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OPEN QUANTUM SYSTEMS AND THE DAMPING OF COLLECTIVE MODES IN DEEP INELASTIC COLLISIONS

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INTRODUCTION

In the last years a large body of experimental data has been accumulated in the field of deep inelastic collisions (DIC)[1-3]. It is known that the essential feature of these collisions is the preservation of the binary character of the system so that the final fragments maintain some resemblance with the initial nuclei. Therefore, we have to discuss the collision dynamics in terms of the degrees of freedom associated with a two center shell model (charge asymmetry, mass asymmetry, neck, fragment deformations and the relative coordinates). The most characteristic feature of these collisions, in addition to charge and mass transfer, is the loss of kinetic energy.

These data allows a vivid discussion between the two extreme theoretical approaches: the transport theories which view this process as being due to independent particle propagation thus stressing the stochasting, random walk nature of the relaxation phenomenon[4] and the quantum mechanical collective theories which view this process as being due to large scale collective modes thus stressing the coherent nature of the relaxation phenomenon[5].

In the first approach the loss of kinetic energy is viewed as a direct process based on the exchange of nucleons[6-7] and in the second one as an indirect process in which collective surface modes and giant resonances are first excited coherently and then damped due to the coupling to the remaining, non-collective degrees of freedom[9].

It is now widely admitted that the introduction of dissipation in quantum mechanics is far from trivial.

One way to solve this problem is to assume that the mechanism of energy loss is similar with the loss of energy of a harmonic oscillator coupled with a large number of other harmonic oscillators. This way can be simulated by a friction term of Kostin type in the Schrödinger equation[10]. Already, on this line the role of collective motion and quantum fluctuations, for charge and mass equilibration in deep inelastic collisions, has been explored[11,12]. Also the influence of neutron degree of freedom on the dynamics of charge equilibration has been studied[13,14].

Another way is to introduce in quantum mechanics forces proportional to the velocity in analogy with the friction force in classical mechanics. It is known that such systems cannot be described by standard Hamiltonian mechanics. Such forces, which cause a decrease in the phase space volume, are more suitably described in the frame of the theory of stochastic processes. In order to prevent the fall in time of any finite volume in phase space into a volume smaller than $(\hbar/2)^n$ a diffusion process (stochastic process) which increases the volume in the phase space is needed. An equilibrium state is a state in which these two opposite tendencies balance. On this line it is necessary to understand how arise such quantum diffusion processes which balance the friction forces and prevent the violation of the uncertainty relations /4/.

We consider that the most appropriate way to introduce dissipation in quantum mechanics, especially for theories which view these processes as large scale damped collective modes is the Lindblad's axiomatic way, which replaces the dynamical group U_t, uniquely determined by its generator H (the Hamiltonian operator of the system) by the completely positive dynamical semigroup Φ_t with bounded generators V_j.

In the present paper, after a short presentation of the Lindblad results we obtained: a generalization of the fundamental constraints on quantum mechanical diffusion coefficients which appear in the corresponding master equations, a generalization of the Hasse pure state condition and a generalized Schrödinger type nonlinear and nonhermitic equation for an open system.

In the next chapters, the Schrödinger, Heisenberg and Weyl-Wigner-Moyal representations of the Lindblad equation are given explicitly. On the basis of these representations, it is shown that various master equations for the damped quantum oscillator used in the literature for the description of the damped collective modes in DIC are particular cases of the Lindblad equation and that the majority of these equations are not satisfying the constraints on quantum mechanical diffusion coefficients. Explicit expressions of the mean values and variances are also given.

Finally we have shown that the solution of the Lindblad equation in the Weyl-Wigner-Moyal representation is of Gaussian type if the initial form of the Wigner function is taken to be a Gaussian corresponding to a coherent wave function.

2. ON QUANTUM MECHANICAL MARKOVIAN MASTER EQUATIONS

The standard quantum mechanics is Hamiltonian. The time evolution of a closed physical system is given by a dynamical group U_t which is uniquely determined by its generator H which is the Hamiltonian operator of the system. The action of the dynamical group U_t on any density matrix ρ, from the set $\mathfrak{D}(\mathcal{H})$ of all density matrices of the quantum system whose corresponding Hilbert space is denoted by $\mathcal{H}$, is defined by

$$\rho(t) = U_t(\rho) = e^{-\frac{i}{\hbar}Ht}\rho e^{\frac{i}{\hbar}Ht} \tag{2.1}$$

for all $t \in (-\infty,\infty)$. If ρ is a pure state (i.e., $\rho^2=\rho$), then $U_t(\rho)$ is a pure state for all $t \in (-\infty,\infty)$. We remind that, according to von Neumann, density operators $\rho \in \mathfrak{D}(\mathcal{H})$ are trace class ($\mathrm{Tr}\rho < \infty$) self-adjoint ($\rho^*=\rho$), positive ($\rho > 0$) operators with $\mathrm{Tr}\rho = 1$. All these properties are conserved by the time evolution defined by U_t.

In the case of open quantum systems the main difficulty consists in finding such time evolutions Φ_t for density operators $\rho(t) = \Phi_t(\rho)$ which preserve these von Neumann conditions for all times. From this requirement it follows that Φ_t must have the following properties

$$\text{i) } \Phi_t(\lambda_1\rho_1+\lambda_2\rho_2) = \lambda_1\Phi_t(\rho_1)+\lambda_2\Phi_t(\rho_2);\ \lambda_1,\lambda_2 \geq 0 \text{ with } \lambda_1+\lambda_2=1, \tag{2.2}$$

i.e., Φ_t must preserve the convex structure of $\mathfrak{D}(\mathcal{H})$.

$$\text{ii) } \Phi_t(\rho^*) = \Phi_t(\rho)^* \tag{2.3}$$

$$\text{iii) } \Phi_t(\rho) > 0 \tag{2.4}$$

$$\text{iv) } \mathrm{Tr}\,\Phi_t(\rho) = 1. \tag{2.5}$$

But these conditions are not enough restrictive in order to give a complete description of the mappings Φ_t as in the case of the time evolutions U_t for closed systems. Even in the last case one has to impose other restrictions to U_t, namely, it must be a group $U_{t+s} = U_t U_s$. Also, it is evident that in this case $U_0(\rho) = \rho$ and $U_t(\rho) \to \rho$ in the trace norm when $t \to 0$.

Other properties of U_t can be more easily expressed for the dual group $\tilde{U}_t$ acting on the observables $A \in \mathfrak{B}(\mathcal{H})$, i.e., on the bounded operators on $\mathcal{H}$

$$\tilde{U}_t(A) = e^{\frac{i}{\hbar}Ht} A e^{-\frac{i}{\hbar}Ht}. \tag{2.6}$$

Then, it is evident that

$$\tilde{U}_t(AB) = \tilde{U}_t(A)\tilde{U}_t(B) \tag{2.7}$$

and

$$\tilde{U}_t(I) = I, \tag{2.8}$$

where I denotes the identity operator on $\mathcal{H}$. Also $\tilde{U}_t(A) \to A$ ultraweakly when $t \to 0$ and U_t is an ultraweakly continuous mapping[15-19]. These mappings have a strong positivity property called complete positivity:

$$\sum_{i,j} B_i^* \bar{U}_t (A_i^* A_j) B_j \geq 0, \quad A_i, B_i \in \mathcal{B}(\mathcal{H}) \tag{2.9}$$

which is an immediate consequence of the above-mentioned property (2.7).

$$\sum_{i,j} B_i^* \bar{U}_t (A_i^* A_j) B_j = \sum_i (\bar{U}_t(A_i) B_i)^* \sum_j (\bar{U}_t(A_j) B_j).$$

Because the detailed physically plausible conditions on the systems, which correspond to these properties, are not known, it is much more convenient to adopt an axiomatic point of view which is based mainly on the simplicity and the success of physical applications. Accordingly[15-19] it is convenient to suppose that the time evolutions Φ_t for open systems are not very different from the time evolutions for closed systems. The simplest dynamics Φ_t which introduces a preferred direction in time, which is characteristic for dissipative processes, is that in which the group condition is replaced by the semigroup condition:

$$\Phi_{t+s} = \Phi_t \Phi_s, \qquad t \geq 0. \tag{2.10}$$

The property (2.7) of the time evolutions for closed systems which is too strong is replaced by the complete positivity condition:

$$\sum_{i,j} B_i^* \tilde{\Phi}_t (A_i^* A_j) B_j \geq 0, \quad A_i, B_i \in \mathcal{B}(\mathcal{H}), \tag{2.11}$$

where $\tilde{\Phi}_t$ denotes the dual of Φ_t acting on $\mathcal{B}(\mathcal{H})$ and is defined by the duality condition:

$$\mathrm{Tr}(\Phi_t(\rho) A) = \mathrm{Tr}(\rho \tilde{\Phi}_t(A)). \tag{2.12}$$

Then the conditions

$$\mathrm{Tr}\, \Phi_t(\rho) = 1 \tag{2.13}$$

and

$$\tilde{\Phi}_t(I) = I \tag{2.14}$$

are equivalent.

Also the conditions:

$$\tilde{\Phi}_t(A) \to A \text{ ultraweakly when } t \to 0, \tag{2.15}$$

and

$$\Phi_t(\rho) \to \rho \text{ in the trace norm when } t \to 0, \tag{2.16}$$

are equivalent.

For the semigroups with the properties (2.13), (2.15) and with a more weak property of positivity than (2.11), namely $A \geq 0 \to \tilde{\Phi}_t(A) \geq 0$, it is well known that there exists a (generally unbounded) mapping $\tilde{L}$ defined on an ultraweakly dense domain such that

$$\lim_{t \to 0} \| \tilde{L}(A) - \frac{1}{t}(\tilde{\Phi}_t(A) - A) \| = 0 \tag{2.17}$$

for A in the domain[15-19]. $\tilde{L}$ is called the generator of $\tilde{\Phi}_t$ and $\tilde{\Phi}_t$ is uniquely determined by $\tilde{L}$. The dual generator of the dual semigroup Φ_t will be denoted by L:

$$\mathrm{Tr}(L(\rho) A) = \mathrm{Tr}(\rho \tilde{L}(A)). \tag{2.18}$$

The evolution equations by which $L(\tilde{L})$ determine uniquely $\Phi_t(\tilde{\Phi}_t)$ are

$$\frac{d\Phi_t(\rho)}{dt} = L(\Phi_t(\rho)) \tag{2.19}$$

and

$$\frac{d\tilde{\Phi}_t(\rho)}{dt} = \tilde{L}(\tilde{\Phi}_t(\rho)) \tag{2.20}$$

respectively in the Schrödinger and Heisenberg picture. These equations replace in the case of open systems the von Neumann-Liouville equations

$$\frac{dU_t(\rho)}{dt} = -\frac{i}{\hbar}[H, U_t(\rho)]. \tag{2.21}$$

and

$$\frac{d\tilde{U}_t(A)}{dt} = \frac{i}{\hbar}[H, \tilde{U}_t(A)], \tag{2.22}$$

respectively.

For any applications the equations (2.19) or (2.20) are only useful if the detailed structure of the generator $L(\tilde{L})$ is known and can be related to the concrete properties of the open systems which are described by such equations.

Such a structural theorem was obtained by Lindblad in[18] for the class of dynamical semigroups $\tilde{\Phi}_t$ which are completely positive and norm continuous, i.e.,

$$\lim_{t\downarrow 0} \| \tilde{\Phi}_t(A) - A \| = 0 \tag{2.23}$$

a condition which is more restrictive than the ultraweak continuity imposed above. For such semigroups the generator $\tilde{L}$ is bounded. In many applications the generator is unbounded. According to Lindblad /18/ the following argument can be used to justify the complete positivity of $\tilde{\Phi}_t$: if the open system is extended in a trivial way to a larger system described in a Hilbert space $\mathcal{H} \otimes \mathcal{K}$ with the time evolution defined by

$$\tilde{W}_t(A \otimes B) = \tilde{\Phi}_t(A) \otimes B, \quad A \in \mathcal{B}(\mathcal{H}), \quad B \in \mathcal{B}(\mathcal{K}), \tag{2.24}$$

then the positivity of the states of the compound system will be preserved by W_t only if $\tilde{\Phi}_t$ is completely positive. With this observation a new equivalent definition of the complete positivity is obtained. $\tilde{\Phi}_t$ is completely positive if $\tilde{W}_t$ is positive for any finite dimensional Hilbert space $\mathcal{K}$.

For completely positive mappings $\tilde{\Phi}_t$ there exists a more direct analogue of (2.7), namely the 2-positivity property

$$\tilde{\Phi}_t(A^*A) \geq \tilde{\Phi}_t(A^*)\tilde{\Phi}_t(A) \tag{2.25}$$

with equality at $t=0$.

This property follows from (2.11) if $i,j = 1,2$ and $A_1 = A$, $B_1 = I$, $A_2 = I$, $B_2 = -\tilde{\Phi}_t(A)$. Differentiation of the inequality (2.25) gives at $t=0$

$$\tilde{L}(A^*A) - \tilde{L}(A^*)A - A^*\tilde{L}(A) \geq 0 \tag{2.26}$$

for all $A \in \mathcal{B}(\mathcal{H})$. A bounded mapping $\tilde{L}: \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ which satisfies $\tilde{L}(I) = 0$, $\tilde{L}(A^*) = \tilde{L}(A)^*$ and (2.26) is called dissipative. Hence the 2-positivity of $\tilde{\Phi}_t$ implies that $\tilde{L}$ is dissipative. Lindblad has shown that conversely the dissipativity of $\tilde{L}$ implies that $\tilde{\Phi}_t$ is 2-positive /18/. $\tilde{L}$ is called completely dissipative if all trivial extensions of $\tilde{L}$:

$$\tilde{R}(A \otimes B) = \tilde{L}(A) \otimes B \tag{2.27}$$

to a compound system described by $\mathcal{H} \otimes \mathcal{K}$ with any finite dimensional Hilbert space $\mathcal{K}$ are dissipative. Then Lindblad /18/ has shown that there exists a one-to-one correspondence between the completely positive norm continuous semigroups $\tilde{\Phi}_t$ and completely dissipative generators $\tilde{L}$. The structural theorem of Lindblad gives then the most general form of a completely dissipative mapping $\tilde{L}$:

Theorem: $\tilde{L}$ is completely dissipative and ultraweakly continuous if and only if it is of the form

$$\tilde{L}(A) = \frac{i}{\hbar}[H, A] + \frac{1}{2\hbar}\sum_j (V_j^*[A, V_j] + [V_j^*, A]V_j), \tag{2.28}$$

where V_j, $\sum_j V_j^* V_j \in \mathcal{B}(\mathcal{H})$, $H \in \mathcal{B}(\mathcal{H})_{s.a.}$.

The dual generator on the state space (Schrödinger picture) is of the form

$$L(\rho) = -\frac{i}{\hbar}[H, \rho] + \frac{1}{2\hbar}\sum_j ([V_j\rho, V_j^*] + [V_j, \rho V_j^*]). \tag{2.29}$$

Equations (2.19) and (2.29) give an explicit form for the most general time-homogeneous quantum mechanical Markovian master equation with a bounded Liouville operator

$$\frac{d\Phi_t(\rho)}{dt} = L(\Phi_t(\rho)) = -\frac{i}{\hbar}[H, \Phi_t(\rho)] + \frac{1}{2\hbar}\sum_j ([V_j\Phi_t(\rho), V_j^*] + [V_j, \Phi_t(\rho)V_j^*]). \tag{2.30}$$

We should like to mention that all Markovian master equations found in the literature are of this form after some rearrangement of terms, even for unbounded generators.

It is also an empirical fact that for many physically interesting situations the time evolutions Φ_t drives the system toward a unique final state $\rho(\infty) = \lim_{t\to\infty}\Phi_t(\rho(0))$ for all $\rho(0) \in \mathcal{T}(\mathcal{H})$.

From the 2-positivity property of $\tilde{\Phi}_t^*$ (2.25) it follows that

$$\tilde{\Phi}_t(\sum_j V_j^* V_j) \geq \sum_j \tilde{\Phi}_t(V_j^*)\,\tilde{\Phi}_t(V_j). \tag{2.31}$$

Because the linear positive mapping $\mathcal{B}(\mathcal{H}) \to \mathbb{C}$ defined by $A \to \mathrm{Tr}\,\rho A$ is completely positive (hence 2-positive) it follows that

$$\mathrm{Tr}(\rho\,\tilde{\Phi}_t(\sum_j V_j^* V_j)) \geq \mathrm{Tr}(\rho\sum_j \tilde{\Phi}_t(V_j^*)\,\tilde{\Phi}_t(V_j)) \geq$$

$$> \sum_j \mathrm{Tr}(\rho\,\tilde{\Phi}_t(V_j^*))\,\mathrm{Tr}(\rho\,\tilde{\Phi}_t(V_j)) \tag{2.32}$$

or by duality

$$\mathrm{Tr}(\Phi_t(\rho)\sum_j V_j^* V_j) \geq \sum_j \mathrm{Tr}(\Phi_t(\rho)V_j^*)\,\mathrm{Tr}(\Phi_t(\rho)V_j). \tag{2.33}$$

This inequality can be considered as a generalization of the inequality (11) from /20/ to any Markovian master equation (2.30).

Now the equality in (2.33) is a necessary and sufficient condition for $\rho(t) = \Phi_t(\rho)$ to be a pure state for all times $t \geq 0$. Indeed, the necessary and sufficient condition for $\rho(t)$ to be a pure state is $\mathrm{Tr}\,\rho(t)^2 = 1$ for all $t \geq 0$ /30/. This condition is equivalent with the condition

$$\frac{d}{dt} \mathrm{Tr}\,\rho(t)^2=0, \text{ for all } t \geq 0, \tag{2.34}$$

i.e., with the condition

$$\mathrm{Tr}\,\rho(t)\, L(\rho(t))=0, \text{ for all } t \geq 0. \tag{2.35}$$

With the explicit form for $L(\rho(t))$ given by (2.30) it follows that

$$\mathrm{Tr}(\rho(t) L(\rho(t))) = \frac{1}{2\hbar} \sum_j (\mathrm{Tr}(\rho(t) V_j \rho(t) V_j^*) - \mathrm{Tr}(\rho(t)^2 V_j^* V_j)). \tag{2.36}$$

But for a pure state $\rho^2(t)=\rho(t)$ and $\rho(t) A \rho(t) = \mathrm{Tr}(\rho(t) A)\,\rho(t)$ for any $A \in \mathcal{B}(\mathcal{H})$. Then (2.36) can be written as

$$\sum_j \mathrm{Tr}(\rho(t) V_j)\, \mathrm{Tr}(\rho(t) V_j^*) = \mathrm{Tr}(\rho(t) \sum_j V_j^* V_j) \tag{2.37}$$

which is exactly the equality situation in (2.33) and is a generalization of the pure state condition[23-25] to all Markovian master equations.

Moreover, if $\rho(t)^2=\rho(t)$ for all $t \geq 0$, there exists a wave function $\psi \in \mathcal{H}$ such that

$$\rho(t)\phi = (\psi(t), \phi)\, \psi(t) \tag{2.38}$$

for any $\phi \in \mathcal{H}$. Then, it is interesting to obtain the evolution equation of $\dot{\phi}(t)$ which follows from the evolution equation (2.30) for $\rho(t)$. From $\rho^2(t) = \rho(t)$ it follows that

$$\frac{d\rho^2(t)}{dt}\phi = L(\rho(t))\,\rho(t)\phi + \rho(t)\, L(\rho(t))\phi = \frac{d\rho(t)}{dt}\phi \tag{2.39}$$

for any $\phi \in \mathcal{H}$ and any $t \geq 0$. Now using the explicit form for L (2.30) and $\rho^2(t) = \rho(t)$; $\rho(t) A \rho(t) = \mathrm{Tr}(\rho(t) A)\, \rho(t)$ it follows that

$$\left(\frac{d\psi(t)}{dt}, \phi\right) \psi(t) + (\psi(t), \phi) \frac{d\psi(t)}{dt} = \frac{i}{\hbar}[\rho(t), H]\phi +$$

$$+ \frac{1}{2\hbar} \sum_j (2\, \mathrm{Tr}(\rho(t) V_j)\, \rho(t) V_j^* - 2\, \mathrm{Tr}(\rho(t) V_j^* V_j)\, \rho(t) +$$

$$+ 2 V_j\, \mathrm{Tr}(\rho(t) V_j^*)\, \rho(t) - V_j^* V_j \rho(t) - \rho(t) V_j^* V_j)\phi =$$

$$= (\psi(t), (\frac{i}{\hbar} H + \frac{1}{\hbar} \sum_j (\psi(t), V_j \psi(t)) V_j^* - \frac{1}{2\hbar}(\psi(t), \sum_j V_j^* V_j \phi(t)) I -$$

$$- \frac{1}{2\hbar} \sum_j V_j^* V_j)\phi)\, \psi(t) + (\psi(t), \phi)(-\frac{i}{\hbar} H + \frac{1}{\hbar} \sum_j (\psi(t), V_j^* \phi(t)) V_j - \tag{2.40}$$

$$- \frac{1}{2\hbar}(\psi(t), \sum_j V_j^* V_j \phi(t)) - \frac{1}{2\hbar} \sum_j V_j^* V_j)\, \psi(t)$$

for any $\phi \in \mathcal{H}$. Hence,

$$\frac{d\psi(t)}{dt} = (-\frac{i}{\hbar} H + \frac{1}{\hbar} \sum_j (\psi(t), V_j^* \psi(t)) V_j - \frac{1}{2\hbar}(\psi(t), \sum_j V_j^* V_j \psi(t)) - \frac{1}{2\hbar} \sum_j V_j^* V_j)\, \psi(t). \tag{2.41}$$

This is an equation of Schrödinger type for $\psi(t)$ with the Hamiltonian

$$H + i \sum_j (\psi(t), V_j^* \psi(t)) V_j - \frac{i}{2}(\psi(t), \sum_j V_j^* V_j \psi(t)) - \frac{i}{2} \sum_j V_j^* V_j \tag{2.42}$$

which is dependent of $\psi(t)$, i.e., this Schrödinger type equation is nonlinear. This result is a generalization to all Markovian master equations of the result obtained for the particular master equations in[21] and [24-25].

3. MASTER EQUATIONS FOR THE DAMPED QUANTUM HARMONIC OSCILLATOR

In this section the case of damped quantum harmonic oscillator is considered in the spirit of the ideas presented in the previous section.

The basic assumption is that the general form (2.29) of a bounded completely dissipative mapping L given by Lindblad theorem[18] is also valid for an unbounded completely dissipative mapping L:

$$L(\rho) = -\frac{i}{\hbar}[H, \rho] + \frac{1}{2\hbar} \sum_j ([V_j \rho, V_j^*] + [V_j, \rho V_j^*]). \tag{3.1}$$

This assumption gives one of the simplest way to construct an appropriate model for this quantum dissipative system[3].

Another simple condition imposed to the operators H, V_j, V_j^* is that they are functions of the basic observables of the one dimensional quantum mechanical system q and p (with $[q,p] = i\hbar I$, where I is the identity operator on $\mathcal{H}$) of such kind that the obtained model is exactly solvable. A precise version for this last condition is that linear spaces spanned by the first degree and respectively second degree noncommutative polynomials in p and q are invariant to the action of the completely dissipative mapping L. This condition implies[17] that V_j are at most the first degree polynomials in p and q and H is at most a second degree polynomial in p and q. These assumptions are of the same kind as those made in classical mechanics when one takes the friction force proportional to the velocity.

Because in the linear space of the first degree polynomials in p and q the operators p and q give a basis, there exist only two $\mathbb{C}$-linear independent operators V_1, V_2 which can be written in the form of

$$V_i = a_i p + b_i q, \quad i = 1, 2 \tag{3.2}$$

with a_i, b_i = 1,2 complex numbers[17]. The constant term is omitted because its contribution to the generator L is equivalent to terms in H linear in p and q which for simplicity are chosen to be zero. Then H is chosen of the form

$$H = \frac{1}{2m} p^2 + \frac{m\omega^2}{2} q^2 + \frac{\mu}{2}(pq + qp). \tag{3.3}$$

With these choices (3.1) becomes

$$L(\rho) = -\frac{i}{\hbar}[H, \rho] + \frac{1}{2\hbar}\sum_j \left([(a_j p + b_j q)\rho, (\bar{a}_j p + \bar{b}_j q)] + [(a_j p + b_j q), \rho(\bar{a}_j p + \bar{b}_j q)]\right), \tag{3.4}$$

where $\bar{a}_j$, $\bar{b}_j$ denote the complex conjugate of a_j, b_j. Then with the notations of[20-23]

$$H_0 = \frac{1}{2m} p^2 + \frac{m\omega^2}{2} q^2; \quad D_{qq} = \frac{\hbar}{2}\sum_{j=1}^{2} |a_j|^2; \quad D_{pp} = \frac{\hbar}{2}\sum_{j=1}^{2} |b_j|^2;$$

$$D_{pq} = D_{qp} = -\frac{\hbar}{2}\,\mathrm{Re}\sum_{j=1}^{2} \bar{a}_j b_j; \quad \lambda = \mathrm{Im}\sum_{j=1}^{2} \bar{a}_j b_j, \tag{3.5}$$

which are different from the Lindblad notations[17], $L(\rho)$ takes the following most symmetric form:

$$L(\rho) = -\frac{i}{\hbar}[H_0, \rho] - \frac{i}{2\hbar}(\lambda + \mu)[q, (\rho p + p\rho)] + \frac{i}{2\hbar}(\lambda - \mu)[p, (\rho q + q\rho)] -$$

$$-\frac{D_{pp}}{\hbar^2}[q,[q,\rho]] - \frac{D_{qq}}{\hbar^2}[p,[p,\rho]] + \frac{D_{pq}}{\hbar^2}[q,[p,\rho]] + \frac{D_{qp}}{\hbar^2}[p,[q,\rho]]. \tag{3.6}$$

Because the following identities are true:

$$\begin{aligned} &[q,[p,\rho]] = [p[q,\rho]] \\ &[q, \rho p + p\rho] + [p, \rho q + q\rho] = -[\rho, pq + qp] \\ &[q, \rho p + p\rho] - [p, \rho q + q\rho] = 2[q, \rho p + p\rho] + [\rho, pq + qp] \\ &[q, \rho p + p\rho] - [p, \rho q + q\rho] = -2[p, \rho q + q\rho] - [\rho, pq + qp] \end{aligned} \tag{3.7}$$

$L(\rho)$ can be put also in the following equivalent form:

$$L(\rho) = -\frac{i}{\hbar}[H_0, \rho] - \frac{i(\lambda - \mu)}{2\hbar}[\rho, pq + qp] - \frac{i\lambda}{\hbar}[q, p\rho + \rho p] - \frac{D_{qq}}{\hbar^2}[p,[p,\rho]] - \frac{D_{pp}}{\hbar^2}[q,[q,\rho]] + \frac{(D_{pq} + D_{qp})}{\hbar^2}[p,[q,\rho]]. \tag{3.8}$$

In this last form (3.8) a direct comparison with Eq.(1) from[20] is possible. It follows that eq.(1) from[20] is a particular case of (3.8) when $\mu = \lambda$. This is a necessary and sufficient condition for L to be translation invariant[17]. Translation invariance means that

$$[p, L(\rho)] = L([p, \rho]). \tag{3.9}$$

In the following general values for λ and μ will be considered, as in Lindblad[17].

Now the first important consequences are the following fundamental constraints on the quantum mechanical diffusion coefficients D_{pp}, D_{qq}, D_{pq}

$$\begin{aligned} &\text{(i)} \; D_{qq} > 0 \\ &\text{(ii)} \; D_{pp} > 0 \\ &\text{(iii)} \; D_{qq} D_{pp} - D_{pq}^2 \geq \frac{\lambda^2 \hbar^2}{4} \end{aligned} \tag{3.10}$$

which are identical with the results of ref.[20] Indeed, the first two inequalities (i) and (ii) follow directly from the definitions (3.5) and the last one (iii) follows from the Schwartz inequality:

$$\left(\mathrm{Re}\sum_{j=1}^{2} \bar{a}_j b_j\right)^2 + \left(\mathrm{Im}\sum_{j=1}^{2} \bar{a}_j b_j\right)^2 \leq \sum_{j=1}^{2} |a_j|^2 \sum_{j=1}^{2} |b_j|^2. \tag{3.11}$$

and the definitions (3.5). The equality in (3.10) (iii) is satisfied if and only if $a_1 = zb_1$, $a_2 = zb_2$, where z is a complex number. Then from (3.5) it follows that $D_{qq} = |z|^2 D_{pp}$, $D_{pq} = D_{qp} = -(\mathrm{Re}\, z) D_{pp}$ and $\lambda = -\frac{2}{\hbar}(\mathrm{Im}\, z) D_{pp}$. Moreover, if the conditions (3.10) are satisfied, then there exists $a_j, b_j \in \mathbb{C}$, $j = 1,2$ so that the diffusion coefficients D_{pp}, D_{qq}, D_{pq} and the friction constant λ are given by (3.5). It follows that the Dekker master equation (see rel.(1) from [20-23]) supplemented with the fundamental constraints (3.10), obtained in [20] from the condition that the time evolution of this master equation does not violate the uncertainty principle at any time, is a particular case of the Lindblad master equation given in [17].

Because the final scope of this paper is the study of the collective coordinates as open systems in heavy-ion collisions it is interesting to consider various master equations studied in connection with this subject in literature.

Thus, the quantum master equation (12) considered in [26] is with notations of the present paper of the following form

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H_0, \rho(t)] - \frac{i\gamma(\omega)}{2m\hbar}[q, p\rho(t) + \rho(t)p] - \frac{\gamma(\omega)T^*(\omega)}{\hbar^2}[q,[q,\rho(t)]], \tag{3.12}$$

i.e., $\lambda = \gamma(\omega)/2m = \mu$ and $D_{qq} = 0$, $D_{pp} = \gamma(\omega)T^*(\omega)$ and $D_{pq} = 0$. Then evidently the fundamental constraints (3.10) are not satisfied.

The quantum master equation (A.36) considered in [25] is

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H_0, \rho(t)] - \frac{i\gamma}{2\hbar}[q,(\rho(t)p + p\rho(t))] - \frac{D}{\hbar^2}[q,[q,\rho(t)]] - \frac{d}{\hbar^2}[q,[p,\rho(t)]], \tag{3.13}$$

where $H_0 = \frac{p^2}{2m} + \frac{1}{2}m(\omega^2 - \bar{k}^2)q^2$.

Hence in this case $\lambda = \mu = \frac{\gamma}{2}$, $D_{pp} = D$, $D_{qq} = 0$, $D_{pq} = D_{qp} = -\frac{d}{2}$ and also the fundamental constraints (3.10) are not fulfilled.

In a more recent paper [28] two kinds of quantum master equations (eq.(5.1) (ansatz I) and eq.(5.6) (ansatz II) of ref. [28]) written for the Wigner transform of the density matrix are obtained. Translated back in equations for the density matrix we get for the ansatz I the following master equation

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] - \frac{i\Gamma}{2\hbar}[q,(\rho(t)p + p\rho(t)] - \frac{D}{2\hbar^2}[q,[q,\rho(t)]] + \frac{B}{\hbar^2}[q,[p,\rho]], \tag{3.14}$$

where $H = H_0 - \frac{Am\omega}{2}q^2 + f(t)q$. This equation can be obtained from the eq.(3.6) by replacing H_0 with H and setting $\lambda = \mu = \frac{\Gamma}{2}$, $D_{pp} = D/2$, $D_{qq} = 0$ and $D_{pq} = D_{qp} = B/2$. Then, the constraints (3.10) are not satisfied. For the ansatz II the corresponding master equation is

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] - \frac{i\Gamma_p^{II}}{2\hbar}[q, \rho(t)p + p\rho(t)] + \frac{i\Gamma_R^{II}}{2\hbar}[p, \rho(t)q + q\rho(t)] - \frac{D_p^{II}}{2\hbar^2}[q,[q,\rho(t)]] - \frac{D_R^{II}}{2\hbar^2}[p,[p,\rho(t)]], \tag{3.15}$$

where $H = H_0 - \frac{A^{II}m\omega}{2}q^2 - \frac{A^{II}}{2m\omega}p^2 + f(t)q$ and $\Gamma_p^{II} = \Gamma_R^{II}$, $D_p^{II}/m\hbar\omega = D_R^{II} m\omega/\hbar$. It follows that this equation obtained from (3.6) by putting H instead of H_0 and $(\lambda + \mu) = \Gamma_p^{II} = \Gamma_R^{II} = (\lambda - \mu)$ (i.e., $\mu = 0$ and $\Gamma_p^{II} = \Gamma_R^{II} = \lambda$) and $D_{pp} = +\frac{D_p^{II}}{2}$, $D_{qq} = +\frac{D_R^{II}}{2}$, $D_{pq} = 0$. In the concrete model developed in [28]

$$\Gamma_p^{II} = \Gamma_R^{II} = \frac{2\pi W_0}{\hbar}\sinh\frac{\hbar\omega}{2kT}, \quad D_p^{II} = 2\pi W_0 m\omega\cosh\frac{\hbar\omega}{2kT}, \quad D_R^{II} = \frac{2\pi W_0}{m\omega}\cosh\frac{\hbar\omega}{2kT}.$$

Evidently $D_{pp} = \pi W_0 m\omega\cosh\frac{\hbar\omega}{2kT} > 0$ and $D_{qq} = \frac{\pi W_0}{m\omega}\cosh\frac{\hbar\omega}{2kT} > 0$ if $W_0 > 0$ and the condition (3.10) (iii) is satisfied for all values of the parameters m, ω, W_0, T because $D_{pq} = 0$ and

$$D_{pp}D_{qq} - \frac{\hbar^2\lambda^2}{4} = (\pi W_0)^2 > 0. \tag{3.16}$$

Now in order to develop further the comparison of the results obtained from the Lindblad master equation (3.6) with the results obtained in literature for other various master equation a concrete particular form of the general equations (2.33), (2.37) and (2.41) will be given.

The following notations will be used in the following:

$$\begin{aligned}
\sigma_q(t) &= \mathrm{Tr}(\rho(t)q) \\
\sigma_p(t) &= \mathrm{Tr}(\rho(t)p) \\
\sigma_{qq}(t) &= \mathrm{Tr}(\rho(t)q^2) - \sigma_q(t)^2 \\
\sigma_{pp}(t) &= \mathrm{Tr}(\rho(t)p^2) - \sigma_p(t)^2 \\
\sigma_{pq}(t) &= \mathrm{Tr}(\rho(t)(\frac{pq + qp}{2})) - \sigma_q(t)\sigma_p(t).
\end{aligned} \tag{3.17}$$

Then $\mathrm{Tr}\rho(t)V_j = \overline{\mathrm{Tr}\rho(t)V_j^*} = a_j\sigma_p(t) + b_j\sigma_q(t)$, hence

$$\sum_j \mathrm{Tr}(\rho(t)V_j^*)\mathrm{Tr}(\rho(t)V_j) = \sum_j(\bar a_j\sigma_p(t)+\bar b_j\sigma_q(t))(a_j\sigma_p(t)+b_j\sigma_q(t)) =$$

$$= \frac{2}{\hbar}D_{qq}\sigma_p(t)^2 + \frac{2}{\hbar}D_{pp}\sigma_q(t)^2 - \frac{4}{\hbar}D_{pq}\sigma_p(t)\sigma_q(t) \qquad (3.18)$$

and

$$\mathrm{Tr}\,\rho(t)\sum_j V_j^*V_j = \frac{2}{\hbar}D_{qq}\mathrm{Tr}(\rho(t)p^2) + \frac{2}{\hbar}D_{pp}\mathrm{Tr}(\rho(t)q^2) -$$

$$- \frac{4}{\hbar}D_{pq}\mathrm{Tr}\left(\rho(t)\frac{(pq+qp)}{4}\right) - i\lambda\,\mathrm{Tr}\,\rho(t)(pq-qp). \qquad (3.19)$$

Then the equality (2.33) becomes

$$D_{qq}\sigma_{pp}(t) + D_{pp}\sigma_{qq}(t) - 2D_{pq}\sigma_{pq}(t) \ge \frac{\hbar^2\lambda}{2}, \qquad (3.20)$$

where the equality corresponds to the eq.(2.37). We should like to remark that the inequality (3.20) has been obtained in ref./20/ (see. eq.(8)) in a completely different way and the equality (3.20) corresponding to the so-called pure state condition was obtained in ref./25/ for the particular master equation (3.13)

$$(D\sigma_{qq}(t) + d\sigma_{pq}(t) = \frac{\hbar^2\gamma}{4}.$$

With the present notations the Hamiltonian of the nonlinear Schrödinger equation (2.41) becomes:

$$H + \lambda(\sigma_p(t)q - \sigma_q(t)p) + i\Big(\lambda\hbar - \frac{D_{qq}}{\hbar}((p-\sigma_p(t))^2 + \sigma_{pp}(t)) -$$

$$- \frac{D_{pp}}{\hbar}((q-\sigma_q(t))^2 + \sigma_{qq}(t)) + \frac{2D_{pq}}{\hbar}\Big(\frac{(p-\sigma_p(t))(q-\sigma_q(t)) + (q-\sigma_q(t))(p-\sigma_p(t))}{2} + \sigma_{pq}(t)\Big). \qquad (3.21)$$

It is interesting to remark that the mean value of this Hamiltonian in the state $\rho(t)$ is equal to the mean value of the Hamiltonian H if the equality in (3.20) is valid. In this last case the Hamiltonian is equal to

$$H + \lambda(\sigma_p(t)q - \sigma_q(t)p) -$$

$$-\frac{i}{\hbar}\Big(D_{qq}(p-\sigma_p(t))^2 + D_{pp}(q-\sigma_q(t))^2 + D_{pq}((p-\sigma_p(t))(q-\sigma_q(t)) + (q-\sigma_q(t))(p-\sigma_p(t))) - \frac{\lambda\hbar^2}{2}\Big). \qquad (3.22)$$

This result, from the physical point of view, is quite natural since the average value of the new Hamiltonian, of the nonlinear Schrödinger equation describing the open system must give the energy of the open system.

In the Heisenberg picture the master equation has the following symmetric form:

$$\frac{d\tilde\Phi_t(A)}{dt} = \tilde L(\tilde\Phi_t(A)) = \frac{i}{\hbar}[H_0,\tilde\Phi_t(A)] - \frac{i}{2\hbar}(\lambda+\mu)([\tilde\Phi_t(A)q]p + p[\tilde\Phi_t(A)q]) +$$

$$+ \frac{i}{2\hbar}(\lambda-\mu)(q[\tilde\Phi_t(A),p] + [\tilde\Phi_t(A),p]q) - \frac{D_{pp}}{\hbar^2}[q,[q,\tilde\Phi_t(A)]] - \qquad (3.23)$$

$$- \frac{D_{qq}}{\hbar^2}[p,[p,\tilde\Phi_t(A)]] + \frac{D_{pq}}{\hbar^2}[p,[q,\tilde\Phi_t(A)]] + \frac{D_{qp}}{\hbar^2}[q,[p,\tilde\Phi_t(A)]].$$

Denoting by A any selfadjoint operator we have

$$\sigma_A(t) = \mathrm{Tr}\,\rho(t)A, \quad \sigma_{AA}(t) = \mathrm{Tr}\,\rho(t)A^2 - \sigma_A(t)^2.$$

It follows that

$$\frac{d\sigma_A(t)}{dt} = \mathrm{Tr}\,L(\rho(t))A = \mathrm{Tr}\,\rho(t)\tilde L(A) \qquad (3.24)$$

and

$$\frac{d\sigma_{AA}(t)}{dt} = \mathrm{Tr}\,L(\rho(t))A^2 - 2\frac{d\sigma_A(t)}{dt}\sigma_A(t) = \mathrm{Tr}\,\rho(t)\tilde L(A^2) - \qquad (3.25)$$

$$- 2\sigma_A(t)\mathrm{Tr}\,\rho(t)\tilde L(A).$$

An important consequence of the precise version of solvability condition formulated at the beginning of the present section is the fact that when A is put equal to p or q in (3.24) and (3.25), then $\frac{d}{dt}\sigma_p(t)$ and $\frac{d}{dt}\sigma_q(t)$ are functions only of $\sigma_p(t)$ and $\sigma_q(t)$ and $\frac{d}{dt}\sigma_{pp}(t)$, $\frac{d}{dt}\sigma_{qq}(t)$ and $\frac{d}{dt}\sigma_{pq}(t)$ are functions only of $\sigma_{pp}(t)$, $\sigma_{qq}(t)$ and $\sigma_{pq}(t)$. This fact allows an immediate determination of the functions of time $\sigma_p(t)$, $\sigma_q(t)$, $\sigma_{pp}(t)$, $\sigma_{qq}(t)$, $\sigma_{pq}(t)$. The results are the following:

$$\frac{d\sigma_q(t)}{dt} = -(\lambda-\mu)\sigma_q(t) + \frac{1}{m}\sigma_p(t)$$

$$\frac{d\sigma_p(t)}{dt} = -m\omega^2\sigma_q(t) - (\lambda+\mu)\sigma_p(t) \qquad (3.26)$$

and

$$\frac{d\sigma_{qq}(t)}{dt} = -2(\lambda-\mu)\sigma_{qq}(t) + \frac{2}{m}\sigma_{pq}(t) + 2D_{qq}$$

$$\frac{d\sigma_{pp}(t)}{dt} = -2(\lambda+\mu)\,\sigma_{pp}(t) - 2m\omega^2\sigma_{pq}(t) + 2D_{pp}$$

$$\frac{d\sigma_{pq}(t)}{dt} = -m\omega^2\sigma_{qq}(t) + \frac{1}{m}\sigma_{pp}(t) - 2\lambda\sigma_{pq}(t) + 2D_{pq}\,. \tag{3.27}$$

From the eq. (3.27) all equations of this type considered in various papers in connection with damping of collective modes in deep inelastic collisions, are obtained as particular cases. For example, equations (6.3) from /28/ correspond to the master eq.(3.14) with $\lambda=\mu=\Gamma/2$; $D_{pp}=D/2$; $D_{qq}=0$; $D_{pq}=D_{qp}=B/2$ and $H=\frac{1}{2m}p^2+\frac{m\omega(\omega-A)}{2}q^2+f(t)q$ and are obtained from (3.27) for these particular values of λ , μ , D_{qq} , D_{pp} , D_{pq} and by setting $m\omega(\omega-A)$ instead of $m\omega^2$. The equations corresponding to the ansatz II in /28/, i.e., to the master equation (3.15) and which are not written in /28/ are

$$\frac{d\sigma_{qq}(t)}{dt} = -2\Gamma_R^{II}\sigma_{qq}(t) + \frac{2(1-\frac{A^{II}}{\omega})}{m}\sigma_{pq}(t) + D_R^{II}$$

$$\frac{d\sigma_{pp}(t)}{dt} = -2\Gamma_P^{II}\sigma_{pp}(t) - 2m\omega(\omega-A^{II})\sigma_{pq}(t) + D_P^{II} \tag{3.28}$$

$$\frac{d\sigma_{pq}(t)}{dt} = -m\omega(\omega-A^{II}) + \frac{1}{m}(1-\frac{A^{II}}{\omega})\,\sigma_{pp}(t) - (\Gamma_P^{II}+\Gamma_R^{II})\sigma_{pq}(t).$$

The integration of the eqs.(3.26) is straightforward. There are two cases: a) the case $\mu>\omega$ (overdamped) and b) the case $\mu<\omega$ (underdamped). If $S(t)$ denotes the vector $\begin{pmatrix}\sigma_q(t)\\ \sigma_p(t)\end{pmatrix}$ and M the 2x2 matrix

$$M = \begin{pmatrix} -(\lambda-\mu) & \frac{1}{m} \\ -m\omega^2 & -(\lambda+\mu) \end{pmatrix} . \tag{3.29}$$

then (3.26) becomes

$$\frac{dS(t)}{dt} = MS(t)\,. \tag{3.30}$$

Now M can be written as $M=N^{-1}FN$ with F a diagonal matrix. It follows that the solution of (3.30) is

$$S(t) = N^{-1}e^{Ft}NS(0). \tag{3.31}$$

In the case of a) with the notation $\nu^2=\mu^2-\omega^2$ the matrices N , N^{-1} and F are given by

$$N = \begin{pmatrix} m\omega^2 & \mu+\nu \\ m\omega^2 & \mu-\nu \end{pmatrix} ; \quad N^{-1} = \frac{1}{2m\omega^2\nu}\begin{pmatrix} -(\mu-\nu) & \mu+\nu \\ m\omega^2 & -m\omega^2 \end{pmatrix} ;$$

$$F = \begin{pmatrix} -(\lambda+\nu) & 0 \\ 0 & -(\lambda-\nu) \end{pmatrix} . \tag{3.32}$$

Then

$$N^{-1}e^{Ft}N = e^{-\lambda t}\begin{pmatrix} \cosh\nu t+\frac{\mu}{\nu}\sinh\nu t & \frac{1}{m\nu}\sinh\nu t \\ -\frac{m\omega^2}{\nu}\sinh\nu t & \cosh\nu t-\frac{\mu}{\nu}\sinh\nu t \end{pmatrix} \tag{3.33}$$

i.e.,

$$\sigma_q(t) = e^{-\lambda t}((\cosh\nu t+\frac{\mu}{\nu}\sinh\nu t)\,\sigma_q(0) + \frac{1}{m\omega}(\sinh\nu t)\,\sigma_p(0))$$

$$\sigma_p(t) = e^{-\lambda t}(-\frac{m\omega^2}{\nu}(\sinh\nu t)\,\sigma_q(0) + (\cosh\nu t-\frac{\mu}{\nu}\sinh\nu t)\,\sigma_p(0)). \tag{3.34}$$

If $\lambda>\nu$, then $\sigma_q(\infty)=\sigma_p(\infty)=0$. If $\lambda<\nu$, then $\sigma_q(\infty)=\sigma_p(\infty)=\infty$. In the case of b) with the notation $\Omega^2=\omega^2-\mu^2$ the matrices N , N^{-1} and F are given by

$$N = \begin{pmatrix} m\omega^2 & \mu+i\Omega \\ m\omega^2 & \mu-i\Omega \end{pmatrix} \tag{3.35}$$

$$N^{-1} = \frac{1}{2im\omega^2\Omega}\begin{pmatrix} -(\mu-i\Omega) & (\mu+i\Omega) \\ m\omega^2 & -m\omega^2 \end{pmatrix} \tag{3.36}$$

$$F = \begin{pmatrix} -(\lambda+i\Omega) & 0 \\ 0 & -(\lambda-i\Omega) \end{pmatrix} . \tag{3.37}$$

Then

$$N^{-1} e^{Ft} N = e^{-\lambda t}\begin{pmatrix} \cos \Omega t + \frac{\mu}{\Omega}\sin \Omega t & \frac{1}{m\Omega}\sin \Omega t \\ -\frac{m\omega^2}{\Omega}\sin \Omega t & \cos \Omega t - \frac{\mu}{\Omega}\sin \Omega t \end{pmatrix} \qquad (3.38)$$

i.e.,

$$\sigma_q(t) = e^{-\lambda t}\left(\left(\cos \Omega t + \frac{\mu}{\Omega}\sin \Omega t\right)\sigma_q(0) + \frac{1}{m\Omega}(\sin \Omega t)\,\sigma_p(0)\right)$$

$$\sigma_p(t) = e^{-\lambda t}\left(-\frac{m\omega^2}{\Omega}(\sin \Omega t)\,\sigma_q(0) + \left(\cos \Omega t - \frac{\mu}{\Omega}\sin \Omega t\right)\sigma_p(0)\right) \qquad (3.39)$$

and $\sigma_q(\infty) = \sigma_p(\infty) = 0$.

In order to integrate the eqs.(3.27) it is convenient to consider the vector

$$X(t) = \begin{pmatrix} m\omega\ \sigma_{qq}(t) \\ \frac{1}{m\omega}\ \sigma_{pp}(t) \\ \sigma_{pq}(t) \end{pmatrix} \qquad (3.40)$$

Then the system of eqs.(3.27) can be written in the form

$$\frac{dX(t)}{dt} = RX(t) + D, \qquad (3.41)$$

where R is the following 3x3 matrix

$$R = \begin{pmatrix} -2(\lambda - \mu) & 0 & 2\omega \\ 0 & -2(\lambda + \mu) & -2\omega \\ -\omega & \omega & -2\lambda \end{pmatrix} \qquad (3.42)$$

and D is the following vector

$$D = \begin{pmatrix} 2m\omega & D_{qq} \\ \frac{2}{m\omega} & D_{pp} \\ 2 & D_{pq} \end{pmatrix} \qquad (3.43)$$

Then there exists a matrix T with property $T^2 = I$ where I is the identity matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and a diagonal matrix K such that $R = TKT$. From this it follows that

$$X(t) = (Te^{Kt}T)X(0) + T(e^{Kt} - I)K^{-1}TD. \qquad (3.44)$$

In the overdamped case $(\mu > \omega)$ the matrices T and K are given by

$$T = \frac{1}{2\nu}\begin{pmatrix} \mu + \nu & \mu - \nu & 2\omega \\ \mu - \nu & \mu + \nu & 2\omega \\ -\omega & -\omega & -2\mu \end{pmatrix} \qquad (3.45)$$

and

$$K = \begin{pmatrix} -2(\lambda - \nu) & 0 & 0 \\ 0 & -2(\lambda + \nu) & 0 \\ 0 & 0 & -2\lambda \end{pmatrix} \qquad (3.46)$$

with $\nu^2 = \mu^2 - \omega^2$.

In the underdamped case $(\mu < \omega)$ the matrices T and K are given by

$$T = \frac{1}{2i\Omega}\begin{pmatrix} \mu + i\Omega & \mu - i\Omega & 2\omega \\ \mu - i\Omega & \mu + i\Omega & 2\omega \\ -\omega & -\omega & -2\mu \end{pmatrix} \qquad (3.47)$$

and

$$K = \begin{pmatrix} -2(\lambda - i\Omega) & 0 & 0 \\ 0 & -2(\lambda + i\Omega) & 0 \\ 0 & 0 & -2\lambda \end{pmatrix} \qquad (3.48)$$

with $\Omega^2 = \omega^2 - \mu^2$.

From (3.44) it follows that

$$X(\infty) = -(TK^{-1}T)D = -R^{-1}D \qquad (3.49).$$

(in the overdamped case the restriction $\lambda > \nu$ is necessary). Then the eq.(3.44) can be written in the form

$$X(t) = (T e^{Kt} T)(X(0) - X(\infty)) + X(\infty). \tag{3.50}$$

Also

$$\frac{dX(t)}{dt} = (TK e^{Kt} T)(X(0) - X(\infty)) = R(X(t) - X(\infty)) \tag{3.51}$$

and

$$\frac{dX(t)}{dt}\Big|_{t=0} = (TKT)(X(0) - X(\infty)) = R(X(0) - X(\infty)). \tag{3.52}$$

The formula (3.49) is remarkable because it gives a very simple connection between the asymptotic values of $\sigma_{qq}(t)$, $\sigma_{pp}(t)$, $\sigma_{pq}(t)$ and the diffusion coefficients D_{qq}, D_{pp}, D_{pq}. As an immediate consequence of (3.49) this connection is the same for both cases - underdamped and overdamped and has the following explicit form:

$$\sigma_{qq}(\infty) = \frac{1}{2(m\omega)^2\lambda(\lambda^2+\omega^2-\mu^2)}\left((m\omega)^2(2\lambda(\lambda+\mu)+\omega^2)D_{qq} + \omega^2 D_{pp} + 2m\omega^2(\lambda+\mu)D_{pq}\right)$$

$$\sigma_{pp}(\infty) = \frac{1}{2\lambda(\lambda^2+\omega^2-\mu^2)}\left((m\omega)^2\omega^2 D_{qq} + (2\lambda(\lambda-\mu)+\omega^2)D_{pp} - 2m\omega^2(\lambda-\mu)D_{pq}\right)$$

$$\sigma_{pq}(\infty) = \frac{1}{2m\lambda(\lambda^2+\omega^2-\mu^2)}\left(-(\lambda+\mu)(m\omega)^2 D_{qq} + (\lambda-\mu)D_{pp} + 2m(\lambda^2-\mu^2)D_{pq}\right). \tag{3.53}$$

These relations show that the asymptotic values $\sigma_{qq}(\infty)$, $\sigma_{pp}(\infty)$, $\sigma_{pq}(\infty)$ do not depend of the initial values $\sigma_{qq}(0)$, $\sigma_{pp}(0)$, $\sigma_{pq}(0)$. In other words,

$$R^{-1} = \frac{-1}{4\lambda(\lambda^2+\omega^2-\mu^2)}\begin{pmatrix} 2\lambda(\lambda+\mu)+\omega^2 & \omega^2 & 2\omega(\lambda+\mu) \\ \omega^2 & 2\lambda(\lambda-\mu)+\omega^2 & -2\omega(\lambda-\mu) \\ -(\lambda+\mu)\omega & (\lambda-\mu)\omega & 2(\lambda^2-\mu^2) \end{pmatrix} \tag{3.54}$$

Conversely, if the relations $D = -RX(\infty)$ are considered, i.e.,

$$\begin{pmatrix} 2m\omega\, D_{qq} \\ \frac{2}{m\omega} D_{pp} \\ 2\, D_{pq} \end{pmatrix} = -\begin{pmatrix} -2(\lambda-\mu) & 0 & 2\omega \\ 0 & -2(\lambda+\mu) & -2\omega \\ -\omega & \omega & -2\lambda \end{pmatrix}\begin{pmatrix} m\omega\, \sigma_{qq}(\infty) \\ \frac{1}{m\omega}\sigma_{pp}(\infty) \\ \sigma_{pq}(\infty) \end{pmatrix} \tag{3.55}$$

then

$$D_{qq} = (\lambda-\mu)\sigma_{qq}(\infty) - \frac{1}{m}\sigma_{pq}(\infty)$$

$$D_{pp} = (\lambda+\mu)\sigma_{pp}(\infty) + m\omega^2\sigma_{pq}(\infty) \tag{3.56}$$

$$D_{pq} = \frac{1}{2}\left(m\omega^2\sigma_{qq}(\infty) - \frac{1}{m}\sigma_{pp}(\infty) + 2\lambda\sigma_{pq}(\infty)\right).$$

Hence, from (3.10) follows the fundamental constraints on $\sigma_{qq}(\infty)$, $\sigma_{pp}(\infty)$; $\sigma_{pq}(\infty)$:

$$D_{qq} = (\lambda-\mu)\sigma_{qq}(\infty) - \frac{1}{m}\sigma_{pq}(\infty) > 0 \tag{3.57}$$

$$D_{pp} = (\lambda+\mu)\sigma_{pp}(\infty) + m\omega^2\sigma_{pq}(\infty) > 0 \tag{3.58}$$

$$\begin{aligned} D_{qq}D_{pp} - D_{pq}^2 &= (\lambda^2-\mu^2)\sigma_{qq}(\infty)\sigma_{pp}(\infty) - \omega^2\sigma_{pq}(\infty)^2 + \\ &+ (\lambda-\mu)m\omega^2\sigma_{qq}(\infty)\sigma_{pq}(\infty) - \frac{(\lambda+\mu)}{m}\sigma_{pp}(\infty)\sigma_{pq}(\infty) - \\ &- \frac{1}{4}(m\omega^2)^2\sigma_{qq}(\infty)^2 - \frac{1}{4m^2}\sigma_{pp}(\infty)^2 - \lambda^2\sigma_{pq}(\infty)^2 + \frac{1}{2}\omega^2\sigma_{qq}(\infty)\sigma_{pp}(\infty) - \\ &- m\omega^2\lambda\sigma_{qq}(\infty)\sigma_{pq}(\infty) + \frac{\lambda}{m}\sigma_{qq}(\infty)\sigma_{pq}(\infty) \geq \frac{\lambda^2\hbar^2}{4}. \end{aligned} \tag{3.59}$$

The constraint (3.59) can be put in a more clear form:

$$\begin{aligned} &4(\lambda^2+\omega^2-\mu^2)(\sigma_{qq}(\infty)\sigma_{pp}(\infty) - \sigma_{pq}(\infty)^2) - \\ &-(m\omega^2\sigma_{qq}(\infty) + \frac{1}{m}\sigma_{pp}(\infty) + 2\mu\sigma_{pq}(\infty))^2 \geq \hbar^2\lambda^2 \end{aligned} \tag{3.60}$$

If $\mu < \omega$ (the underdamped case), then $\lambda^2+\omega^2-\mu^2 > \lambda^2$. If $\mu > \omega$ (the overdamped case), then $0 \leq \lambda^2+\omega^2-\mu^2 < \lambda^2$ ($\lambda > \nu$) and the constraint (3.60) is more strong, then the uncertainty inequality $\sigma_{qq}(\infty)\sigma_{pp}(\infty) - \sigma_{pq}(\infty)^2 \geq \hbar^2/4$. Also from the inequality (3.20) which must be valid for all values of $t \in (0,\infty)$ it follows that

$$D_{qq}\sigma_{pp}(\infty) + D_{pp}\sigma_{qq}(\infty) - 2D_{pq}\sigma_{pq}(\infty) \geq \frac{\hbar^2\lambda}{2}. \tag{3.61}$$

Using (3.56) this inequality is equivalent with the uncertainty inequality $\sigma_{qq}(\infty)\sigma_{pp}(\infty) - \sigma_{pq}(\infty)^2 \geq \hbar^2/4$. A restriction connecting the initial values $\sigma_{qq}(0)$, $\sigma_{pp}(0)$, $\sigma_{pq}(0)$ with the asymptotic values $\sigma_{pp}(\infty)$, $\sigma_{qq}(\infty)$, $\sigma_{pq}(\infty)$ is also obtained:

$$D_{qq}\sigma_{pp}(0) + D_{pp}\sigma_{qq}(0) - 2D_{pq}\sigma_{pq}(0) \geq \frac{\hbar^2\lambda}{2}. \tag{3.62}$$

More explicitly

$$\begin{aligned}&\lambda(\sigma_{qq}(\infty)\sigma_{pp}(0) + \sigma_{pp}(\infty)\sigma_{qq}(0) - 2\sigma_{pq}(0)\sigma_{pq}(\infty)) - \mu(\sigma_{qq}(\infty)\sigma_{pp}(0) - \sigma_{pp}(\infty)\sigma_{qq}(0))\\ &- \frac{1}{m}(\sigma_{pq}(\infty)\sigma_{pp}(0) - \sigma_{pp}(\infty)\sigma_{pq}(0)) +\\ &+ m\omega^2(\sigma_{pq}(\infty)\sigma_{qq}(0) - \sigma_{qq}(\infty)\sigma_{pq}(0)) \geq \frac{\hbar^2\lambda}{2}.\end{aligned} \tag{3.63}$$

As was remarked in [20] if for a fixed value t_0 of t

$$\sigma_{qq}(t_0) = \frac{D_{qq}}{\lambda}, \quad \sigma_{pp}(t_0) = \frac{D_{pp}}{\lambda}, \quad \sigma_{pq}(t_0) = \frac{D_{pq}}{\lambda}, \tag{3.64}$$

then the inequality (3.20) in t_0 and also the uncertainty inequality are both equivalent to (3.10) (iii), i.e., this is an admissible choice. In the particular case of $t_0 = \infty$ the following relations are obtained from (3.64) and (3.56):

$$m\omega D_{qq} = \frac{1}{m\omega} D_{pp}, \quad D_{pq} = -m\mu D_{qq} = -\frac{\mu}{m\omega^2} D_{pp}. \tag{3.65}$$

Then, the fundamental constraint (3.10) (iii) implies that

$$m^2(\omega^2 - \mu^2) D_{qq}^2 \geq \frac{\lambda^2\hbar^2}{4}. \tag{3.66}$$

which can be satisfied only if $\omega > \mu$ (underdamped case). If the asymptotic state is a Gibbs state

$$\rho_G(\infty) = e^{-H_0/kT} / \mathrm{Tr}(e^{-H_0/kT})$$

then

$$\sigma_{qq}(\infty) = \frac{\hbar}{2m\omega}\coth\frac{\hbar\omega}{2kT}; \ \sigma_{pp}(\infty) = \frac{\hbar m\omega}{2}\coth\frac{\hbar\omega}{2kT}; \ \sigma_{pq}(\infty) = 0 \tag{3.67}$$

and

$$D_{pp} = \frac{(\lambda+\mu)}{2}\hbar m\omega\coth\frac{\hbar\omega}{2kT}; \ D_{qq} = \frac{(\lambda-\mu)}{2}\frac{\hbar}{m\omega}\coth\frac{\hbar\omega}{2kT}; \ D_{pq} = 0 \tag{3.68}$$

and the fundamental constraints (3.10) are satisfied only if $\lambda > \mu$ and [17]:

$$(\lambda^2 - \mu^2)\left(\coth\frac{\hbar\omega}{2kT}\right)^2 \geq \lambda^2. \tag{3.69}$$

If the initial state is the ground state of the harmonic oscillator, then

$$\sigma_{qq}(0) = \frac{\hbar}{2m\omega}, \quad \sigma_{pp}(0) = \frac{m\hbar\omega}{2}, \quad \sigma_{pq}(0) = 0. \tag{3.70}$$

Then (3.63) becomes

$$\lambda\left(\sigma_{qq}(\infty)m\omega + \frac{\sigma_{pp}(\infty)}{m\omega}\right) - \mu\left(\sigma_{qq}(\infty)m\omega - \frac{\sigma_{pp}(\infty)}{m\omega}\right) \geq \hbar\lambda. \tag{3.71}$$

For example, in the case (3.67) this implies $\coth\frac{\hbar\omega}{2kT} \geq 1$ which is always valid.

Another example is taken from [28] for the solutions corresponding to the ansatz II: $\mu = 0$, $\lambda = \Gamma$, $D_{pp} = D/2$, $D_{qq} = \frac{D}{2(m\omega)^2}$ (for simplicity we put $A^{II} = 0$). Then

$$\begin{aligned}&\sigma_{qq}(\infty) = \frac{D}{2(m\omega)^2\Gamma}; \ \sigma_{pp}(\infty) = \frac{D}{2\Gamma}; \ \sigma_{pq}(\infty) = 0;\\ &\sigma_{qq}(\infty)m\omega + \frac{\sigma_{pp}(\infty)}{m\omega} = \frac{D}{m\omega\Gamma},\end{aligned} \tag{3.72}$$

and (3.71) becomes

$$D/\Gamma \geq \hbar m\omega \tag{3.73}$$

which is the same constraint obtained from the fundamental constraint $\frac{D^2}{4(m\omega)^2} \geq \frac{\hbar^2\Gamma^2}{4}$. It follows also that the Heisenberg uncertainty inequality $\sigma_{qq}(\infty)\sigma_{pp}(\infty) \geq \hbar^2/4$ is also satisfied by (3.72).

For the case corresponding to the ansatz I, $\mu = \lambda = \Gamma/2$, $D_{pp} = D/2$, $D_{qq} = 0$, $D_{pq} = B/2$ it follows that

$$\sigma_{qq}(\infty) = \frac{D}{2(m\omega)^2\Gamma} + \frac{B}{m\omega^2}; \ \sigma_{pp}(\infty) = \frac{D}{2\Gamma}; \ \sigma_{pq}(\infty) = 0$$

$$\sigma_{qq}(\infty)\, m\omega + \frac{\sigma_{pp}(\infty)}{m\omega} = \frac{D}{m\omega\Gamma} + \frac{B}{\omega}; \quad \sigma_{qq}(\infty)\, m\omega - \frac{\sigma_{pp}(\infty)}{m\omega} = \frac{B}{\omega}. \qquad (3.74)$$

so that (3.71) gives again the inequality (3.73) in spite of the fact that in this case the fundamental constraint is not fulfilled. For (3.74) the Heisenberg inequality implies

$$B \geq -\frac{1}{m^2 \omega \hbar \sinh \frac{\hbar\omega}{kT}}. \qquad (3.75)$$

For the master equation (3.12) which is taken from [26] the following values for $\sigma_{qq}(\infty)$, $\sigma_{pp}(\infty)$, $\sigma_{pq}(\infty)$ follow from (3.53):

$$\sigma_{qq}(\infty) = \frac{T^*}{m\omega^2} = \frac{\hbar}{2m\omega} \coth \frac{\hbar\omega}{2kT}$$

$$\sigma_{pp}(\infty) = mT^* = \frac{m\omega\hbar}{2} \coth \frac{\hbar\omega}{2kT} \qquad (3.76)$$

$$\sigma_{pq}(\infty) = 0,$$

i.e., the values corresponding to a Gibbs state ρ_G at $t=\infty$.

Now, the explicit time dependence of $\sigma_{qq}(t)$, $\sigma_{pp}(t)$, $\sigma_{pq}(t)$ will be given for both under- and overdamped cases. From (3.50) it follows that in order to obtain this explicit time dependence it is necessary to obtain the matrix elements of $Te^{Kt}T$. In the overdamped case $(\mu > \omega)$, $\nu^2 = \mu^2 - \omega^2$ we have

$$Te^{Kt}T = \frac{e^{-2\lambda t}}{2\nu^2} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

with

$$\left.\begin{aligned}
a_{11} &= (\mu^2+\nu^2)\cosh 2\nu t + 2\mu\nu \sinh 2\nu t - \omega^2 \\
a_{12} &= (\mu^2-\nu^2)\cosh 2\nu t - \omega^2 \\
a_{13} &= 2\omega(\mu \cosh 2\nu t + \nu \sinh 2\nu t - \mu) \\
a_{21} &= (\mu^2-\nu^2)\cosh 2\nu t - \omega^2 \\
a_{22} &= (\mu^2+\nu^2)\cosh 2\nu t - 2\mu\nu \sinh 2\nu t - \omega^2 \\
a_{23} &= 2\omega(\mu \cosh 2\nu t - \nu \sinh 2\nu t - \mu) \\
a_{31} &= -\omega(\mu \cosh 2\nu t + \nu \sinh 2\nu t - \mu) \\
a_{32} &= -\omega(\mu \cosh 2\nu t - \nu \sinh 2\nu t - \mu) \\
a_{33} &= -2(\omega^2 \cosh 2\nu t - \mu^2).
\end{aligned}\right\} \qquad (3.77)$$

In the underdamped case $(\mu < \omega)$, $\Omega^2 = \omega^2 - \mu^2$ we have

$$Te^{Kt}T = -\frac{e^{-2\lambda t}}{2\Omega^2} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

with

$$\left.\begin{aligned}
b_{11} &= (\mu^2-\Omega^2)\cos 2\Omega t - 2\mu\Omega \sin 2\Omega t - \omega^2 \\
b_{12} &= (\mu^2+\Omega^2)\cos 2\Omega t - \omega^2 \\
b_{13} &= 2\omega(\mu \cos 2\Omega t - \Omega \sin 2\Omega t - \mu) \\
b_{21} &= (\mu^2+\Omega^2)\cos 2\Omega t - \omega^2 \\
b_{22} &= (\mu^2-\Omega^2)\cos 2\Omega t + 2\mu\Omega \sin 2\Omega t - \omega^2 \\
b_{23} &= 2\omega(\mu \cos 2\Omega t + \Omega \sin 2\Omega t - \mu) \\
b_{31} &= -\omega(\mu \cos 2\Omega t - \Omega \sin 2\Omega t - \mu) \\
b_{32} &= -\omega(\mu \cos 2\Omega t + \Omega \sin 2\Omega t - \mu) \\
b_{33} &= -2(\omega^2 \cos 2\Omega t - \mu^2)
\end{aligned}\right\} \qquad (3.78)$$

4. THE EXPLICIT ACTION OF THE DYNAMICAL SEMIGROUP ON THE WEYL OPERATORS

As was shown by Lindblad in [17] the equations of motion (3.23) written for Weyl operators $(A = W(\xi,\eta) = e^{\frac{i}{\hbar}(\eta q - \xi p)})$ can be integrated in a very simple and elegant way. This fact is important from both points of view: practical and theoretical. From practical point of view, it gives in a new way explicit formulas for $\sigma_{qq}(t)$, $\sigma_{pp}(t)$, $\sigma_{pq}(t)$ and moreover, it gives the action of the dynamical semigroup $\tilde{\Phi}_t$ generated by (3.23) on any polynomial in the noncommutative variables p and q. Because $\tilde{\Phi}_t(AB) \neq \tilde{\Phi}_t(A)\tilde{\Phi}_t(B)$ it is not sufficient to know $\tilde{\Phi}_t(p)$ and $\tilde{\Phi}_t(q)$ as in the case of dynamical groups $\tilde{U}_t(AB) = \tilde{U}_t(A)\tilde{U}_t(B)$, where evidently it is sufficiently to know the action of $\tilde{U}_t$ on p and q in order to know this action on all noncommutative polynomials. From theoretical point of view, the explicit action of $\tilde{\Phi}_t$ on Weyl operators $W(\xi,\eta)$ allows, as was shown by Lindblad in the Appendix of [17], to give a direct proof for the fact that the semigroup $\tilde{\Phi}_t$ is indeed a semigroup of completely positive mappings. This assertion cannot be considered as a consequence of the structural theorem of Lindblad because $\tilde{L}$ is unbounded.

Firstly, some formulas for Weyl operators which will be used in the following are given [17,18,30]:

$$W(\xi,\eta) = e^{-\frac{i}{\hbar}\xi p}\, e^{\frac{i}{\hbar}\eta q}\, e^{\frac{i\xi\eta}{2\hbar}} \tag{4.1}$$

$$W(\xi,\eta) = e^{\frac{i}{\hbar}\eta q}\, e^{-\frac{i}{\hbar}\xi p}\, e^{-\frac{i\xi\eta}{2\hbar}} \tag{4.2}$$

and

$$W(\xi_1,\eta_1)\,W(\xi_2,\eta_2) = e^{-\frac{i}{2\hbar}(\xi_1\eta_2-\xi_2\eta_1)}\, W(\xi_1+\xi_2,\eta_1+\eta_2) \tag{4.3}$$

The following consequences of (4.1), (4.2) and (4.3) are essentially used

$$W(\xi,\eta)\,p\,W(-\xi,-\eta) = p-\eta I \tag{4.4}$$

$$W(\xi,\eta)\,q\,W(-\xi,-\eta) = q-\xi I, \tag{4.5}$$

(I is the identity operator on $\mathcal{H}$)

$$[p, W(\xi,\eta)] = \eta W(\xi,\eta) \tag{4.6}$$

$$[q, W(\xi,\eta)] = \xi W(\xi,\eta) \tag{4.7}$$

$$[p^2, W(\xi,\eta)] = \eta(pW(\xi,\eta) + W(\xi,\eta)p) \tag{4.8}$$

$$[q^2, W(\xi,\eta)] = \xi(qW(\xi,\eta) + W(\xi,\eta)q) \tag{4.9}$$

$$[pq, W(\xi,\eta)] = \eta W(\xi,\eta)q + \xi p\,W(\xi,\eta) \tag{4.10}$$

$$[qp, W(\xi,\eta)] = \xi W(\xi,\eta)p + \eta q W(\xi,\eta) \tag{4.11}$$

$$\frac{\partial W(\xi,\eta)}{\partial\xi} = -\frac{i}{2\hbar}(pW(\xi,\eta) + W(\xi,\eta)p). \tag{4.12}$$

$$\frac{\partial W(\xi,\eta)}{\partial\eta} = \frac{i}{2\hbar}(qW(\xi,\eta) + W(\xi,\eta)q). \tag{4.13}$$

Then, we can write as Lindblad [17]

$$\tilde{\Phi}_t(W(\xi,\eta)) = W(\xi(t),\eta(t))\,e^{g(t)}\,, \tag{4.14}$$

where $\xi(0)=\xi$, $\eta(0)=\eta$, $g(0)=0$ and

$$\frac{d\xi(t)}{dt} = -(\lambda+\mu)\,\xi(t) - \frac{1}{m}\eta(t) \tag{4.15}$$

$$\frac{d\eta(t)}{dt} = m\omega^2\xi(t) - (\lambda-\mu)\,\eta(t) \tag{4.16}$$

$$\frac{dg(t)}{dt} = -\frac{1}{\hbar^2}(D_{pp}\xi(t)^2 + D_{qq}\eta(t)^2 - 2D_{pq}\xi(t)\eta(t)). \tag{4.17}$$

The proof of this fact is straightforward: with the use of above properties of Weyl operators (4.6)-(4.13):

$$\frac{d\tilde{\Phi}_t(W(\xi,\eta))}{dt} = e^{g(t)}\Big(-\frac{i}{2\hbar}(pW(\xi(t),\eta(t)) + W(\xi(t),\eta(t))p\,\frac{d\xi(t)}{dt} + \frac{i}{2\hbar}(qW(\xi(t),\eta(t)) + W(\xi(t),\eta(t))q\,\frac{d\eta(t)}{dt} + W(\xi(t),\eta(t))\frac{dg(t)}{dt}\Big) \tag{4.18}$$

and

$$\tilde{L}(\tilde{\Phi}_t(W(\xi,\eta))) = \frac{i}{2\hbar}((\lambda+\mu)\xi(t) + \frac{1}{m}\eta(t))(pW(\xi(t),\eta(t)) + W(\xi(t),\eta(t))p) + \frac{i}{2\hbar}(m\omega^2\xi(t) - (\lambda-\mu)\eta(t))(qW(\xi(t),\eta(t)) + W(\xi(t),\eta(t))q) - \frac{1}{\hbar^2}(D_{pp}\xi(t)^2 + D_{qq}\eta(t)^2 - 2D_{pq}\xi(t)\eta(t))\,W(\xi(t),\eta(t)). \tag{4.18'}$$

Then the equivalence between the master equation

$$\frac{d\tilde{\Phi}_t(W(\xi,\eta))}{dt} = \tilde{L}(\tilde{\Phi}_t(W(\xi,\eta))) \tag{4.19}$$

and the differential equations (4.15), (4.16) and (4.17) is evident.

Now the explicit determination of $\tilde{\Phi}_t(W(\xi,\eta))$ as a function of t reduces to the integration of differential equations (4.15-4.17).

The equations (4.15) and (4.16) can be obtained from the equations (3.26) by the following identifications $\sigma_p(t) = m\omega\,\xi(t)$ and $\sigma_q(t) = \frac{\eta(t)}{m\omega}$, i.e., the vector $V(t) = \begin{pmatrix}\xi(t)\\ \eta(t)\end{pmatrix}$ is related to the vector $S(t) = \begin{pmatrix}\sigma_q(t)\\ \sigma_p(t)\end{pmatrix}$ by the matrix $O = \begin{pmatrix}0 & \frac{1}{m\omega}\\ m\omega & 0\end{pmatrix}$

$$S(t) = O\,V(t). \tag{4.20}$$

Because $O^2 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ it follows from (3.31) that

$$V(t) = O\,S(t) = (ON^{-1}e^{Ft}NO)\,V(0) \tag{4.21}$$

and from (3.33) it follows that

$$ON^{-1}e^{Ft}NO = e^{-\lambda t}\begin{pmatrix} \cosh\nu t - \frac{\mu}{\nu}\sinh\nu t & -\frac{1}{m\nu}\sinh\nu t \\ \frac{m\omega^2}{\nu}\sinh\nu t & \cosh\nu t + \frac{\mu}{\nu}\sinh\nu t \end{pmatrix} \tag{4.22}$$

The following notations will be also used

$$ON^{-1}e^{Ft}NO = \begin{pmatrix} \alpha(t) & \beta(t) \\ \gamma(t) & \delta(t) \end{pmatrix}. \tag{4.23}$$

Then

$$\xi(t) = \alpha(t)\,\xi(0) + \beta(t)\,\eta(0)$$
$$\eta(t) = \gamma(t)\,\xi(0) + \delta(t)\,\eta(0) \tag{4.24}$$

and

$$\frac{dg(t)}{dt} = -\frac{1}{\hbar^2}\left(\frac{dA(t)}{dt}\xi(0)^2 + \frac{dB(t)}{dt}\eta(0)^2 + 2\frac{dC(t)}{dt}\xi(0)\,\eta(0)\right), \tag{4.25}$$

where

$$\frac{dA(t)}{dt} = D_{pp}\,\alpha(t)^2 + D_{qq}\,\gamma(t)^2 - 2D_{pq}\,\alpha(t)\,\gamma(t)$$

$$\frac{dB(t)}{dt} = D_{pp}\,\beta(t)^2 + D_{qq}\,\delta(t)^2 - 2D_{pq}\,\beta(t)\,\delta(t) \tag{4.26}$$

$$\frac{dC(t)}{dt} = D_{pp}\,\alpha(t)\,\beta(t) + D_{qq}\,\gamma(t)\,\delta(t) - D_{pq}\,(\alpha(t)\,\delta(t) + \beta(t)\,\gamma(t)).$$

Instead of integration of (4.26) it is preferable to make firstly the connection with the section 3 and, if this will be done, the solution of (4.26) will be written more simply using the notations of Section 3. For this the action of the dynamical semigroup $\tilde\Phi_t$ on p, q and p^2, q^2 is determined in the following.

From

$$\tilde\Phi_t(W(\xi(0),\eta(0))) = W(\xi(t),\eta(t))\,e^{g(t)} = e^{\frac{i}{\hbar}(\eta(t)q - \xi(t)p) + g(t)} \tag{4.27}$$

and from (4.24) it follows that

$$\tilde\Phi_t(W(\xi(0),\eta(0))) = e^{\frac{i}{\hbar}(\eta(0)(\delta(t)q - \beta(t)p) - \xi(0)(-\gamma(t)q + \alpha(t)p)) + g(t)} \tag{4.28}$$

But from (4.12) and (4.13) it follows that

$$\frac{\partial W(\xi(0),\eta(0))}{\partial\xi(0)}\Big|_{\xi(0)=\eta(0)=0} = -\frac{i}{\hbar}p \tag{4.29}$$

and

$$\frac{\partial W(\xi(0),\eta(0))}{\partial\eta(0)}\Big|_{\xi(0)=\eta(0)=0} = \frac{i}{\hbar}q \tag{4.30}$$

respectively. Then from (4.28) and (4.30), (4.29) the action of Φ_t on q and p respectively is obtained:

$$\tilde\Phi_t(q) = \delta(t)q - \beta(t)p; \quad \tilde\Phi_t(p) = -\gamma(t)q + \alpha(t)p \tag{4.31}$$

or with a matrix notation

$$\begin{pmatrix} \tilde\Phi_t(q) \\ \tilde\Phi_t(p) \end{pmatrix} = N^{-1}e^{Ft}N\begin{pmatrix} q \\ p \end{pmatrix} \tag{4.32}$$

which is the expected result because from this by taking the mean values the equation (3.31) is reobtained. An immediate consequence of (4.31) is $[\Phi_t(q), \Phi_t(p)] = i\hbar I e^{-2\lambda t}$. Also from (4.12) and (4.13) it follows that

$$\frac{\partial^2 W(\xi(0),\eta(0))}{\partial\xi(0)^2}\Big|_{\xi(0)=\eta(0)=0} = -\frac{1}{\hbar^2}p^2 \tag{4.33}$$

$$\frac{\partial^2 W(\xi(0),\eta(0))}{\partial\eta(0)^2}\Big|_{\xi(0)=\eta(0)=0} = -\frac{1}{\hbar^2}q^2 \tag{4.34}$$

and

$$\frac{\partial^2 W(\xi(0),\eta(0))}{\partial\xi(0)\,\partial\eta(0)}\Big|_{\xi(0)=\eta(0)=0} = \frac{1}{2\hbar^2}(pq + qp) \tag{4.35}$$

Then from (4.28) and (4.34), (4.33) and (4.35) it follows respectively:

$$\tilde{\Phi}_t(q^2) = \tilde{\Phi}_t(q)^2 + 2B(t) \tag{4.36}$$

$$\tilde{\Phi}_t(p^2) = \tilde{\Phi}_t(p)^2 + 2A(t) \tag{4.37}$$

and

$$\tilde{\Phi}_t\left(\frac{pq+qp}{2}\right) = \tilde{\Phi}_t(p)\,\tilde{\Phi}_t(q) - 2C(t) \tag{4.38}$$

Now by definition, for any state $\rho \in \mathfrak{T}(\mathcal{H})$:

$$\sigma_{qq}(t) = \mathrm{Tr}\,\rho\tilde{\Phi}_t(q^2) - (\mathrm{Tr}\,\rho\,\tilde{\Phi}_t(q))^2 , \tag{4.39}$$

$$\sigma_{pp}(t) = \mathrm{Tr}\,\rho\tilde{\Phi}_t(p^2) - (\mathrm{Tr}\,\rho\tilde{\Phi}_t(p))^2 \tag{4.40}$$

and

$$\sigma_{pq}(t) = \mathrm{Tr}\,\rho\tilde{\Phi}_t\left(\frac{pq+qp}{2}\right) - (\mathrm{Tr}\,\rho\tilde{\Phi}_t(p))(\mathrm{Tr}\,\rho\tilde{\Phi}_t(q)). \tag{4.41}$$

Then, the equations (4.36), (4.37), (4.38) and (4.31) give respectively

$$\sigma_{qq}(t) = \delta(t)^2\sigma_{qq}(0) + \beta(t)^2\sigma_{pp}(0) - 2\delta(t)\,\beta(t)\,\sigma_{pq}(0) + 2B(t) \tag{4.42}$$

$$\sigma_{pp}(t) = \gamma(t)^2\sigma_{qq}(0) + \alpha(t)^2\sigma_{pp}(0) - 2\gamma(t)\,\alpha(t)\,\sigma_{pq}(0) + 2A(t) \tag{4.43}$$

and

$$\sigma_{pq}(t) = -\delta(t)\,\gamma(t)\,\sigma_{qq}(0) - \beta(t)\,\alpha(t)\,\sigma_{pp}(0) + (\delta(t)\,\alpha(t) + \beta(t)\gamma(t))\sigma_{pq}(0) - 2C(t). \tag{4.44}$$

Because $\alpha(\infty) = \beta(\infty) = \delta(\infty) = \gamma(\infty) = 0$ it follows from (4.42), (4.43) and (4.44) that:

$$\sigma_{qq}(\infty) = 2B(\infty), \quad \sigma_{pp}(\infty) = 2A(\infty), \quad \sigma_{pq}(\infty) = -2C(\infty). \tag{4.45}$$

From the comparison of (4.42), (4.43) and (4.44) with (3.44) one has the following relations:

$$Te^{Kt}T = \begin{pmatrix} \delta(t)^2 & (\beta(t)\,m\omega)^2 & -2\delta(t)\beta(t)\,m\omega \\ \left(\frac{\gamma(t)}{m\omega}\right)^2 & \alpha(t)^2 & \frac{-2\,\alpha(t)\,\gamma(t)}{m\omega} \\ -\frac{\delta(t)\gamma(t)}{m\omega} & -\alpha(t)\,\beta(t)\,m\omega & \alpha(t)\,\delta(t) + \beta(t)\gamma(t) \end{pmatrix} \tag{4.46}$$

and

$$\begin{pmatrix} 2B(t)\,m\omega \\ 2\frac{A(t)}{m\omega} \\ -2C(t) \end{pmatrix} = T(e^{Kt} - I)K^{-1}TD. \tag{4.47}$$

Their validity can be verified by direct calculations. But from (4.26) and (4.46) it is evident that

$$\begin{pmatrix} \frac{d}{dt}(2m\omega B(t)) \\ \frac{d}{dt}\left(\frac{2A(t)}{m\omega}\right) \\ \frac{d}{dt}(-2C(t)) \end{pmatrix} = Te^{Kt}\,TD \tag{4.48}$$

Then (4.47) is obtained by integration from (4.48).

An interesting consequence of these observations is that the time dependence of the variances $\sigma_{qq}(t)$, $\sigma_{pp}(t)$, $\sigma_{pq}(t)$ decomposes in a classical part given by $Te^{Kt}T\,X(0)$ (because the relation (4.46) is exactly the classically expected relation between the $\sigma_{qq}(t)$, $\sigma_{pp}(t)$, $\sigma_{pq}(t)$ and $\sigma_q(t)$, $\sigma_p(t)$) and a quantum part given by $T(e^{Kt} - I)K^{-1}TD$. Exactly this quantum part governs the asymptotic behaviour of the variances when $t \to \infty$.

From this point of view it is interesting and will be also useful to put the above results in a new form. Let us denote by $\sigma(t)$ the correlation matrix

$$\sigma(t) = \begin{pmatrix} m\omega\sigma_{qq}(t) & \sigma_{pq}(t) \\ \sigma_{pp}(t) & \frac{1}{m\omega}\sigma_{pp}(t) \end{pmatrix} \tag{4.49}$$

Then from (4.42)-(4.43) it follows that

$$\sigma(t) = \mathfrak{R}(t)\,\sigma(0)\,\mathfrak{R}^{T}(t) + Z(t). \tag{4.50}$$

where

$$\mathcal{R}(t) = \begin{pmatrix} \delta(t) & -m\omega\beta(t) \\ -\frac{\gamma(t)}{m\omega} & \alpha(t) \end{pmatrix}, \tag{4.51}$$

$\mathcal{R}^T(t)$ is the matrix obtained from $\mathcal{R}(t)$ by transposition, and

$$Z(t) = 2\begin{pmatrix} m\omega B(t) & -C(t) \\ -C(t) & \frac{A(t)}{m\omega} \end{pmatrix}. \tag{4.52}$$

Then,

$$Z(\infty) = \sigma(\infty) \tag{4.53}$$

and from comparison of (4.46), (4.47) and (4.50) it follows that

$$Z(t) = -\mathcal{R}(t)\,\sigma(\infty)\,\mathcal{R}^T(t) + \sigma(\infty), \tag{4.54}$$

i.e.,

$$\sigma(t) = \mathcal{R}(t)\,(\sigma(0) - \sigma(\infty))\,\mathcal{R}^T(t) + \sigma(\infty). \tag{4.55}$$

Because,

$$\frac{d\mathcal{R}(t)}{dt} = Y\mathcal{R}(t) \tag{4.56}$$

where

$$Y = \begin{pmatrix} -(\lambda - \mu) & \omega \\ -\omega & -(\lambda + \mu) \end{pmatrix} \tag{4.57}$$

it follows that

$$\frac{d\sigma(t)}{dt} = Y(\sigma(t) - Z(t)) + (\sigma(t) - Z(t))\,Y^T + \frac{dZ(t)}{dt} \tag{4.58}$$

and the comparison with (3.27) gives

$$\frac{dZ(t)}{dt} = \mathfrak{D} + YZ(t) + Z(t)\,Y^T, \tag{4.59}$$

where

$$\mathfrak{D} = 2\begin{pmatrix} m\omega D_{qq} & D_{pq} \\ D_{pq} & \frac{D_{pp}}{m\omega} \end{pmatrix} \tag{4.60}$$

From (4.54) it follows that

$$\frac{dZ(t)}{dt} = Y(Z(t) - \sigma(\infty)) + (Z(t) - \sigma(\infty))\,Y^T \tag{4.61}$$

and the comparison of (4.61) with (4.59) gives

$$Y\sigma(\infty) + \sigma(\infty)Y^T = -\mathfrak{D}. \tag{4.62}$$

From (4.48) it follows that

$$\frac{dZ(t)}{dt} = \mathcal{R}(t)\,\mathfrak{D}\,\mathcal{R}^T(t). \tag{4.63}$$

Now, from (4.25)

$$g(t) = -\frac{1}{\hbar^2}(A(t)\,\xi(0)^2 + B(t)\,\eta(0)^2 + 2C(t)\,\xi(0)\,\eta(0)) \tag{4.64}$$

or

$$g(t) = -\frac{1}{\hbar^2}\left(\frac{\eta(0)}{\sqrt{m\omega}}, \sqrt{m\omega}\,\xi(0)\right)\begin{pmatrix} m\omega B(t) & -C(t) \\ -C(t) & \frac{A(t)}{m\omega} \end{pmatrix}\begin{pmatrix} \frac{\eta(0)}{\sqrt{m\omega}} \\ \sqrt{m\omega}\,\xi(0) \end{pmatrix} \tag{4.65}$$

If the vector $\left(\frac{\eta(0)}{\sqrt{m\omega}}, \sqrt{m\omega}\,\xi(0)\right)$ is denoted $V(0)$, then

$$g(t) = -\frac{1}{2\hbar^2}\,V(0)\,Z(t)\,V(0)^T \tag{4.66}$$

or from (4.54)

$$g(t) = \frac{1}{2\hbar^2}\left(V(0)\,\mathcal{R}(t)\,\sigma(\infty)\,\mathcal{R}(t)^T\,V(0)^T - V(0)\,\sigma(\infty)\,V(0)^T\right). \tag{4.67}$$

Usinf the relations (3.53) and (3.56) the following interesting formulas are obtained

$$(\lambda^2 + \omega^2 - \mu^2)\det\sigma(\infty) = \frac{1}{4}\det\mathfrak{D} + \frac{1}{\lambda^2}\left(\frac{1}{2m}D_{pp} + \frac{m\omega^2}{2}D_{qq} + \mu D_{pq}\right)^2 \tag{4.68}$$

and

$$(\lambda^2+\omega^2-\mu^2)\det\sigma(\infty) = \frac{1}{4}\det\mathfrak{D} + \tag{4.69}$$
$$+ \left(\frac{1}{2m}\sigma_{pp}(\infty) + \frac{1}{2}m\omega^2\sigma_{qq}(\infty) + \mu\sigma_{pq}(\infty)\right)^2.$$

Comparison of (4.68) with (4.69) gives

$$\frac{1}{2m}\sigma_{pp}(\infty) + \frac{1}{2}m\omega^2\sigma_{qq}(\infty) + \mu\sigma_{pq}(\infty) = \frac{1}{\lambda}\left(\frac{1}{2m}D_{pp} + \frac{m\omega^2}{2}D_{qq} + \mu D_{pq}\right). \tag{4.70}$$

But, the left-hand side of (4.70) is exactly the asymptotic mean value of the energy of the open harmonic oscillator. Hence (4.70) gives the value of $E(\infty)$ as a function of diffusion coefficients:

$$E(\infty) = \frac{1}{\lambda}\left(\frac{1}{2m}D_{pp} + \frac{m\omega^2}{2}D_{qq} + \mu D_{pq}\right). \tag{4.71}$$

Another expression for $E(\infty)$ which also follows from (4.68) and (4.69) is

$$E(\infty) = \left((\lambda^2+\omega^2-\mu^2)\det\sigma(\infty) - \frac{1}{4}\det\mathfrak{D}\right)^{1/2}. \tag{4.72}$$

The reality condition for $E(\infty)$ implies

$$\det\sigma(\infty) \geq \frac{1}{4(\lambda^2+\omega^2-\mu^2)}\det\mathfrak{D} \geq \frac{\lambda^2\hbar^2}{4(\lambda^2+\omega^2-\mu^2)} \tag{4.73}$$

which is more restrictive for the overdamped case ($\omega < \mu$) than $\det\sigma(\infty) \geq \hbar^2/4$.

5. THE MASTER EQUATION IN THE WEYL-WIGNER-MOYAL REPRESENTATION

The Weyl-Wigner-Moyal representation is a remarkable phase-space representation of the quantum mechanics. Roughly speaking, a phase space representation of the quantum mechanics is a mapping from the Hilbert space operators to the functions on the classical phase space which is such that if A is mapped onto $f_A(x,y)$ and ρ is mapped onto $f_\rho(x,y)$, then:

$$\mathrm{Tr}(\rho A) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_\rho(x,y)\, f_A(x,y)\, dx\, dy. \tag{5.1}$$

In reality, it is not exactly so, because the Weyl mapping is a mapping from the functions on the phase space to the Hilbert

space operators. This mapping W was defined by Weyl [31] in the following way:

$$W(f) = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x\eta-y\xi)} f(x,y)\, dx\, dy\right) W(\xi,\eta)\, d\xi\, d\eta. \tag{5.2}$$

From this it follows very formally that for any $\rho \in \mathfrak{D}(\mathcal{H})$

$$\mathrm{Tr}\,\rho W(f) = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x\eta-y\xi)}\,\mathrm{Tr}(\rho W(\xi,\eta))\, d\xi\, d\eta\right) dx\, dy \tag{5.3}$$

and that (5.3) can be put in the standard form (5.1) if the following function on the phase space is associated to any $\rho \in \mathfrak{D}(\mathcal{H})$

$$f_\rho(x,y) = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x\eta-y\xi)}\,\mathrm{Tr}(\rho W(\xi,\eta))\, d\xi\, d\eta. \tag{5.4}$$

The mapping $\rho \to f_\rho$ defined by (5.4) is exactly the Wigner mapping [29,32] which is the dual of the Weyl mapping $f \to W(f)$ (hence it can be denoted by $\tilde{W}$) and $f_\rho = \tilde{W}(\rho)$ is the Wigner function corresponding to the quantum state $\rho \in \mathfrak{D}(\mathcal{H})$. But the quantum nature of the expectation value is not lost because $f_\rho(x,y)$ is not a probability distribution on the phase space, taking positive and negative values. The fact that the Wigner function is positive only for the wave functions given by Gauss-Cornu functions was proved rigorously [33].

In the following, the phase space representation of the master equation (3.6) is obtained by using the Wigner mapping (5.4). Denoting by

$$f(x,y,t) = f_{\rho(t)}(x,y) = f_{\Phi_t(\rho)}(x,y) \tag{5.5}$$

it follows from the definition (5.4) that

$$f(x,y,t) = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x\eta-y\xi)}\,\mathrm{Tr}(\rho(t)\, W(\xi,\eta))\, d\xi\, d\eta. \tag{5.6}$$

Then

$$\frac{\partial f(x,y,t)}{\partial t} = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x\eta-y\xi)}\,\mathrm{Tr}(L(\rho(t))W(\xi,\eta))\, d\xi\, d\eta \tag{5.7}$$

and by duality

$$\frac{\partial f(x,y,t)}{\partial t} = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{+\infty} e^{-\frac{i}{\hbar}(x\eta-y\xi)}\,\mathrm{Tr}(\rho(t)\,\tilde{L}(W(\xi,\eta)))\, d\xi\, d\eta. \tag{5.8}$$

But from (3.23) in the case $t = 0$, $A = W(\xi, \eta)$ and from the equations (4.6)-(4.13) it follows that

$$L(W(\xi,\eta)) = -\frac{\eta}{m}\frac{\partial W(\xi,\eta)}{\partial \xi} + m\omega^2 \xi \frac{\partial W(\xi,\eta)}{\partial \eta} - (\lambda-\mu)\eta\frac{\partial W(\xi,\eta)}{\partial \eta} - (\lambda+\mu)\xi\frac{\partial W(\xi,\eta)}{\partial \xi} - \frac{D_{qq}}{\hbar^2}\eta^2 W(\xi,\eta) - \frac{D_{pp}}{\hbar^2}\xi^2 W(\xi,\eta) + \frac{2D_{pq}}{\hbar^2}\xi\eta W(\xi,\eta). \tag{5.9}$$

Putting this in (5.8) and denoting $Tr(\rho(t) W(\xi,\eta))$ by $G(\xi,\eta,t)$, (5.8) becomes:

$$\frac{\partial f(x,y,t)}{\partial t} = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} d\xi\, d\eta\, e^{-\frac{i}{\hbar}(x\eta - y\xi)} \times \Big(-\frac{\eta}{m}\frac{\partial G(\xi,\eta,t)}{\partial \xi} + m\omega^2\xi\frac{\partial G(\xi,\eta,t)}{\partial \eta} - (\lambda-\mu)\eta\frac{\partial G(\xi,\eta,t)}{\partial \eta} - (\lambda+\mu)\xi\frac{\partial G(\xi,\eta,t)}{\partial \xi} - \frac{D_{qq}}{\hbar^2}\eta^2 G(\xi,\eta,t) - \frac{D_{pp}}{\hbar^2}\xi^2 G(\xi,\eta,t) + \frac{2D_{pq}}{\hbar^2}\xi\eta G(\xi,\eta,t)\Big). \tag{5.10}$$

Using the well-known identities for the Fourier transformation

$$\int_{-\infty}^{\infty} e^{\frac{iy\xi}{\hbar}} \frac{\partial G(\xi,\eta,t)}{\partial \xi} d\xi = -\frac{iy}{\hbar}\int_{-\infty}^{\infty} e^{\frac{iy\xi}{\hbar}} G(\xi,\eta,t)\, d\xi \tag{5.11}$$

$$\int_{-\infty}^{\infty} e^{\frac{iy\xi}{\hbar}} \xi G(\xi,\eta,t)\, d\xi = -i\hbar\frac{\partial}{\partial y}\int_{-\infty}^{\infty} e^{\frac{iy\xi}{\hbar}} G(\xi,\eta,t)\, d\xi \tag{5.12}$$

$$\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} \frac{\partial G(\xi,\eta,t)}{\partial \eta} d\eta = \frac{ix}{\hbar}\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} G(\xi,\eta,t)\, d\eta \tag{5.13}$$

$$\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} \eta G(\xi,\eta,t)\, d\eta = i\hbar\frac{\partial}{\partial x}\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} G(\xi,\eta,t)\, d\eta \tag{5.14}$$

$$\int_{-\infty}^{\infty} e^{\frac{iy\eta}{\hbar}} \xi \frac{\partial G(\xi,\eta,t)}{\partial \xi} d\xi = -\frac{\partial}{\partial y}\Big(y\int_{-\infty}^{\infty} e^{\frac{iy\xi}{\hbar}} G(\xi,\eta,t)\, d\xi\Big) \tag{5.15}$$

$$\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} \eta \frac{\partial G(\xi,\eta,t)}{\partial \eta} d\eta = -\frac{\partial}{\partial x}\Big(x\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} G(\xi,\eta,t)\, d\eta\Big) \tag{5.16}$$

$$\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} \eta^2 G(\xi,\eta,t)\, d\eta = -\hbar^2\frac{\partial^2}{\partial x^2}\int_{-\infty}^{\infty} e^{\frac{-ix\eta}{\hbar}} G(\xi,\eta,t)\, d\eta \tag{5.17}$$

$$\int_{-\infty}^{\infty} e^{\frac{iy\xi}{\hbar}} \xi^2 G(\xi,\eta,t)\, d\xi = -\hbar^2\frac{\partial^2}{\partial y^2}\int_{-\infty}^{\infty} e^{\frac{iy\xi}{\hbar}} G(\xi,\eta,t)\, d\xi \tag{5.18}$$

the equation (5.10) is transformed in the following evolution equation for the Wigner function

$$\frac{\partial f(x,y,t)}{\partial t} = -\frac{y}{m}\frac{\partial f(x,y,t)}{\partial x} + m\omega^2 x\frac{\partial f(x,y,t)}{\partial y} + (\xi-\mu)\frac{\partial}{\partial x}(xf(x,y,t)) + (\lambda+\mu)\frac{\partial}{\partial y}(yf(x,y,t)) + D_{qq}\frac{\partial^2 f(x,y,t)}{\partial x^2} + D_{pp}\frac{\partial^2 f(x,y,t)}{\partial y^2} + 2D_{pq}\frac{\partial^2 f(x,y,t)}{\partial x\partial y}. \tag{5.19}$$

This equation looks very classical. In fact, it is exactly an equation of the Fokker-Planck type. But attention, not every function $f(x,y,0)$ on the phase space is the Wigner transform of a density operator. Hence, the quantum mechanics appears now in the restrictions imposed by this last condition on the initial condition $f(x,y,0)$ for the equation (5.19). Unfortunately, these conditions are not known explicitly.

Because the most frequently used choice for $f(x,y,0)$ is a Gaussian function and because the equation (5.19) preserves this Gaussian type, i.e., $f(x,y,t)$ is also a Gaussian function, the differences between the quantum mechanics and classical mechanics are completely lost in this representation of the master equation. This is a possible explanation for the frequently occurred ambiguities on this subject in literature.

The master equation (5.19) is directly comparable with the master equations (5.1) and (5.6) from [7,28]. Both these equations can be obtained from the equation (5.19) if the values of the parameters m, ω, λ, μ, D_{qq}, D_{pp}, D_{pq} take the particular values indicated in §3 (see the eqs. (3.14) and (3.15), respectively).

Now from (5.6) by duality it follows that

$$f(x,y,t) = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x\eta - y\xi)}\, Tr(\rho(0)\Phi_t(W(\xi,\eta)))\, d\xi\, d\eta \tag{5.20}$$

and from the results of §4 it follows that

$$f(x,y,t) = \frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x\eta - y\xi)}\, Tr(\rho(0)\, W(\xi(t),\eta(t))e^{\beta(t)})\, d\xi\, d\eta. \tag{5.21}$$

where

$$\xi(0)=\xi,\quad \eta(0)=\eta,\quad g(t)=-\frac{1}{\hbar^2}(A(t)\xi^2+B(t)\eta^2+2C(t)\xi\eta),$$

$$\xi(t)=\alpha(t)\xi+\beta(t)\eta \quad \text{and} \quad \eta(t)=\gamma(t)\xi+\delta(t)\eta .$$

If the initial state $\rho(0)$ is a pure state corresponding to a coherent wave function $\rho(0)\phi=(\psi_{\sigma_q(0),\sigma_p(0)},\phi)\psi_{\sigma_q(0),\sigma_p(0)}$ centered in $x=\sigma_q(0)$, $y=\sigma_p(0)$, i.e., if $\psi_{\sigma_q(0),\sigma_p(0)}(x)=(W(\sigma_q(0),\sigma_p(0))\psi_0)(x)$ where $\psi_0(x)=(2\pi\sigma_{qq}(0))^{-1/4}\exp(-x^2/4\sigma_{qq}(0))$, then

$$\mathrm{Tr}(\rho(0)W(\xi,\eta))=(\psi_{\sigma_q(0),\sigma_p(0)},W(\xi,\eta)\psi_{\sigma_q(0),\sigma_p(0)})=$$
$$=(\psi_0,W(-\sigma_q(0),-\sigma_p(0))W(\xi,\eta)W(\sigma_q(0),\sigma_p(0))\psi_0). \tag{5.22}$$

From (4.3) it follows that

$$W(-\sigma_q(0),-\sigma_p(0))W(\xi,\eta)W(\sigma_q(0),\sigma_p(0))=W(\xi,\eta)e^{\frac{i}{\hbar}(\sigma_q(0)\eta-\sigma_p(0)\xi)}, \tag{5.23}$$

i.e.,

$$\mathrm{Tr}(\rho(0)W(\xi,\eta))=e^{\frac{i}{\hbar}(\sigma_q(0)\eta-\sigma_p(0)\xi)}(\psi_0,W(\xi,\eta)\psi_0). \tag{5.24}$$

But for any wave function ψ from (4.2) it follows that

$$(W(\xi,\eta)\psi)(x)=e^{-\frac{i\xi\eta}{2\hbar}+\frac{i}{\hbar}x\eta}(e^{-\frac{i}{\hbar}\xi p}\psi)(x) \tag{5.25}$$

and because $p=-i\hbar\frac{d}{dx}$ the final result is

$$(W(\xi,\eta)\psi)(x)=e^{-\frac{i\xi\eta}{2\hbar}+\frac{i}{\hbar}x\eta}\psi(x-\xi). \tag{5.26}$$

Then

$$(\psi_0,W(\xi,\eta)\psi_0)=\frac{1}{(2\pi\sigma_{qq}(0))^{1/2}}\int_{-\infty}^{\infty}e^{-\frac{x^2}{4\sigma_{qq}(0)}}e^{-\frac{i\xi\eta}{2\hbar}+\frac{i}{\hbar}x\eta}e^{-\frac{(x-\xi)^2}{4\sigma_{qq}(0)}}dx \tag{5.27}$$

and with the well-known formula

$$\int_{-\infty}^{+\infty}e^{-ax^2+bx}dx=\sqrt{\frac{\pi}{a}}e^{b^2/4a} \tag{5.28}$$

the final result is

$$(\psi_0,W(\xi,\eta)\psi_0)=e^{-\frac{\xi^2}{8\sigma_{qq}(0)}-\frac{\sigma_{qq}(0)}{2\hbar^2}\eta^2}. \tag{5.29}$$

With the notation $\sigma_{pp}(0)=\frac{\hbar^2}{4\sigma_{qq}(0)}$ from (5.29) and (2.29) it follows that

$$\mathrm{Tr}(\rho(0)W(\xi,\eta))=e^{-\frac{\xi^2}{8\sigma_{qq}(0)}-\frac{\eta^2}{8\sigma_{pp}(0)}+\frac{i}{\hbar}(\sigma_q(0)\eta-\sigma_p(0)\xi)} \tag{5.30}$$

and

$$\mathrm{Tr}(\rho(0)W(\xi(t),\eta(t)))=\exp(-(\frac{\alpha(t)^2}{8\sigma_{qq}(0)}+\frac{\gamma(t)^2}{8\sigma_{pp}(0)})\xi^2-$$
$$-(\frac{\beta(t)^2}{8\sigma_{qq}(0)}+\frac{\delta(t)^2}{8\sigma_{pp}(0)})\eta^2-(\frac{\alpha(t)\beta(t)}{4\sigma_{qq}(0)}+\frac{\gamma(t)\delta(t)}{4\sigma_{pp}(0)})\xi\eta+ \tag{5.31}$$
$$+\frac{i}{\hbar}(\sigma_q(0)\gamma(t)-\sigma_p(0)\alpha(t))\xi+\frac{i}{\hbar}(\sigma_q(0)\delta(t)-\sigma_p(0)\beta(t))).$$

Now the Wigner function

$$f(x,y,t)=\frac{1}{(2\pi\hbar)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d\xi d\eta\, e^{-\frac{i}{\hbar}(x\eta-y\xi)+g(t)}\mathrm{Tr}(\rho(0)W(\xi(t),\eta(t))) \tag{5.32}$$

can be analytically calculated because the integrand is an exponential function with the exponent having a quadratic form in ξ and η

$$-\frac{1}{\hbar^2}[(A(t)+\frac{\hbar^2\alpha(t)^2}{8\sigma_{qq}(0)}+\frac{\hbar^2\gamma(t)^2}{8\sigma_{pp}(0)})\xi^2+(B(t)+\frac{\hbar^2\beta(t)^2}{8\sigma_{qq}(0)}+\frac{\hbar^2\delta(t)^2}{8\sigma_{pp}(0)})\eta^2+$$
$$+2(C(t)+\frac{\hbar^2\alpha(t)\beta(t)}{8\sigma_{qq}(0)}+\frac{\hbar^2\gamma(t)\delta(t)}{8\sigma_{pp}(0)})\xi\eta]+\frac{i}{\hbar}(\sigma_q(0)\gamma(t)-\sigma_p(0)\alpha(t)+y)\xi+ \tag{5.33}$$
$$+\frac{i}{\hbar}(\sigma_q(0)\delta(t)-\sigma_p(0)\beta(t)-x)\eta.$$

From the well known formula

$$\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty}e^{-\sum_{k,\ell=1}^{n}a_{k\ell}x_kx_\ell+\sum_{m=1}^{n}b_mx_m}dx_1dx_2\dots dx_n=\frac{\pi^{\frac{n}{2}}}{\sqrt{\Delta}}\exp(\frac{1}{4\Delta}\sum_{k,\ell}b_kb_\ell\frac{\partial\Delta}{\partial a_{k\ell}}) \tag{5.34}$$

with $\Delta = \det(a_{k\ell})$ it follows for $n = 2$ and with the obvious identifications that

$$f(x,y,t) = \frac{\pi}{(2\pi\hbar)^2\sqrt{\Delta}} \exp\{ -\frac{1}{4\hbar^4\Delta}((A(t) + \frac{\hbar^2\alpha(t)^2}{8\sigma_{qq}(0)} + \frac{\hbar^2\gamma(t)^2}{8\sigma_{pp}(0)})(x-\sigma_q(0)\delta(t)+$$

$$+\sigma_p(0)\beta(t))^2 + (B(t) + \frac{\hbar^2\beta(t)^2}{8\sigma_{qq}(0)} + \frac{\hbar^2\delta(t)^2}{8\sigma_{pp}(0)})(y+\sigma_q(0)\gamma(t) - \sigma_p(0)\alpha(t))^2 - \tag{5.35}$$

$$-2(C(t) + \frac{\hbar^2\alpha(t)\beta(t)}{8\sigma_{qq}(0)} + \frac{\hbar^2\gamma(t)\delta(t)}{8\sigma_{pp}(0)})(x-\sigma_q(0)\delta(t)+\sigma_p(0)\beta(t))(y+\sigma_q(0)\gamma(t)-\sigma_p(0)\alpha(t)))\},$$

With the help of this Wigner function the coordinate and momentum probability distribution are defined respectively by

$$P(x,t) = \int_{-\infty}^{\infty} f(x,y,t)\,dy \tag{5.36}$$

and

$$P(y,t) = \int_{-\infty}^{\infty} f(x,y,t)\,dx\,. \tag{5.37}$$

From (5.35) and with use of (5.34) for $n = 1$ it follows respectively

$$P(x,t) = \frac{1}{\sqrt{2\pi\sigma_{qq}(t)}} \exp(-\frac{(x-\sigma_q(t))^2}{2\sigma_{qq}(t)}) \tag{5.38}$$

with the notations defined in Sec.3 and 4

$$\sigma_q(t) = \delta(t)\sigma_q(0) - \beta(t)\sigma_p(0)$$
$$\sigma_{qq}(t) = \delta(t)^2\sigma_{qq}(0) + \beta(t)^2\sigma_{pp}(0) + 2B(t) \tag{5.39}$$

and

$$P(y,t) = \frac{1}{\sqrt{2\pi\sigma_{pp}(t)}} \exp(-\frac{(y-\sigma_p(t))^2}{2\sigma_{pp}(t)})\,. \tag{5.40}$$

with

$$\sigma_p(t) = \alpha(t)\sigma_p(0) - \gamma(t)\sigma_q(0)$$

$$\sigma_{pp}(t) = \alpha(t)^2\sigma_{pp}(0) + \gamma(t)^2\sigma_{qq}(0) + 2A(t)\,. \tag{5.41}$$

With these notations the Wigner function takes the following more suggestive form:

$$f(x,y,t) = \frac{1}{2\pi\sqrt{\sigma_{pp}(t)\sigma_{qq}(t) - \sigma_{pq}^2(t)}} \exp\{ - \frac{1}{2(\sigma_{pp}(t)\sigma_{qq}(t) - \sigma_{pq}(t)^2)} \times$$

$$\times(\sigma_{pp}(t)(x-\sigma_q(t))^2 + \sigma_{qq}(y-\sigma_p(t))^2 - 2\sigma_{pq}(t)(x-\sigma_q(t))(y-\sigma_p(t)))\}, \tag{5.42}$$

where

$$\sigma_{pq}(t) = \gamma(t)\delta(t)\sigma_{qq}(0) + \alpha(t)\beta(t)\sigma_{pp}(0) + 2C(t)\,. \tag{5.43}$$

Evidently

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,t)\,dx\,dy = 1 \tag{5.44}$$

and

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y,t)\,dx\,dy = \sigma_q(t) \tag{5.45}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f(x,y,t)\,dx\,dy = \sigma_p(t) \tag{5.46}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x,y,t)\,dx\,dy = \sigma_{qq}(t) \tag{5.47}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 f(x,y,t)\,dx\,dy = \sigma_{pp}(t) \tag{5.48}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f(x,y,t)\,dx\,dy = \sigma_{pq}(t)\,. \tag{5.49}$$

6. CONCLUSIONS

In the present paper we have solved the problem of the damping of only one collective coordinate. The generalization to many dimensions is straighforward. Consequently a full quantum description of large scale damped collective modes could now be formulated.

We consider that the next problems which have to be solved are related to the three dimensional systems like the opening of a spin system and of a rigid body, or the inclusion of more collective coordinates like charge asymmetry, mass asymmetry and the neck degrees of freedom. Also a very important problem is to describe correctly the decays where not only the energy is changed but also the number of particles.

We should like to mention that, by considering one dimensional quantum equations in the new collective coordinates (charge asymmetry, mass asymmetry and neck) we already obtained large successes in the description of some phenomena related to the collision dynamics. Thus the maximum of the fusion cross sections for producing new elements were related not only to the minimum excitation energy but also to the relative stability to the charge and mass transfer in the presence of the shell effects of the colliding partners [34-37].

The existence of a plateau in the width distributions for the equilibration of the neutron to proton ratio of the fragments from binary dissipative heavy ion collisions could be interpreted in a natural way as the zero point vibrational motion in this degree of freedom [11-12]. We hope that also the mass transfer in DIC at the top of the nuclear barrier will reveal some quantum mechanical characteristics.

Also the introduction of the neck coordinate allowed us to describe the α-decay as a superasymmetric fission process [38-40] and to predict the existence of new decay modes intermediate between alpha decay and fission [41].

The experimental discovery of ^{14}C and ^{24}Ne emissions from some nuclei [42-48] supports our quantum mechanical description of collision dynamics. Recently, we interpreted one of the two components observed in the spontaneous fission of the very heavy elements as an evidence for Sn-emission and the cold fragmentation of the ^{233}U after neutron capture as a heavy cluster emission with mass around ≈100 [49].

Finally we should like to stress that the collective fluctuations have not been revealed with clarity by experiment. Now it is clear that, due to the similarity of the equations and solutions in both extreme theoretical approaches; transport theories and quantum collective theories the effects are similar. We consider that is premature to conclude, like the majority of the recent papers [1], that the present data suggest that the dynamical evolution of the dinuclear system may be seen as an independent particle exchange process constrained by the underlying potential energy surface (PES).

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Сэндулеску А., Скутару Х. E4-85-705

Открытые квантовые системы и затухание коллективных мод в глубоконеупругих столкновениях

В рамках теории Линдблада для открытых квантовых систем были получены следующие результаты: обобщение соотношения для ограничения коэффициентов диффузии, обобщенное условие для существования чистых состояний и обобщенное уравнение типа Шредингера для открытых систем. Были получены также в явном виде представления Шредингера, Гейзенберга и Вайла-Вигнера-Мойала уравнения Линдблада. На основе этих представлений было показано, что многие уравнения для гармонического осциллятора с диссипацией энергии, использованные в литературе для описания затухания коллективных мод, представляют собой частные случаи уравнения Линдблада и большинство этих уравнений не выполняет соотношения для ограничения коэффициентов диффузии, вытекающих из принципа неопределенности. Решение дифференциальных уравнений для дисперсий представлено в компактной форме, опирающейся на прямой расчет дисперсий с помощью зависящего от времени оператора Вайля. Решение уравнения Линдблада в Вайля-Вигнера-Мойала представлении имеет форму гауссиана, если начальная форма функции Вигнера взята в виде гауссиана, соответствующего когерентной волновой функции.

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Open Quantum Systems and the Damping of Collective Modes in Deep Inelastic Collisions

In the framework of the Lindblad theory for open quantum systems the following results are obtained: a generalization of the fundamental constraints on quantum mechanical diffusion coefficients which appear in the corresponding master equations, a generalization of the Hasse pure state condition and a generalized Schrödinger type nonlinear equation for an open system. Also, the Schrödinger, Heisenberg and Weyl-Wigner-Moyal representations of the Lindblad equation are given explicitly. On the basis of these representations, it is shown that various master equations for the damped quantum oscillator used in the literature for the description of the damped collective modes in DIC are particular cases of the Lindblad equation and that the majority of these equations are not satisfying the constraints on quantum mechanical diffusion coefficients. The solutions of the differential equations for the variances are put in a new synthetic form, suggested by a direct computation of the variances from the time dependent Weyl operators. The solution of the Lindblad equation in the Weyl-Wigner-Moyal representation is of Gaussian type if the initial form of the Wigner function is taken to be a Gaussian corresponding to a coherent wave function.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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