup>/MeV confirm the conclusion on the collectivity of  $3^+$  states in these nuclei. It is seen from table 1 that the energies of  $3_1^+$  and  $3_2^+$  states are qualitatively described in the Er, Yb and Hf isotopes. If in <sup>168</sup>Er  $B(E4)$  is equal to 0.8 spu, in <sup>168-174</sup>Yb and <sup>174,176,178</sup>Hf it takes the values from 1.0 to 1.7 spu.

In <sup>172,174</sup>Yb there are two  $K^\pi=3^+$  states with the wave functions consisting of the mixing of two-quasiparticle neutron and proton configurations. This is shown in table 1. The mixing of the neutron  $nn512\uparrow+521\downarrow$  and proton  $pp404\downarrow-411\downarrow$  configurations in  $3^+$  states in <sup>172</sup>Yb has experimentally been observed in ref.<sup>13/</sup>. The value of the neutron component has been confirmed in ref.<sup>12/</sup>. The results of calculations of the contribution of neutron and proton configurations to the  $3^+$  states in <sup>172</sup>Yb coincide with the experimental data<sup>12,13/</sup>.

The hexadecapole collective  $K^\pi=4^+$  states have been observed experimentally<sup>20,21/</sup> in Os isotopes. The results of calculations presented in table 1 indicate that beginning from <sup>178</sup>Hf the collectivity of  $4^+$  states increases. In <sup>186,188</sup>Os we have  $B(E4)=4$  spu that are

Table 1. Results of calculations for low-lying  $K^\pi=3^+$  and  $4^+$  states in rare-earth nuclei

Nucleus	$K^\pi$	Experiment		Calculations in the QPM	
		E, MeV	structure	E, MeV, B(E4), spu.	structure
1	2	3	4	5	6
<sup>158</sup> Gd	$4_1^+$	1.380	(t, $\alpha$ ), $pp411\uparrow+413\downarrow$ is large	1-2	441 95%, {221,221} 3.3%
	$4_2^+$	1.920	(d,p), $nn521\uparrow+523\downarrow$ is large	0.4	441: $pp413\downarrow+411\uparrow$ 85% $nn523\downarrow+521\uparrow$ 13%
	$4_1^+$	1.184		1.7	442 98%
<sup>158</sup> Dy	$4_1^+$	1.895	$\log ft=4.9$ ; $nn521\uparrow+523\downarrow$ is large	0.01	442: $nn523\downarrow+521\uparrow$ 87% $pp413\downarrow+411\uparrow$ 12%
	$4_1^+$	1.694	$\log ft=4.7$ ; $nn521\uparrow+523\downarrow$ is large	1.2	441 93%, 442 3%, {221,221} 1%
<sup>160</sup> Dy	$4_2^+$	2.096	$\log ft=5.8$	0.3	441: $nn523\downarrow+521\uparrow$ 53% $pp413\downarrow+411\uparrow$ 46%
	$4_1^+$	1.535	(d,t), $nn521\uparrow+523\downarrow$ is large	2.2	441 92%, 442 6%
<sup>162</sup> Dy	$4_1^+$	1.535	(d,t), $nn521\uparrow+523\downarrow$ is large	0.4	441: $pp413\downarrow+411\uparrow$ 36% $nn523\downarrow+521\uparrow$ 28%
	$4_1^+$	1.535	(d,t), $nn521\uparrow+523\downarrow$ is large	1.7	441 98%, {221,221} 0.6%
				0.1	441: $nn521\uparrow+523\downarrow$ 85%
				2.0	442 85%, {201,442} 6%
				0.1	442: $nn642\uparrow+651\uparrow$ 74% $nn523\downarrow+521\uparrow$ 8%
				1.7	441 98%
				0.2	441: $nn523\downarrow+521\uparrow$ 92%

continuation of table 1

1	2	3	4	5	6
<sup>164</sup> Dy	4 <sub>1</sub> <sup>+</sup>	2.194	logft=5.6	2.2	441 99%
				0.3	441: pp413↓+411↑ 66%
					nn523↓+521↑ 30%
	4 <sub>2</sub> <sup>+</sup>	2.206	logft=5.0: { pp523↑-411↑ + + nn633↑+523↓ } is large	2.6	{321,761} 100%
<sup>166</sup> Er	4 <sub>1</sub> <sup>+</sup>	1.976		2.0	321: pp523↑-411↑ 73%
					761: nn523↓+633↑ 100%
					441 95%, {221,221} 0.8%
					441: nn523↓+521↑ 52%
					nn633↑+660↑ 8%
<sup>168</sup> Er	3 <sub>1</sub> <sup>+</sup>	1.653	(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub>	1.6	431 99%
				0.8	431: nn512↑+521↓ 90%
	4 <sub>1</sub> <sup>+</sup>	2.055	no large two-phonon components	2.0	441 94%, {221,221} 1%
				3.3	441: nn514↓+521↓ 18%
					pp413↓+411↑ 9%
<sup>170</sup> Er	3 <sub>1</sub> <sup>+</sup>	1.217	(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub>	1.2	431 99%
				0.6	431: nn512↑+521↓ 94%
<sup>168</sup> Yb	3 <sub>1</sub> <sup>+</sup>	1.452	(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub> (α,2n) } nn 20%	1.6	431 98%
			g <sub>k</sub> - factor / pp 80%	1.6	431: pp404↓-411↓ 61%
			(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub>		nn512↑+521↓ 13%
<sup>170</sup> Yb	3 <sub>1</sub> <sup>+</sup>	<1.470		1.3	431 99%
				1.4	431: nn512↑+521↓ 66%
					pp404↓-411↓ 24%

continuation of table 1

1	2	3	4	5	6
<sup>172</sup> Yb	3 <sub>1</sub> <sup>+</sup>	1.172	(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub> (d,t), (d,p), nn512↑+521↓ 75% (p,α) } nn512↑+521↓ 73% magn.mom. } pp404↓-411↓ 27%	1.3	431 99%
			(d,t),(d,p), nn512↑+521↓ is noticeable (p,α), pp404↓-411↓ 26%	1.3	431: nn512↑+521↓ 68%
					pp404↓-411↓ 20%
	3 <sub>2</sub> <sup>+</sup>	1.663		1.6	432 100%
				0.4	432: pp404↓-411↓ 52%
					nn512↑+521↓ 30%
<sup>174</sup> Yb	4 <sub>1</sub> <sup>+</sup>	2.073	(p,α), pp404↑+411↓ is noticeable	1.94	441 99%
				3*10 <sup>-4</sup>	441: pp404↑+411↓ 99%
	3 <sub>1</sub> <sup>+</sup>	1.606	(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub>	1.4	431 99%
				1.3	431: nn514↑-521↓ 45%
					pp404↑-411↓ 39%
	3 <sub>2</sub> <sup>+</sup>	2.016	(p,t)	1.8	432 100%
				0.1	432: nn514↓-521↓ 52%
					pp404↓-411↑ 40%
<sup>174</sup> Hf	3 <sub>1</sub> <sup>+</sup>	1.303	(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub>	1.3	431 99%
				1.0	431: nn512↑+521↓ 75%
<sup>176</sup> Hf	3 <sub>1</sub> <sup>+</sup>	1.578	(d,d') is large for 4 <sup>+</sup> 3 <sub>1</sub> (d,t'), nn514↓-521↓ is large	1.3	431 99%
				1.2	431: nn514↑-521↓ 72%
					pp404↓-411↓ 12%
<sup>178</sup> Hf	4 <sub>1</sub> <sup>+</sup>	1.888	(d,t), nn514↑+521↑ is large	1.7	441 100%
				0.01	441: nn514↑+521↓ 99%
	4 <sub>1</sub> <sup>+</sup>	1.514	(d,p)	1.8	441 95%, {221,221} 3%
				0.4	441: nn514↑+510↑ 89%



1	2	3	4	5	6
	$3_1^+$	1.828		1.7	431 99%
$184_{\text{W}}$	$3_1^+$	1.425		1.0	431: pp404 $\downarrow$ -411 $\downarrow$ 79%
$186_{\text{Os}}$	$4_1^+$	1.352	B(E4) is large in ( $\alpha, \alpha'$ )	1.2	431 96%
				0.8	431: nn503 $\uparrow$ +510 $\uparrow$ 91%
				1.2	441 90%, {201,441} 3%, {221,221} 1%
				4.0	441: pp402 $\uparrow$ +402 $\downarrow$ 56%
					nn514 $\uparrow$ +510 $\uparrow$ 13%
$188_{\text{Os}}$	$4_1^+$	1.280	B(E4) is large in ( $\alpha, \alpha'$ )	1.0	441 89%, {201,441} 3%, {221,221} 1%
				4.0	441: pp402 $\uparrow$ +402 $\downarrow$ 52%

probably overestimated. For a better description of the experimental data one should reduce  $\mathcal{X}_c^{(44)}$  by (5-10)%. Note that  $^{188}\text{Os}$  is a transitional nucleus and the description of  $K_n^\pi=0_2^+$  and  $2_1^+$  states in it is rough, whereas the  $4_1^+$  state is not very collective and can be described sufficiently well.

Thus, the general picture of hexadecapole  $3^+$  and  $4^+$  states in the region  $158 \leq A \leq 188$  is the following. The Gd and Dy isotopes contain almost two-quasiparticle  $4^+$  states with an energy from 1.1. to 2.2 MeV. The nuclei  $^{168,170}\text{Er}$ ,  $^{168,170,172,174}\text{Yb}$  and  $^{174,176,178}\text{Hf}$  have collective  $3^+$  states with  $B(E4)=0.8 \div 1.7$  spu. In heavy Hf and W isotopes the collectivity of  $4^+$  states increases and it appears to be large in Os isotopes. The hexadecapole states can correctly be described with the fixed values of the constants  $\mathcal{X}_c^{(43)}$  and  $\mathcal{X}_c^{(44)}$ .

The calculations reproduce the dominating two-quasiparticle components of  $3_1^+, 4_1^+$  states obtained experimentally as well as the mixing of neutron and proton two-quasiparticle components of  $3^+$  states in Yb isotopes <sup>/12,13/</sup>. It should be noted that in those nuclei where the low-lying  $3^+$  and  $4^+$  states have not been observed experimentally the calculated energies of the lowest two-quasiparticle states with  $K^\pi = 3^+$  and  $4^+$  are usually higher than 2 MeV.

All the calculated states  $4_1^+$  and  $4_2^+$  have small two-phonon components, especially {221,221}. The present calculations confirm the conclusion made in refs. <sup>/27,28/</sup> about the absence of collective two-phonon states in deformed nuclei.

#### 4. Influence of hexadecapole forces on the states with $K=2^+$

Hexadecapole forces with  $\lambda_\mu=42$  may contribute to the states with  $K^\pi=2^+$ .

In the calculations for the states with  $K^\pi=2^+$  we have simultaneously taken into account both the quadrupole  $\lambda_\mu=22$  and hexadecapole  $\lambda_\mu=42$  forces. The state energies were found from the secular equation of the fourth order, the amplitudes of the two-quasiparticle components of one-phonon states comprise quadrupole and hexadecapole parts as it is shown in table 2. Since value of the constant  $\mathcal{X}_c^{(42)}$  is unknown the calculations were made for some values of  $\mathcal{X}_c^{(42)}$  in the region  $0 \leq \mathcal{X}_c^{(42)} \leq 0.015 \text{ fm}^2/\text{MeV}$  (as has been mentioned above, one should expect that  $\mathcal{X}_c^{(42)} < \mathcal{X}_c^{(43)}$ , where  $\mathcal{X}_c^{(43)} = 0.015 \text{ fm}^2/\text{MeV}$ ). The constant  $\mathcal{X}_c^{(22)}$  was chosen by fitting the  $K_n^\pi=2_1^+$  state energy.

The results of calculations for the  $K_n^\pi=2_1^+$  state in  $^{168}\text{Er}$  are shown in table 2. It is seen that with increasing hexadecapole constant  $\mathcal{X}_c^{(42)}$ , the collectivity of states decreases and its structure

Table 2. Contribution of hexadecapole forces with  $\lambda\mu=42$  to  $K_n^\pi=2_1^+$  state in  $^{168}\text{Er}$ . The  $B(E2, 0^+0_1 \rightarrow 2^+2_1)$ -values, contributions to the state normalization of the dominating neutron  $nn5234-5214$  and proton  $pp4114+4114$  components, quadrupole ( $\lambda\mu=22$ ) and hexadecapole ( $\lambda\mu=42$ ) parts of the amplitudes of these components

$\mathcal{K}_0^{(2)}$	$\mathcal{K}_0^{(4)}$	B(E2) a.u.	nn5234-5214			pp4114+4114		
			22	42	Contrib. to normal %	22	42	Contrib. to normal %
0.023	0	5.9	1	-	20%	1	-	48%
0.020	0.010	4.5	0.65	0.35	28%	0.80	0.20	42%
0.019	0.012	4.0	0.57	0.43	31%	0.70	0.30	41%
0.016	0.015	2.9	0.39	0.61	34%	0.49	0.51	37%

changes. At  $\mathcal{K}_0^{(4)} > 0.010 \text{ fm}^2/\text{MeV}$  hexadecapole forces contribute greatly to the amplitudes of two-quasiparticle components.

Recently, it has been shown that experimental results on the excitation of the  $I^\pi K = 4^+2$  state of  $\gamma$ -vibrational band in  $^{168}\text{Er}$  in  $(\bar{p}, \bar{p}')$  reaction can be explained only if hexadecapole forces are introduced<sup>[26]</sup>. These data may be considered as an experimental evidence of the influence of hexadecapole forces on the properties of  $K_n^\pi=2^+$  states. The results of our calculations obtained at reasonable values of  $\mathcal{K}_0^{(4)}$  are in qualitative agreement with experiment<sup>[26]</sup>.

### 5. Conclusion

The low-lying nonrotational hexadecapole states with  $K_n^\pi=3^+$  and  $4^+$  in the region  $158 \leq A \leq 188$  are described within the QPNM at fixed values of the constants of the hexadecapole interaction  $\mathcal{K}_0^{(4)} = 0.015 \text{ fm}^2/\text{MeV}$  and  $\mathcal{K}_0^{(4)} = 0.022 \text{ fm}^2/\text{MeV}$ . The description is in qualitative agreement with available experimental data. It is shown that in  $4_1^+$  and  $4_2^+$  states the two-phonon components, in particular  $\{221, 221\}$ , are small. This confirms the conclusion made in refs.<sup>[27, 28]</sup> on the absence of collective two-phonon states in deformed nuclei. The results of calculations for  $K_n^\pi=2^+$  states in  $^{168}\text{Er}$ , in which the forces with  $\lambda\mu=22$  and  $42$  have simultaneously been taken into account, are in qualitative agreement with experimental data<sup>[26]</sup> on a considerable contribution of hexadecapole forces to  $K_n^\pi=2^+$  states.

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Гексадекапольные состояния в деформированных ядрах

В рамках квазичастично-фононной модели ядра при фиксированных значениях констант гексадекапольного взаимодействия получено качественно правильное описание низколежащих неротационных гексадекапольных состояний с  $K^\pi=3^+$  и  $4^+$  в области  $158 \leq A \leq 188$ . Показано, что в  $4_1^+$  и  $4_2^+$ -состояниях двухфононные компоненты малы ( $<10\%$ ). Продемонстрировано, что гексадекапольные силы могут давать заметный вклад в  $K^\pi=2^+$ -состояния.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Hexadecapole States in Deformed Nuclei

The low-lying nonrotational hexadecapole states with  $K^\pi=3^+$  and  $4^+$  in the  $158 \leq A \leq 188$  region are described within the quasiparticle-phonon model at fixed values of the hexadecapole interaction constants. The description is in qualitative agreement with the available experimental data. It is shown that the two-phonon components are small ( $<10\%$ ) in  $4_1^+$  and  $4_2^+$  states. It is found that hexadecapole forces considerably influence the properties of  $K^\pi=2^+$  states.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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