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St.Mrówczyński

EQUATION OF STATE OF NUCLEAR MATTER OF NUCLEONS AND DIBARYONS

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There have been numerous interesting theoretical considerations of nuclear matter at densities higher than the normal one. Pion condensation, density isomers or quark-gluon plasma have been discussed as possible phases of cold (temperature close to zero) nuclear matter. See, e.g., the review [1] and references therein. It is supposed that such states of nuclear matter can be realized in the hearts of neutron stars or in heavy ion collisions.

In this paper we briefly consider the phase which is somewhere between the pure nucleon gas and quark-gluon plasma. Namely, we discuss the earlier studied [2,3] phase of dibaryons, i.e. elementary particles of double baryon charge. There are numerous experimental candidates for such states, see the review [4] and Refs.5-10. On the other hand, some results of high energy nuclear experiments and, in particular, the EMC effect have led many authors to the conclusion of significant admixture of dibarions in nuclei, see, e.g., [11-13]. The evidence for multi-quark states is also highly desired from the point of view of the QCD.

To omit the possible misunderstandings let us discuss the differences between deuterons and dibaryons. The deuteron is a bound state of two nucleons and internucleon distances in the deuteron and in nuclear matter at normal density are very close. So, the deuteron cannot be treated as an elementary particle in the problem considered. In fact, it is known [14], that the deuterons cannot exist in the nuclear matter due to the Mott mechanism. In contrast to the deuteron the dibaryon is elementary particle with a mass greater than the double nucleon mass. In the MIT bag model [15] the dibaryon is a six-quark bag of the radius of $(M/m)^{1/3}\,r$, where M and m are the masses of dibaryon and nucleon, and r is the nucleon radius. Because the dibaryon radius is close to the nucleon one, the dibaryons should be treated in the same way as nucleons, i.e. as elementary particles. The question arises to what extend such treatment is justified. At the densities where interhadron distances are close to the hadron diameter one has to describe the nuclear matter at a quark level and the nucleons and dibaryons cannot be longer treated as structureless. The quantitative estimation of such densities is given at the end of our paper.

In our previous paper [3], the later on called paper I*, we

*In paper I a misprint has occurred. In formula (3) the coefficient 1/2 should be placed in front of $\sum_p D_p^{g}$.

have discussed the possibility of the existence of dibaryon admixture in nuclei. It has been found that such an admixture is very probable if the dibaryon mass does not significantly exceed the double nucleon mass. In this paper we study the evolution of nuclear matter when the density increases.

One may wonder why we take into account the exotic dibaryons while familiar mesons and isobars are omitted. At zero temperature the most energetically favourable configurations are realized only. The dibaryons as bosons occur in the system as a Bose condensate. This means that the kinetic energy of all dibaryons is zero. It is, of course, not possible for isobars which are fermions. On the other hand, a larger amount of energy per unit of baryon charge is needed to generate the lightest isobar $\Delta(1232)$ than that to generate the dibaryon with a mass smaller than 2564 MeV. In fact, many experimental candidates for dibaryons with such a mass have been found [4-10]. The existence of real (nonvirtual) pions in cold nuclear matter is energetically unfavourable, except the matter where the difference of Fermi energies of neutron and proton components is greater than the pion mass.

Our considerations of energetically favourable nonnucleon admixtures in nuclear matter may be invalid at densities significantly higher than the normal one since the nucleon-nucleon interaction can essentially affect a simple picture of nuclear matter as a strongly degenerated weakly interacting gas of nucleons.

For the obvious reasons our arguments in favour of an important role of dibaryons hold for any particles with an even number of baryon charge although the mass of such bosons with the baryon number equal to k has to be close to k masses of nucleon. This remark is important in the context of some papers, see, e.g., [16-18], where a significant admixture of twelve-quark states in nuclei has been advocated.

The description of nuclear matter changes totally at a temperature of some tens of MeV. In this case one is forced to take into account many isobars and mesons for realistic description, see, e.g., [19-21]. The point is that, in contrast to the zero temperature case, the energetical favour factor is not crucial since the state of the system is determined by a minimum of Helmholtz energy.

Firstly, we consider an ideal gas of nucleons (fermions) and dibaryons (bosons) in chemical equilibrium at zero temperature. Dibaryons occur in the system as a Bose-Einstein condensate. The internal energy of the gas reads

$$U = (B - 2D)\left[m + \frac{3}{5}\frac{p_{max}^2}{2m}\right] + DM \qquad /1/$$

with B the total baryon number of the system, D the number of dibaryons, V the volume of the the system and $p_{max} = [2m(\mu - m)]^{1/2}$, μ is the chemical potential of nucleons and [3]

$$\mu = \begin{cases} M/2 & \text{for } \rho > \rho_c \\ E_F + m & \text{for } \rho < \rho_c , \end{cases} \qquad /2/$$

where $\rho = B/V$ is the density of the baryon number and E_F is the Fermi energy

$$E_F = \frac{p_F^2}{2m}, \quad p_F = \left(\frac{3}{2}\pi^2\rho\right)^{1/3}, \quad \rho_c = \frac{2}{3\pi^2}[m(M^2 - 2m)]^{2/3} .$$

It is assumed that M>2m. The first term in (1) gives the energy related to nucleon masses. The second one describes the kinetic energy of nucleons and the third one relates to dibaryon masses.

The number of dibaryons found in paper I is

$$D = \begin{cases} 0 & \text{for } \rho < \rho_c \\ \frac{B}{2}\left\{1 - \frac{2}{3\pi}[m(M-2m)]^{3/2}\rho^{-1}\right\} & \text{for } \rho > \rho_c . \end{cases} \qquad /3/$$

Let us observe that for the ideal gas the nuclear matter changes to a pure dibaryon condensate at infinite density only. (This limit is, of course, of no physical meaning.) For finite densities there are always nucleons in the system.

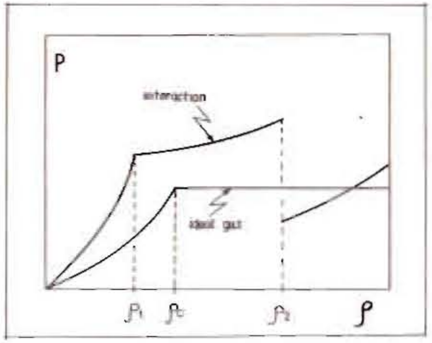
At zero temperature the internal energy coincides with the Helmholtz free energy. So, the pressure is determined by the relation

$$P = -\frac{\partial U}{\partial V} . \qquad /4/$$

With the help of formulae (1), (3) and (4) one finds the equation of state

$$P = \begin{cases} \frac{1}{5}\left(\frac{3}{2}\pi^2\right)^{2/3} m^{-1}\rho^{5/3} & \text{for } \rho < \rho_c \\ \frac{2}{15\pi^2} m^{3/2}(M-2m)^{5/2} & \text{for } \rho > \rho_c . \end{cases} \qquad /5/$$

The above equation is illustrated in the figure. The constant value of pressure for $\rho > \rho_c$ is characteristic for phase transition regions.



The pressure versus density of baryon charge.

Let us now discuss how the results are modified due to the interaction. Short-range repulsive forces are the most important for the properties of nuclear matter [22]. These forces can be represented by the delta-like potential. Repeating the considerations from the paper I, one finds the internal energy at zero temperature which deviates from (1) in three points: The chemical potential differs from (2). The dibaryon concentration formula (3) is modified. The potential energy term occurs

$$U_{pot} = \frac{3\pi a(B-2D)^2}{2mV} + \frac{2\pi\tilde{a}(B-2D)D}{m_R V} + \frac{2\pi a_D D^2}{MV}, \qquad /6/$$

where a, $\tilde{a}$ and a_D are the scattering lengths (diameters of hard core potentials) in nucleon-nucleon, nucleon-dibaryon and dibaryon-dibaryon interactions; m_R is the reduced mass of a nucleon-dibaryon system. Under the assumption that the radius of a hard core in the third power is proportional to the particle mass, $\tilde{a}$ and a_D can be expressed through a [3], the value of which is known, $a = 0.4$ fm [22]. To obtain the formula (6) it has been assumed that the numbers of nucleons with opposite spins are equal and that the dibaryon degeneration factor equals unity.

The most important result of the interaction is the occurrence of the pure dibaryon phase at the densities higher than

$$\rho_2 \simeq \frac{0.36}{\pi} m(M-2m)\,a^{-1} .$$

On the other hand, below the density

$$\rho_1 \simeq \frac{2}{3\pi^2}[m(M-2m)]^{3/2}\,\{1 - \frac{2.6}{\pi}[m(M-2m)]^{1/2}\,a\}$$

there are no dibaryons in the system. Both values of critical densities and all resulats concerning the interacting gas are valid in the lowest order of the dimensionless parameter $p_F a$. It has been also assumed that the dibaryon mass is close to the double nucleon mass, i.e. $M - 2m \ll M$. As is shown below, our considerations are reasonable for such light dibaryons only.

As in the case of ideal gas, the equation of state has been found from the relation (4)

$$P = \begin{cases} \frac{1}{5}(\frac{3}{2}\pi^2)^{2/3}\, m^{-1}\rho^{5/3} + \frac{3}{2}\pi m^{-1} a\rho^2 & \text{for } \rho_1 > \rho \\ \frac{2}{15\pi^2} m^{3/2}(M-2m)^{5/2} - \frac{0.045}{\pi^3} m^2 (M-2m)^3 a + 0.31\pi m^{-1} a\rho^2 & \text{for } \rho_2 > \rho > \rho_1 \\ 0.31\pi m^{-1} a\rho^2 & \text{for } \rho > \rho_2 . \end{cases} \qquad /7/$$

Two comments of a technical character are in order. 1) The calculations are simplified if one notices that only the volume-dependent terms of internal energy are needed to deduce the pressure. 2) In paper I we have found the chemical potentials and dibaryon concentration for the limiting cases $D \to B/2$ and $D \to 0$ only. It occurs that the volume-dependent terms of internal energy and, consequently, the equation of state can be found for any value of D in the first order of the $p_F a$ parameter.

The equation of state (7) is illustrated in the figure. At $\rho = \rho_1$ the pressure is a continuous function of density while at $\rho = \rho_2$ one observes discontinuity. To understand the physical reason for this jump of pressure, we have to return to the equation of state for the ideal gas (5). For $\rho > \rho_c$ the pressure is constant as a function of density in spite of the fact that the number of nucleons decreases as density increases. Because the dibaryons are at rest, the nucleons are responsible for the existence of the finite value, P^0, of preassure. In the case of interacting gas (where repulsive forces contribute to the pressure), the transition to the pure dibaryon phase is associated with the pressure decrease by the value related to the motion of nucleons. This value equals P^0 when $a \to 0$. It will be shown in our next paper that the discontinuity of the pressure as a function of density occurs at zero temperature only. The point is that at finite temperature there is no transition to the pure dibaryon phase. The finite temperature corrections to the equation of state are not considered here because the problem demands a special treatment due to a dominant role of collective excitations characteristic for systems of bosons [22].

The equation of state (7) is much softer than that of nucleon gas. The importance of this fact for neutron-star physics has been briefly commented in paper I.

Let us estimate the critical densities ρ_1 and ρ_2. For a dibaryon mass interval of 2000-2050 MeV, where numerous experimental candidates have been found, we get $\rho_1 \approx 1\text{-}2\rho_0$ and $\rho_2 \approx 5\text{-}7\rho_0$, $\rho_0 = 0.17$ fm^{-3}. Let us now discuss validity of our results. For the baryon density equal to $6\rho_0$ the average inter-dibaryon distance in the dibaryon gas is about 1.25 fm. (one should remember that a dibaryon carries double baryon charge).

In the model of the hard-sphere interaction, which is used in this paper, the scattering length coincides with the hard-sphere diameter. Because a = 0.4 fm [22] the dibaryon diameter is about 0.5 fm. According to the above estimation it is reasonable to treat the dibaryons as elementary particles (bosons) even at the densities around $6\rho_0$, because the ratio of the dibaryon diameter to the average distance between dibaryons in the dibaryon gas is 0.4. The system of dibaryons resembles the liquid ^{4}He, where the value of the ratio of interest is about 0.6 [22]. In spite of the fact the ratio is not much less than unity, as a first approximation the helium atoms are treated as hard spheres with the electron structure neglected. On the basis of such simplified model the essential properties of the liquid helium, like the phonon excitations at the temperature close to zero, can be explained.

It is probable that the value of the dibaryon diameter given above is underestimated. (The MIT bag model gives higher hadron radii.) Then our considerations are limited to the lower densities. On the other hand, the properties of nuclear matter can be dramatically changed in the density interval under considerations due to the phase transitions to, e.g., quark-gluon plasma. In such a case the speculations presented in this paper are quite inadequate to the high density nuclear matter.

We conclude as follows. The equation of state studied here is of practical importance if the critical density ρ_1 is smaller or very close to the normal nuclear one. It demands the existence of a dibaryon with a mass which exceeds the double nucleon mass by a value of about 100 MeV [3]. As was mentioned above there are some experimental candidates for such states. Then the softness of the equation of state (7) manifests itself at moderate densities where our considerations are reasonable. Anyway these considerations are rather of a qualitative than a quantitative character because of a very simplified form of interaction and the validity of our results in the lowest order of $p_F a$ parameter.

As was discussed previosly, our results for dibaryons can be trivially modified for any particles with an even number k of baryon charge. As examples, we give the formulae for concentration of "k-baryons" with a mass M and the respective equation of state for ideal gas approximation:

$$K = \begin{cases} 0 & \text{for} \quad k(m+E_F) < M \\ \frac{B}{k}\left[1-\left(\frac{M-km}{kE_F}\right)^{3/2}\right] & \text{for} \quad km < M < k(m+E_F), \end{cases}$$

$$P = \begin{cases} \frac{1}{5}\left(\frac{3}{2}\pi^2\right)^{2/3} m^{-1}\rho^{5/3} & \text{for} \quad \rho < \rho_c^k \\ \frac{2}{15\pi} m^{-1}\left[2m\left(\frac{M}{k}-m\right)\right]^{5/2} & \text{for} \quad \rho > \rho_c^k, \end{cases}$$

$$\rho_c^k = \frac{2}{3\pi^2}\left[2m\left(\frac{M}{k}-m\right)\right]^{3/2} .$$

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Уравнение состояния ядерной материи, состоящей из нуклонов и дибарионов

Рассматривается ядерная материя, состоящая из нуклонов и дибарионов, т.е. элементарных частиц с двойным барионным зарядом. Получено уравнение состояния такой материи при нулевой температуре. Обсуждено приближение идеального газа, а затем рассмотрена роль взаимодействия, которое включено с помощью дельтаобразного потенциала. Обсуждены особенности и возможные физические следствия полученного уравнения.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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Equation of State of Nuclear Matter of Nucleons and Dibaryons

It is considered the nuclear matter consisting of nucleons and dibaryons, i.e., elementary particles of double baryon charge. The equation of state of such matter at zero temperature is found. The ideal gas approximation is considered and then it is discussed the role of interaction which is included by means of delta-like potential. The peculiarities and possible physical consequences of the equation of state are considered.

The investigation has been performed at the Laboratory of High Energies, JINR.

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