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GENERATION OF SQUEEZED STATES OF LIGHT VIA FOUR-PHOTON PROCESS IN SYSTEM OF THREE-LEVEL ATOMS

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There has been much recent interest in the generation of squeezed or two-photon coherent states [1-3]. Such states have less noise than a coherent state in one of the field quadratures and have potential applications in optical communication systems [4] and gravitational radiation detectors [5].

A number of nonlinear optical systems susceptible to produce squeezed states has been analyzed theoretically. These include degenerate parametric oscillators [6], resonance fluorescence [8], degenerate four-wave mixing [7], optical bistability [9], free-electron lasers [11], the Jaynes-Cumming model [10] and the two-photon processes [12]

In this letter we present the generation of squeezed states of light via four-photon processes in systems of three-level atoms (see the figure). The collective effects and effects of an atomic reservoir are accounted for. The condition for receiving the optimum squeezing is defined. The N three-level atoms, concentrated in a region small compared to the wave length of all the relevant radiation modes interact with three driving modes $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$ with frequencies Ω_1, Ω_2 and Ω_3 respectively. The external fields $\vec{E}_1$ and $\vec{E}_2$ are assumed intense and can be treated classically. For simplicity the field $\vec{E}_1$ is assumed to be in resonance with the level separation $\omega_2 - \omega_1 = \omega_{21}$ (system with $\hbar = 1$) and the field $\vec{E}_2$ is assumed to be in resonance with $\omega_3 - \omega_2 = \omega_{32}$. The transition between the upper $|3\rangle$ and lower levels $|1\rangle$ is the two-photon process and let a and a^+ be annihilation and creation operators of the mode $\vec{E}_3$

The coherence part of the Hamiltonian in the rotating-wave approximation and interaction picture is

$$H_{coh} = \delta a^+ a + g_{31}^{(2)} (J_{31} a^2 + a^{+2} J_{13}) + G_{21} (J_{21} + J_{12}) + G_{32} (J_{32} + J_{23}), \qquad (1)$$

where $$\delta = \frac{1}{2} (2\Omega_3 - \omega_3 + \omega_1)$$

$$J_{ij} = \sum_{K=1}^{N} \sigma_{ij}^{(K)} . \qquad (2)$$

Here $\sigma_{ij}^{(K)} = |i\rangle_K \, {}_K\langle j| \qquad (i, j = 1, 2, 3)$

G_{21} and G_{32} are the electric dipole interaction matrix elements, $-\vec{d}_{21}\vec{E}_1$ and $-\vec{d}_{32}\vec{E}_2$ respectively. Here $\vec{d}$ is the electric dipole operator for the system. The $g_{31}^{(2)}$ is the matrix element for the two-photon transition $|3\rangle \rightarrow |1\rangle$.

Considering the operators a and a^+ in Hamiltonian (1) as C-numbers and using the Markovian approximation one finds the master equation for the atomic system as [13]

$$\frac{\partial \rho}{\partial t} = - i \left[H_{coh} , \rho \right] + \left. \frac{\partial \rho}{\partial t} \right|_A \doteq L \rho , \qquad (3)$$

where the dissipative term for the atoms is

$$\left. \frac{\partial \rho}{\partial t} \right|_A = - \gamma_{21} (J_{21} J_{12} \rho - 2 J_{12} \rho J_{21} + \rho J_{21} J_{12}) - \gamma_{32} (J_{32} J_{23} \rho - 2 J_{23} \rho J_{32} + \rho J_{32} J_{23}) - \gamma_{31} (J_{31} J_{13} \rho - 2 J_{13} \rho J_{31} + \rho J_{31} J_{13}) .$$

The terms γ_{ij} are the transition rates caused by the atomic reservoirs from level $|i\rangle$ to $|j\rangle$.

Atomic coherence phenomena can perhaps be illustrated with greater lucidity by introducing the schwinger representation for angular momentum [14] $J_{ij} = C_i^+ C_j$, where C_i obey boson commutation relations. The operators C_i and C_i^+ can be considered the annihilation and creation operators of the atoms on the level $|i\rangle$

In the case of sufficiently intense $G = (G_{21}^2 + G_{32}^2)^{1/2}$ so that

$$G \gg N \gamma_{ij} \; , \; g_{31}^{(2)} |\vec{E}_3| \qquad (4)$$

it is possible to develop an approximation scheme that enables us to obtain analytic results.

After performing the canonical transformation

$$C_3 = - \frac{\sin\alpha}{\sqrt{2}} Q_1 + \cos\alpha \, Q_2 + \frac{\sin\alpha}{\sqrt{2}} Q_3$$

$$C_2 = \frac{1}{\sqrt{2}} Q_1 + \frac{1}{\sqrt{2}} Q_3 \qquad (5)$$

$$C_1 = - \frac{\cos\alpha}{\sqrt{2}} Q_1 - \sin\alpha \, Q_2 + \frac{\cos\alpha}{\sqrt{2}} Q_3 .$$

where $$tg\,\alpha = \frac{G_{32}}{G_{21}}$$

one can find that the Liouville operator L appearing in equation (3) splits into two components L_0 and L_1. The component L_0 is slowly varying in time whereas L_1 contains rapidly oscillating terms at frequencies G, $2G$ and $4G$. For intense fields $\vec{E}_1$ and $\vec{E}_2$, so that the condition (4) is fulfilled, it is reasonable to make the secular approximation [15], i.e., to retain only the slowly varying part. Corrections to the results obtained in this fashion will be of an order of

$$(N \gamma_{ij} / G)^2 \quad \text{or} \quad (g_{31}^{(2)} / \vec{E}_3 /)^2 .$$

Making the secular approximation, one can find a stationary solution of the master equation

$$\tilde{\rho} = \Omega^{-1} \sum_{R=0}^{N} x^R \sum_{M=-R,-R+2,\dots}^{R} |R, M\rangle\langle M, R| . \qquad (6)$$

where

$$X = \frac{\gamma_{32}\cos^2\alpha + \gamma_{31}\cos^4\alpha}{\gamma_{21}\sin^2\alpha + \gamma_{31}\sin^4\alpha}$$

$$\Omega^{-1} = (X-1)^2/[(N+1)X^{N+2} - (N+2)X^{N+1} + 1] .$$

$\tilde{\rho} = U\rho U^{+}$, where U is a unitary operator representing the canonical transformation (5).

$|R,M\rangle$ is an eigenstate of the operators $\hat{R} = R_{11} + R_{33}$ $\Delta_3 = R_{33} - R_{11}$ and of the operator of the total number of atoms

$$\hat{N} = J_{11} + J_{22} + J_{33} = R_{11} + R_{22} + R_{33} .$$

Here $R_{ij} = Q_i^{+} Q_j \qquad (i,j = 1,2,3)$.

Now we return to Hamiltonian (1) Retaining only the slowly varying terms (secular approximation), one can find the equations for the operators a and a^{+}

$$\dot{a}(t) = -i\,\delta\, a(t) - i\, B(t)\, a^{+}(t)$$
$$\dot{a}^{+}(t) = i\,\delta\, a^{+}(t) + i\, B(t)\, a(t) . \qquad (7)$$

where $B(t) = g_{31}^{(2)} \sin\alpha\cos\alpha\,(\hat{N} - 3R_{22}(t))$.

Ignoring the slow time dependence of $R_{22}(t)$(stationary stage), we can write the solution of equations (7) in the form

$$a(t) = \mu(t)\, a + \nu(t)\, a^{+}, \qquad (8)$$

where
$$\mu(t) = \cos(\varepsilon t) - i\,\frac{\delta}{\varepsilon}\,\sin(\varepsilon t)$$
$$\nu(t) = -i\,\frac{B}{\varepsilon}\,\sin(\varepsilon t)$$
$$\varepsilon^2 = \delta^2 - B^2 .$$

The Hermitian amplitude operators a_1 and a_2 are defined (in the interaction picture) as

$$a = a_1 + i a_2 .$$

When the field $\vec{E}_3$ is initially in a coherent state $|\alpha\rangle$, the variances of the operators $a_1(t)$ and $a_2(t)$ for large time have the form.

$$(\Delta a_1(t))^2 = \frac{1}{4}\left[1 - 2\,\frac{\delta\langle B\rangle_s - \langle B^2\rangle_s}{\delta^2 - \langle B^2\rangle_s}\sin^2\!\left(\sqrt{\delta^2 - \langle B^2\rangle_s}\; t\right)\right] \qquad (9)$$

$$(\Delta a_2(t))^2 = \frac{1}{4}\left[1 - 2\cdot\frac{\delta\langle B\rangle_s + \langle B^2\rangle_s}{\delta^2 - \langle B^2\rangle_s}\sin^2\!\left(\sqrt{\delta^2 - \langle B^2\rangle_s}\; t\right)\right], \qquad (10)$$

where $\langle A\rangle_s$ indicates an expectation value of operator A in stationary state (6). In relation (9) the correlators of a higher than second order are factorized.

From relation (6) one can fined

$$\langle R_{22}\rangle_s = \frac{N X^{N+2} - (N+2)X^{N+1} + (N+2)X - N}{(N+1)X^{N+3} + (2N+3)X^{N+2} + (N+2)X^{N+1} + X - 1} \qquad (11)$$

$$\langle R_{22}^2\rangle_s = \frac{N X^{N+3} - 4X^{N+2} - (N+2)X^{N+1} + (N+2)^2 X^2 - 2(N^2+2N-1)X + N^2}{(1-X)^2[(N+1)X^{N+2} - (N+2)X^{N+1} + 1]} \qquad (12)$$

It is easy to shown for relations (9-12) that squeezing is presented when

$$|\delta| > \langle B^2\rangle_s / |\langle B\rangle_s| \qquad (13)$$

In the case of $\delta = 0$ (no detuning) the condition (13) is not satisfied and squeezing is absent. One can find from relations (11-12) that $\langle B\rangle_s = 0$ if $X = 1$ and in this case the squeezing is absent too.

The optimum variances of operators a_1 or a_2 are achievable when

$$|\delta| = \frac{\langle B^2\rangle_s}{|\langle B\rangle_s|} + \sqrt{\left(\frac{\langle B^2\rangle_s}{\langle B\rangle_s}\right)^2 - \langle B^2\rangle_s}$$

and equal the value:

$$\frac{1}{4}\,\frac{(\Delta B)^2 + \sqrt{\langle B\rangle_s^2(\Delta B)^2 + (\Delta B)^4}}{\langle B\rangle_s^2 + (\Delta B)^2 + \sqrt{\langle B\rangle_s^2(\Delta B)^2 + (\Delta B)^4}} \qquad (14)$$

when $\sin^2\left(\sqrt{\delta^2 - \langle B^2\rangle_s}\,t\right)$.

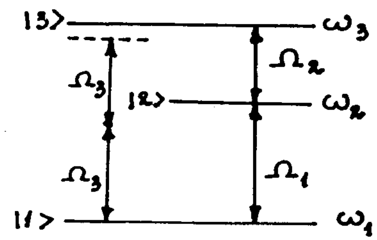
Here $(\Delta B)^2 = \langle B^2\rangle_s - \langle B\rangle_s^2$, $(\Delta B)^4 = ((\Delta B)^2)^2$.

One can show from relation (14) that squeezing tends to the limited value if

$$(\Delta B)^2 \ll \langle B\rangle_s^2. \qquad (15)$$

It is easy to see from relations (11-12) that condition (15) is satisfied when

$x > 1$, $N \gg 1$ so that $x^N \gg N$ or
$x < 1$, $N \gg 1$ so that $x^N \ll \frac{1}{N}$
or in the case of $x \gg N$ or $x \ll \frac{1}{N}$.



Three-level system of atoms interacting with the three monochromatic applied fields via four-photon process.

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Генерация сжатого состояния света
через четырехфотонный процесс
в системе трехуровневых атомов

Исследована генерация сжатого состояния света через четырехфотонный процесс в системе трехуровневых атомов. Сформулировано условие получения оптимального сжатия света.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Generation of Squeezed States of Light
via Four-Photon Process
in System of Three-Level Atoms

The generation of squeezed states of light via four-photon processes in systems of three-level atoms is presented. The condition for receiving an optimum squeezing is defined.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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