

Объединенный институт ядерных исследований дубна

E4-85-728

R.V.Jolos, S.P.Ivanova, R.Pedrosa, V.G.Soloviev

INVESTIGATION
OF THE TWO-PHONON POLE SHIFT
IN DEFORMED NUCLEI
BY THE BOSON EXPANSION METHOD

Submitted to "TMO"

# 1. Introduction

The problem of two-phonon collective states in doubly even deformed nuclei is still unclear. The Born-Mottelson model implies twophonon collective states in deformed nuclei; this has been pointed out once more in ref. 11. According to the interacting boson model. the low-lying states should include those with large two-boson components 2/. The study of two-phonon states within the quasiparticle-phonon nuclear model has shown'4/ that the Pauli principle, i.e. the inclusion of exact commutation relations between phonons, shifts the two-phonon poles towards larger excitation energies. The relevant secular equations have been solved in ref. /5/. The Pauli principle shifts the centroid energies of collective two-phonon states by 1-2 MeV. At excitation energies of 3-4 MeV the collective two-phonon strength should be distributed over many nuclear levels. On this basis it has been concluded 5/ that there are no collective two-phonon states in doubly even deformed nuclei. In describing the two-phonon collective states in deformed nuclei the quasiparticle-phonon nuclear model is in contradiction with the Bohr-Mottelson and interacting boson models. A disagreement between the quasiparticle-phonon nuclear model and the interacting boson model has been reported/6/ in describing other nonrotational states in deformed nuclei. According to ref. /7/, in doubly even nuclei the Fauli principle does not shift considerably the energies of two-phonon states, which is in agreement with experimental data on the two-phonon collective states in spherical nuclei.

The analysis of experimental data  $^{(8)}$  has provided evidence for the absence of two-phonon collective states in deformed nuclei. As follows from recent experimental data on  $^{168}{\rm Er}/9,10^{\prime}$ , the levels that have been treated as two-phonon ones have a dominating one-phonon component. The absence of two-phonon quadrupole states with an energy less than 2 MeV in  $^{168}{\rm Er}$  is explained  $^{(1,11)}$  by a large anharmonicity of  $\beta$  - and  $\sqrt{\phantom{a}}$ -vibrations due to a three-axis ellipsoid form of a



nucleus. The absence of two-phonon octupole states in some isotopes of Ra, Th and U<sup>/12</sup>/is attributed in many papers to the existence of a stable octupole deformation. It is to be noted that the new experimental data on <sup>168</sup>Er of refs./9,13/ are in good agreement with the calculations within the quasiparticle-phonon nuclear model and with those of the structure of one-phonon states/14/ and disagree with the calculations within the interacting boson model.

Since the problem of two-phonon collective states in doubly even deformed nuclei is very important, it is necessary to study the behaviour of two-phonon states by utilizing another mathematical formalism. Am important role of the Pauli principle in two-phonon components of the wave functions necessitates calculations with phonons constructed of the operators of "true" bosons. Whether a large shift of two-phonon poles occurs in this statement of the problem is the aim of the present paper.

## 2. Model Hamiltonian

We consider doubly even deformed nuclei. The Hamiltonian of the quasiparticle-phonon nuclear model contains an average field (Saxon-Woods potential), pairing interaction, multipole-multipole and spin-multipole - spin-multipole isoscalar and isovector forces. For simplicity we shall use the multipole-multipole isoscalar forces. After Bogolubov's  $\mathcal{U}$ ,  $\mathcal{V}$ -transformation the model Hamiltonian expressed through the quasiparticle operators is

$$H = \sum_{q \in S} \mathcal{E}(q) d_{q} d_{q} d_{q} d_{q} - \sum_{\lambda \mu \in S} \mathcal{X}_{o}^{(\lambda \mu)} Q_{\lambda \mu \in S}^{\dagger} Q_{\lambda - 5 \mu}, \qquad (1)$$
where  $\mathcal{X}_{o}^{(\lambda \mu)}$  is the constant of isoscalar multipole forces,  $\mathcal{E}(q)$  are one-quasiparticle energies,  $q \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states,  $d_{q} \in S$  and  $d_{q} \in S$  are quantum numbers of single-particle states.

The multipole moment operator  $\lambda$  with projection M and the projection sign  $\delta$  has the form 15/:

$$Q_{\lambda\mu\delta} = \frac{1}{2} \sum_{q_1 q_2} (f^{\lambda\mu}(q_1 q_2) u_{q_1 q_2}^{(+)} (A^{\dagger}(q_1 q_2, \mu \delta)) + \\ + A(q_1 q_2, \mu - \delta)) + \overline{f}(q_1 q_2) u_{q_1 q_2}^{(+)} (\overline{A}^{\dagger}(q_1 q_2, \mu \delta)) + \\ \overline{A}(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2) B(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta)) + 2v_{q_1 q_2}^{(-)} (f^{\lambda\mu}(q_1 q_2, \mu \delta)) + \\ A(q_1 q_2, \mu - \delta$$

$$+f^{\lambda\mu}(q_1q_2)B(q_1q_2,\mu_5)),$$
 (2)

where  $U_{q_1q_2}^{(+)} = U_{q_1}V_{q_2} + V_{q_1}U_{q_2}$ ;  $V_{q_1q_2}^{(-)} = U_{q_1}U_{q_2} - V_{q_1}V_{q_2}$ ,  $f^{\lambda\mu}(q_1q_2)$ ,  $f^{\lambda\mu}(q_1q_2)$  are the single-particle matrix elements. The operators A, A, B and B are expressed through the operators  $A_{qq}$ ,  $A_{qq}$ .

$$A^{+}(q_{1}q_{2};\mu \sigma) = \sum_{\sigma'} \delta_{\sigma'(\kappa_{1}-\kappa_{2}),\sigma\mu'} \delta^{-} d_{q_{1}\sigma'}^{+} d_{q_{2}-\sigma'}^{+}, \quad (3)$$

$$\bar{A}^{+}(q_{1}q_{2};\mu_{0}) = \sum_{\sigma'} \delta_{\sigma'}(\kappa_{1}+\kappa_{2}), \delta_{\mu} d_{q_{2}}^{+} \sigma' d_{q_{1}}^{+} \sigma' , \qquad (3')$$

B 
$$(q, q_1, \mu 6) = \sum_{\sigma'} \delta_{\sigma'(\kappa_1 - \kappa_2)}, \sigma_{\mu} \alpha_{q_1 \sigma'}^{\dagger} \alpha_{q_2 \sigma'}$$
 (4)

# 3. Boson transformation of the Hamiltonian

By analogy with the two-quasiparticle operators we shall consider the ideal boson operators

 $b_{q,\sigma_1}^+, q_2, \sigma_2$ ,  $b_{q_1,\sigma_1}, q_2, \sigma_2$ , satisfying the following commutation relations:

$$[\theta_{q_1,\sigma_1,q_2,\sigma_2}, \theta_{q_1',\sigma_1',q_2',\sigma_2'}] = \delta_{q_1,q_1'} \delta_{\sigma_1,\sigma_1'} \delta_{q_1,q_2'} \delta_{\sigma_2,\sigma_2'} - (5)$$

$$\begin{bmatrix} b_{q_1} \delta_{1_1}, q_2 \delta_{2_1}, b_{q'_1} \delta_{1_1}' q'_2 \delta_{2_1}' \end{bmatrix} = \begin{bmatrix} b_{q_1} \delta_{1_1}, q_2 \delta_{2_1}, \delta_{1_1}' \delta_{1_1}' \delta_{1_2}' \delta_{1_1}' \end{bmatrix} = 0$$
and the condition

(6)

$$b_{q6}, q'6' = -b_{q'6'}, q6.$$
 (7)

Analogously to (3), (3'), (4) and (4') we may introduce the ideal boson operators with an appropriate sign of the momentum projection

$$\beta^{+}(q_{1}q_{2}, \mu \delta) = \sum_{\sigma'} \delta_{\sigma'}(\kappa_{1} - \kappa_{2}), \delta_{\mu} \cdot \delta' \delta_{q_{1}\sigma'}, q_{2} - \delta'$$
(8)

$$\vec{b}^{+}(q_{1}q_{2}, \mu \delta) = \sum_{\sigma'} \delta_{\sigma'}(\kappa_{1} + \kappa_{2}), \delta \mu \quad \delta_{q_{2}\sigma'}^{+}, q_{1}\delta'. \tag{81}$$

It follows from (5) and (6) that the operators (8) and (8') satisfy the following commutation relations:

$$\begin{bmatrix} \mathcal{B}(q,q_2,\mu_{\delta}), \mathcal{B}^{\dagger}(q,q_2',\mu'_{\delta}') \end{bmatrix} = \delta_{\delta\delta}' \delta_{\mu\mu'} (\delta_{q,q_1'} \delta_{q,q_2'} \delta_{q,q_2'} \delta_{q,q_2'} \delta_{q,q_2'})^{(9)}$$

$$\begin{bmatrix} \overline{\mathcal{B}}(q,q_2,\mu_{\delta}), \overline{\mathcal{B}}^{\dagger}(q,q_2',\mu'_{\delta}') \end{bmatrix} = \delta_{\delta\delta}' \delta_{\mu\mu'} (\delta_{q,q_1'} \delta_{q,q_2'} \delta_{q,q_2'} - \delta_{q,q_2'} \delta_{q,q_2})^{(9)}$$

Now we find the boson images of the operators  $\mathcal{A}$ ,  $\overline{\mathcal{A}}$ ,  $\mathcal{A}^+$ ,  $\overline{\mathcal{A}}^+$ ,  $\mathcal{B}$  and  $\overline{\mathcal{B}}$  satisfying the commitation relations for these operators up to the terms quadratic in  $\mathcal{B}$ ,  $\mathcal{E}^+$ ,  $\overline{\mathcal{E}}$  and  $\overline{\mathcal{B}}^+$ . The relevant boson images have the form

$$\begin{array}{l}
A^{+}(q_{1}q_{2}, \mu \delta) = b^{+}(q_{1}q_{2}; \mu \delta) + z_{1}b^{+}(q_{1}q_{2}, \mu \delta)P + \\
+ z_{2}\sum_{\sigma} \delta_{\sigma}'(\kappa_{1} \cdot \kappa_{2}), \delta \mu \cdot \delta' \cdot R^{+}(q_{1}\sigma', q_{2} - \delta') + \cdots, (10) \\
\bar{A}^{+}(q_{1}q_{2}, \mu \delta) = b^{+}(q_{1}q_{2}, \mu \delta) + z_{1}\bar{b}^{+}(q_{1}q_{2}; \mu \delta)P + \\
+ z_{2}\sum_{\sigma'} \delta_{\sigma}'(\kappa_{1} + \kappa_{2}), \delta \mu R^{+}(q_{2}\delta', q_{1}\delta') + \cdots, (11)
\end{array}$$

$$\overline{B}(q, q_2, \mu \delta) = \sum_{\sigma'} \delta_{\sigma'}(\kappa_1 + \kappa_2)_{,\sigma \mu} \cdot \delta' B(q, \sigma', q_2 - \delta') , \qquad (13)$$

where

$$P = \sum_{q_1,q_2,\mu_5} \ell^+(q_1,q_2,\mu_5) \ell(q_1,q_2,\mu_6) + \\
+ \sum_{q_1,q_2,\mu_5} \bar{\ell}^+(q_1,q_2,\mu_6) \bar{\ell}(q_1,q_2,\mu_6), \\
R^+(q_1,\sigma_1,q_2,\sigma_2) = \sum_{q_1,q_2',\sigma_1',\sigma_2'} \ell_{q_1}\sigma_1, \sigma_1', \ell_{q_2}'\sigma_2, \ell_{q_2'}'\sigma_2', \ell_{q_2'$$

The coefficients  $\mathcal{X}_1$  and  $\mathcal{X}_2$  are determined from the requirement that the commutation relations between bilinear combinations of fermion operators are fulfiled. If in these commutation relations the terms quadratic in  $\mathcal{X}_1$  and  $\mathcal{X}_2$  are retained, then  $\mathcal{X}_1 = -\frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right)$ ,  $\mathcal{X}_2 = -\frac{1}{\sqrt{3}}$ . If only the linear terms are considered, then  $\mathcal{X}_2 = 0$  and  $\mathcal{X}_2 = -\frac{1}{\sqrt{3}}$ .

### 4. The RPA phonons

Now we pass from the operators  $b^+(q_1q_2,\mu\delta)$ ,  $\overline{b}^+(q_1q_2,\mu\delta)$  to the operators  $Q_{\lambda\mu\delta}^+$  diagonalizing a part of the Hamiltonian quadratic in the boson operators. For this purpose we use the linear transformation

$$Q_{\lambda\mu i\delta} = \frac{1}{2} \sum_{q_1 q_2} \left\{ (q_1 q_2) \mu_{\sigma} \left\{ (q_1 q_2) \mu_{\sigma} \right\} - q_1 q_2 \delta^{+} \left\{ (q_1 q_2) \mu_{\sigma} \right\} \right\} + \overline{q_1 q_2} \delta^{+} \left\{ (q_1 q_2) \mu_{\sigma} \right\} - \overline{q_1 q_2} \delta^{+} \left\{ (q_1 q_2) \mu_{\sigma} \right\} + \overline{q_1 q_2} \delta^{+} \left\{ (q_1 q_2) \mu_{\sigma} \right\} - \overline{q_1 q_2} \delta^{+} \left\{ (q_1 q_2) \mu_{\sigma} \right\} \right\},$$
(14)

where the boson operators  $Q_{g6}$ ,  $Q_{g6}^{\dagger}$  satisfy the commutation relations

The commutation relations (15) results in the known within the RPA relations of orthogonality for the amplitudes  $\Psi$ ,  $\overline{\Psi}$ ,  $\overline{\Psi}$  and  $\overline{\Psi}$ .

Using the latter one can easily express the operators  $\ell$ ,  $\ell$ ,  $\ell$  and  $\ell$  through the operators  $Q^{\dagger}$  and Q. Substituting these expressions into Hamiltonian (1), we obtain the system Hamiltonian given in terms of ideal boson operators and containing the second, third and fourth order terms in the boson operators  $Q_{qq}^{\dagger}$ ,  $Q_{qq}^{\dagger}$ :

$$\begin{split} H &= \sum_{qq'} \{ \sum_{q} \mathcal{E}(q) (\Psi_{qq'}^{g} \Psi_{qq'}^{g'} + \Psi_{qq'}^{g} \Psi_{qq'}^{g'} - 2D_{g} D_{g'} \mathcal{X}_{\lambda\mu} \} Q_{g}^{\dagger} Q_{g} \\ &+ \sum_{qq'} \{ \sum_{qq'} \mathcal{E}(q) \Psi_{qq'}^{g} \Psi_{qq'}^{g'} - D_{g} D_{g'} \mathcal{X}_{\lambda\mu} \} (Q_{g}^{\dagger} Q_{g'}^{\dagger} + Q_{g'} Q_{g}) + \end{split}$$

$$+\frac{1}{4}\sum_{gg'g_{1}}(\mathcal{X}_{\lambda\mu}D_{g}D_{g'}\mathcal{A}(g',g_{1})+\mathcal{X}_{\lambda_{1}\mu_{1}}D_{g},D_{g'}\mathcal{A}(g',g_{1}))(Q_{g}^{+}Q_{g,+}^{+}+Q_{g}^{+}Q_{g_{1}}^{+}+Q_{g}Q_{g}^{+})+\frac{1}{4}\sum_{gg'g_{1}g_{2}}\mathcal{X}_{\lambda\mu}D_{g}D_{g'}(K(g_{2}g',gg_{1}))+K(g_{2}g',g_{1}g_{1}))(Q_{g}^{+}Q_{g_{2}}+h.c.)+(16)$$

$$+\frac{1}{2}\sum_{gg'g_{1}g_{2}g_{3}}(\mathcal{X}_{\lambda\mu}D_{g}D_{g'}K(g_{3}g',g_{1}g_{1})+\mathcal{X}_{\lambda_{2}\mu_{2}}D_{g}D_{g'}K(g_{1}g_{2}',g_{3}g))+\mathcal{X}_{\lambda_{2}\mu_{2}}D_{g}D_{g'}K(g_{1}g_{2}',g_{3}g))$$

$$\times Q_{g_{1}}^{+}Q_{g_{2}}^{+}Q_{g_{3}}Q_{g}-\frac{1}{\sqrt{2}}\sum_{gg_{1}g_{2}}U_{g_{1}g_{2}}(g)(Q_{g_{1}}^{+}Q_{g_{2}}^{+}Q_{g}+h.c)$$

$$+\frac{1}{2}\sum_{gg_{1}g_{2}}\mathcal{X}_{gg}D_{g'}(g)(Q_{g_{1}}^{+}Q_{g_{2}}^{+}Q_{g}+h.c)$$

+ + + \( \sum\_{99,9,9,9} \mathbb{X}\_{\mathbb{A}\mu} \Dg [\Dg, 9, 9, 9, (9) K (9, 39, 9, 9, 2) + \Dg, 9, K (9, 39, 9, 9, 4) +

+ Dq,94 K(q49; 9,92)](Q, Q, Q, Q, + h.c.)+ = = = = = = Dq ×

×[89,94(9)K(929,9493)+89,94(9)K(9,9,9493)](Qg, Qg, Qg, +h.c),

$$D_{g} = \sum_{qq'} f^{\lambda\mu}(qq') U_{qq'}^{(+)} Z_{qq'}^{g}, \qquad (17)$$

$$\Delta g_1 g_2(g) = \sum_{qq'q} f(qq') v_{qq'}^{(-)} (\psi_{qq'}^{g_1} \psi_{q'q_1}^{g_2} + \psi_{q'q_1}^{g_1} \psi_{qq_1}^{g_2}), \quad (18)$$

$$g_{1}g_{2}(g) = \sum_{qq'q_{1}} f^{\prime\prime}(qq') v_{qq'}^{(-)} \psi_{qq'}^{g_{1}} \psi_{qq_{1}}^{g_{2}},$$
 (19)

$$U_{g_1g_2}(g) = 2\sqrt{2} \mathcal{X}^{\lambda\mu}(D_g f_{g_1g_2}(g) + D_{g_1} \Delta_{gg_2}(g_1)),$$
 (20)

$$A(g,g_1) = \sum_{gg'g,g'} \sum_{g_2} (\psi_{gg'}^g - \psi_{gg'}^g) [Z_{gg_1}^{g_1} \psi_{gg'}^g \psi_{gg'}^g + Q_{gg'}^g] + (21)$$

For compactness, only the main terms of the third end fourth order in  $Q_q^+$ ,  $Q_q$  are given in (16). With the same sim, the terms containing the amplitudes  $\Psi$  and  $\Psi$  are omitted in (17)-(21) since the general structure of the coefficients is obvious from the above expressions and the quantum number 6 is not singled out explicitly. Moreover, we have used  $x_1 = 0$  and  $x_2 = -\frac{1}{a}$ .

The first two terms in (16) are the RPA Hamiltonian. Requiring it diagonal, we obtain the well-known expressions for the amplitudes  $\mathcal{Q}_q^{f}$  and  $\mathcal{Q}_{qq}^{f/16}$ . In our case, the terms quadratic in the operators  $\mathcal{Q}_q^{f}$ ,  $\mathcal{Q}_q^{f}$  are also given in the third and fourth terms of (16). They have been obtained in reducing to the normal form the third and fourth order terms in H. Their consideration brings us beyond the scope of the RPA. In the RPA we get  $D_g = \frac{\tau}{x^{\lambda\mu}\sqrt{2y_g}}$  and

The fifth term in (16) describes the energy shift of the twophonon pole. It is interesting to compare the results of this paper with those of refs. 4,5/ in which the effect of the Pauli principle on the properties of two-phonon states was taken into account in a different way. The comparison shows that up to the main terms containing only the amplitudes  $\ell_{qq'}^g$  the results of both the considerations for  $\mathcal{H}(g_1g_2|g_3')$  coincide.

The diagonal part of the four-boson term in (16) leads to the following expression for the energy shift of the two-phonon pole  $\Delta \omega_{q_1q_2}$ :

$$\Delta \omega_{g_1g_2} = -\frac{1}{4(1+\delta_{g_1g_2})} \sum_{i'} \left\{ \frac{K(g_2g_1|\lambda_1\mu_1 i'g_2)}{x^{\lambda_1\mu_1} V y_{g_1} V_{\lambda_1\mu_1 i'}} + \frac{K(g_1\lambda_2\mu_2 i' | g_2g_1)}{x^{\lambda_2\mu_2} V y_{g_2} V_{\lambda_2\mu_2 i'}} \right\},$$
where

\* O ( M1 M2; K1 K2 K3 K4) +

plus the terms containing small amplitudes  $(q_{g'})$ .

Here  $\mathcal{O}(y_1y_2,K_1K_2K_3K_4)$  comprisses the sum of the Kronecker symbols. The general form of  $\mathcal{K}(g',g'_2)g_2g_3$  is given in refs. 15, 15/. The comparison of expression (22) with the relevant formula from ref. 15/

indicates that both the methods of calculation provide similar expressions for the two-phonon energy shift.

The present investigation has confirmed the conclusion about the absence of collective two-phonon states in deformed nuclei.

#### References:

- 1. Bohr A., Mottelson B.R., Physica Scripta, 1982, v. 25, p. 28.
- Warner D.D., Casten P.F., Davidson W.F., Phys.Rev.C, 1981, v. 24,
   p. 1713. Casten R.F., Warner D.D., Phys.Rev.C, 1982, v. 25,
   p. 2019; Phys.Rev.Lett., 1982, v. 48, p. 666.
- 3. Соловьев В.Г., ЭЧАЯ, 1978, т. 9, с. 810.
- 4. Джолос Р.В., Молина Х.Л., Соловьев В.Г., ТМД, 1979, т. 40, с. 245. Jolos R.V., Molina J.L., Soloviev V.G., Z.Phys.A, 1980, v. 295, p. 147.
- Soloviev V.G., Shirikova N.Yu., Z.Phys.A, 1981, v. 301, p. 293.
   Соловьев В.Г., Ширикова Н.Ю., ЯФ, 1982, т. 37, с. 1976.
- 6. Соловьев В.Г., Пысьма в шэтф, 1984, т. 40, с. 398,
- 7. Соловьев В.Г., Стоянов Ч., Николаева Р., Изв. АН СССР, сер.физ., 1983, т. 47, с. 2082.
- 8. Peker L.K., Hamilton J.H., Puture Directions in Studies of Nuclei far from Stability, ed. J.Hamilton, Amsterdam, Oxford, N.Y.North-Holland P.C., p. 323, 1980.
- 9. Kleppinger E.W., Yates S.W., Phys.Rev.C, 1983, v. 28, p. 943.
- 10. Davidson W.F. et al., Phys.Lett.B, 1983, v. 130, p. 161.
- Dumitrescu T.S., Hamamoto I., Nucl. Phys. A, 1982, v. 383, p. 205.
   Matsuo M., Prog. Theor. Phys. 1984, v. 72, p. 666.
- 12. Kurcewicz W. et al., Nucl. Phys. A, 1976, v. 270, p. 175.
- Davidson W.F., Dixon W.R., Storey R.S., Can. Journ. Phys., 1984, v. 62, p. 1538.
  - Burke D.G. et al., Preprint Hamilton, Ontario, Canada, 1985.
- Тригорьев Е.П., Соловьев В.Г., Структура четных деформированных ядер., М., Наука, 1974.
- 15. Соловьев В.Г., ТМФ, 1982, т. 53, с. 399.
- 16. Соловьев В.Г., Теория сложных ядер, М., Наука, 1971.

Received by Publishing Department on October 9, 1985. Джолос Р.В., и др. Е4-85-728 Изучение сдвига двухфононных полюсов в деформированных ядрах методом бозонных разложений

На основе метода бозонных разложений построены новые фононы. Гамильтониан модели выражен через операторы новых фононов вплоть до членов четвертого порядка. Рассчитан сдвиг двухфононного полюса. С точностью до главных членов он совпадает со сдвигом, рассчитанным с RPA фононами при строгом учете принципа Паули. Тем самым подтверждено заключение об отсутствин коллективных двухфононных состояний в четно-четных деформированных ядрах.

Работа выполнена в Лаборатории теоретической физикн ОИЯИ

Препринт Объединенного института ядерных исследований. Дубна 1985

Jolos R.V. et al. E4-85-728 Investigation of the Two-Phonon Pole Shift in Deformed Nuclei by the Boson Expansion Method

New phonons are constructed by the boson expansion method. The model Hamiltonian is expressed through the new phonon operators up to the fourth order terms. The two-phonon pole shift is calculated. It coincides up to the main terms with that calculated with the RPA phonons and strict inclusion of the Pauli principle.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985