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CHARGE SYMMETRY BREAKING EFFECTS FROM PHASE SHIFT ANALYSIS OF ELASTIC $\pi^{\pm}\,{}^4He$ SCATTERING

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Pion - nucleus interaction provides new possibilities for studying charge symmetry breaking (CSB) effects owing to the existence of two pions of opposite charges.

One interesting consequence of the difference between the u- and d-quark masses is that there must exist a difference between the pure hadronic cross sections of π^+ and π^- scattering on nuclei with zero isospin. The authors of refs. /1-4/ claim to have discovered such a charge symmetry breaking effect in $\pi^{\pm}d$, ^{3}He, ^{3}H elastic scattering. The observed difference between the Coulomb corrected cross sections of π^+ and π^--nuclei scattering were interpreted as a manifestation of different masses and widths of the Δ_{33}-isobar states excited in π^+ and π^--nuclei scattering /1-3/.

Obtained estimates of the splittings of the Δ_{33}-states are in suitable agreement with predictions of the models which take into account the different quark composition of Δ_{33}-resonances /5/. Nevertheless, there remain some doubts concerning the conclusion on CSB /6,7/.

These doubts arise from the fact that for any firm conclusion about CSB one must calculate properly not only Coulomb corrections but the pion - nuclear amplitude with very high precision, unattainable at this moment. So it would be better to search for CSB as much model independentely way as possible. The phase shift analysis (PSA) is rather convenient method for that purpose. It is easier to perform PSA of elastic $\pi\,^4$He scattering due to a simple spin-isospin structure of ^{4}He and a smaller number of free parameters needed for the description of $\pi\,^4$He scattering as compared with $\pi\,^{12}$C or $\pi\,^{16}$O-scattering because only few partial amplitudes take part in $\pi\,^4$He scattering. An indication of CSB in that case will be discovering of significant difference between the pure hadronic phases extracted from $\pi^+\,^4$He and $\pi^-\,^4$He scattering.

The amplitude of $\pi\,^4$He scattering was parametrized as usual:

$$f_{\pi\,^4He}(\theta) = f_C(\theta) + f_{CN}(\theta), \qquad (1)$$

here $f_c(\theta)$ is the Coulomb amplitude where the nonpoint charge distribution in ^{4}He and pion was taken into account as in /8,9/.

The main difficulty in the search for CSB-effects in PSA is to determine pure hadronic phase shifts. We used the formalism of approximate treating for external Coulomb corrections which has been developed in /10-13,7/. This approach allows a rather good simultaneous description of π^+ and π^-A elastic scattering both in the case of heavy nuclei /11-13/, and for $\pi^{\pm}d$ scattering /7/.

According to /10/ the total phase shift $\Delta^{(\pm)}_{tot}$, in the nuclear amplitude $f_{CN}(\Theta)$

$$f^{\pm}_{CN}(\Theta)=\frac{1}{k}\sum_{\ell}^{L}(2\ell+1)e^{2i\tilde{\sigma}^{\pm}_{\ell}}\frac{e^{2i\Delta^{(\pm)}_{tot,\ell}}-1}{2i}P_{\ell}(\cos\theta) \qquad (2)$$

is

$$\Delta^{(\pm)}_{tot,\ell}=\delta_{H,\ell}+\delta^{(\pm)}_{R,\ell}+i\left[\omega_{H,\ell}+\omega^{(\pm)}_{R,\ell}\right]. \qquad (3)$$

Here $\tilde{\sigma}^{\pm}_{\ell}$ are the Coulomb phase shifts $\tilde{\sigma}^{\pm}_{\ell}=\arg\left[\Gamma(\ell+1+i\xi)\right]$, ξ is the Coulomb parameter $\xi=\pm\alpha/\beta$, where $\alpha=1/137$, $\beta=V^{lab}_{\pi}/c$, $P_{\ell}(\cos\Theta)$ are the Legendre polinomials. Phases $\delta^{(\pm)}_{R,\ell}$ and $\omega^{(\pm)}_{R,\ell}$ are the reduced phases which represent the Coulomb distortion of the nuclear potential. By its physical meaning $\delta^{(\pm)}_{R,\ell}$ and $\omega^{(\pm)}_{R,\ell}$ correspond to external Coulomb corrections /2/. $\delta_{H,\ell}$ and $\omega_{H,\ell}$ are pure hadronic phase shifts which must be equal for π^{+} and π^{-}A scattering if charge symmetry exists.

As was shown in /7/

$$\delta^{(\pm)}_{R,\ell}=\hat{\alpha}_{\ell}(k)\left[\frac{d\delta_{H,\ell}}{dk}+\frac{\sin 2\delta_{H,\ell}\cdot\cosh 2\omega_{H,\ell}}{2k}\right],$$
$$\omega^{(\pm)}_{R,\ell}=\hat{\alpha}_{\ell}(k)\left[\frac{d\omega_{H,\ell}}{dk}+\frac{\cos 2\delta_{H,\ell}\cdot\sinh 2\omega_{H,\ell}}{2k}\right], \qquad (4)$$

where

$$\hat{\alpha}_{\ell}(k)=\frac{2\mu Z_{\pi}Z_{^4He}\alpha k}{\pi}\int_{0}^{\infty}dk'\frac{k'^2}{k^2-k'^2}\int_{-1}^{+1}\frac{P_{\ell}(\cos\theta)\mathcal{F}_{^4He}(q^2)\mathcal{F}_{\pi}(q^2)}{q^2}d\cos\theta , \qquad (5)$$

here μ is the reduced mass of π^{4}He, $q^2=k^2+k'^2-2kk'\cos\theta$, $\mathcal{F}_{^4He}(q^2)$ and $\mathcal{F}_{\pi}(q^2)$ are the formfactors of ^{4}He and of pion.

We calculate the derivatives $d\delta_{H,\ell}/dk$ and $d\omega_{H,\ell}/dk$ from (4) in the framework of the unitarity approach to πA scattering based on the law of evolution of a system with respect to the coupling constant (CCE-method)/14/. In this approach one can obtain a good description of the low energy π^{4}He scattering (see Fig. 1) /15/.

PSA was done in few steps. At first, the pure hadronic phase shifts $\delta_{H,\ell}$ and $\omega_{H,\ell}$ were determined by fitting the differential cross sections simultaneously for π^{+} and π^{-} ^{4}He scattering. After that the ambiguities of PSA were analysed. We considered all Barrelet type ambiguities /16/ by finding the zeros z_i of $f_{CN}(\Theta)$ in the complex plane $z=\cos\Theta$ and investigating all possible permutations $z_i\rightarrow z_i^{*}$. Of the 8 feasible sets of phase shifts which were analysed only one physical set turned out to be satisfactory; the rejected

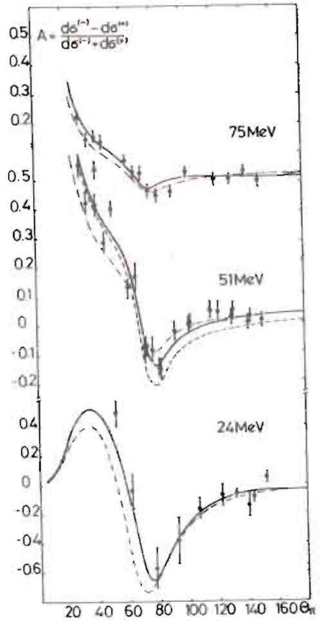

Fig. 1. Angular dependence of the asymmetry parameter $A(\Theta)$ in $\pi^{\pm}$ ^{4}He elastic scattering at a) T_{π} = 24 MeV, b) 51 MeV and c) 75 MeV. Continuous lines represent PSA results, dot-dashed lines represent the calculation in CCE-model /15/. The dashed line at 51 MeV corresponds to PSA-fit when $\delta^{(\pm)}_{R,\ell}-\omega^{(\pm)}_{R,\ell}=0$. Experimental data are from /17-19/.

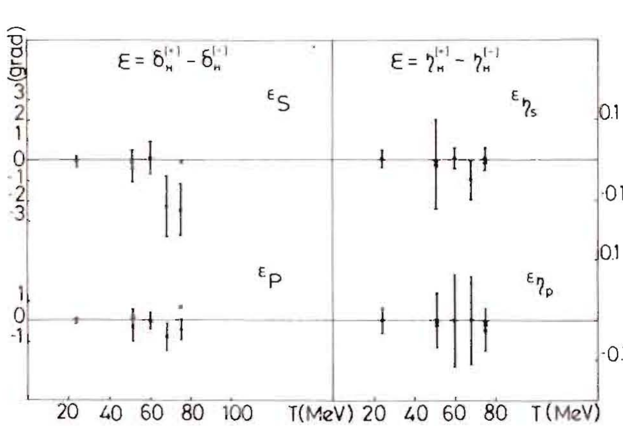

Fig.2. Difference $\varepsilon_{\ell}=\delta^{(+)}_{H,\ell}-\delta^{(-)}_{H,\ell}$ between pure hadronic phases $\delta_{H,\ell}$ obtained separately from π^{+} and π^{-} ^{4}He differential cross sections for S and P-waves. On the right-hand side the corresponding difference $\varepsilon_{\eta,\ell}=\eta^{(+)}_{\ell}-\eta^{(-)}_{\ell}$ for inelasticity parameters $\eta_{\ell}=\exp(-2\omega_{H,\ell})$ is shown. The small crosses at 51 MeV and 75 MeV correspond to the pion-helium phase difference induced by mass and width splittings of the Δ_{33}-resonances (Δm = 6 MeV, $\Delta\Gamma$ = 8 MeV) in the P_{33}-wave.

sets violated unitarity or had greater χ^2/NDF than the chosen solution. In Table 1 we collect the obtained phase shifts and the values of σ_{tot} and σ_{el}. In Fig. 1 the angular dependence of the asymmetry parameter $A(\theta) = (d\sigma^-/d\Omega - d\sigma^+/d\Omega)/(d\sigma^-/d\Omega + d\sigma^+/d\Omega)$ is shown. It must be stressed that despite the fact that the magnitude of $\delta^{(\pm)}_{R,\ell}$ and $\omega^{(\pm)}_{R,\ell}$ is not large (about 10% of $\delta_{H,\ell}$, $\omega_{H,\ell}$) the role of these phases is rather important. If reduced phases $\delta^{(\pm)}_{R,\ell}$, $\omega^{(\pm)}_{R,\ell}$ are "turned off" the quality of the fit falls drastically and χ^2/NDF increases 1.5-2 time (see Fig. 1).

For a more detailed search for CSB we performed the PSA for π^+ and π^- ^{4}He scattering data separately. Parametrization of $f_{CN}(\theta)$ and Coulomb corrections (3)-(5) were the same as earlier, ambiguities were re-analysed for control. Due to large errors of the phase shifts at $T_\pi > 100$ MeV (see Table 1) we performed CSB analysis only for good accuracy low energy data.

Fig. 2 shows the difference ε_ℓ between pure hadronic phases obtained from π^+ and π^- ^{4}He scattering separately. It is clear that no significant difference exists. That indicates at the validity of charge symmetry in the energy interval considered.

We estimated the possible CSB effect in the $\pi^\pm$ ^{4}He phase shifts due to the mass splitting of Δ_{33}-resonances. For that purpose we calculated phase shifts of π^4He elastic scattering in the framework of the CCE-model when the input πN P_{33}-wave was slightly distorted to imitate the mass difference of Δ_{33}-resonances. For imitation of Δ^{++}_{33} or Δ^{-}_{33} we slightly shifted the parameters of the mass and width from the standard one. The splitting of the isobars was only $\Delta m = 6$ MeV by mass and $\Delta\Gamma = 8$ MeV by widths. This procedure did not distort the whole energy dependence of P_{33} phase shifts. It led, for example, to such a small difference between P_{33} phases at 75 MeV as $\Delta P_{33} = 0.35^\circ$. It is instructive that this difference is amplified in pion-helium scattering - the corresponding value in hadronic phases $\varepsilon_1 = \delta^{(+)}_{H,1} - \delta^{(-)}_{H,1}$ grows up to $\varepsilon_1 = 0.72^\circ$ at T = 75 MeV. (It is shown in Fig. 2 by crosses). In principle, such a difference between hadronic phases may be detected in the PSA if the differential cross sections of elastic scattering will be measured with 1-2% precision. It is important to measure $d\sigma/d\Omega$ in a wide angular interval.

In conclusion, we have performed a simultaneous PSA of elastic π^+ and π^- ^{4}He scattering with allowance for external Coulomb corrections. We have obtained quite a good description of the angular dependence of the asymmetry parameter $A(\theta)$. No significant differences between pure hadronic phase shifts deduced separately from π^+ and π^- ^{4}He scattering have been found.

Table 1. Pure hadronic phase shifts $\delta_{H,\ell}$ and inelasticity parameters $\eta_{H,\ell} = \exp(-2\omega_{H,\ell})$ from PSA of $\pi^\pm$ ^{4}He elastic scattering. The errors were determined in the sense of $\chi^2 + 1$.

T (MeV)	δ_S (grad)	η_S	δ_P (grad)	η_P	δ_D (grad)	η_D	δ_F (grad)	η_F	σ_{el} (mb)	σ_{tot} (mb)	χ^2/NDF	Ref.
24.	-4.47 + .08 - .09	.91 +.007 -.003	2.77 ±.06	1.00 +.0 -.005	.23 ±.08	.984 ±.008			11.0 ±.5	72.5 ±17.3	1.3	17,19
51.	-7.65 + .06 - .07	.871 +.006 -.002	8.73 ±.04	1.00 +.0 -.005	1.03 ±.04	.998 ±.023			28.98 ±.20	50.1 ±8.8	3.5	18.19
60.	-9.66 ± .20	.785 +.017 -.008	10.32 +.16 -.11	1.00 +.0 -.009	1.25 ±.07	.976 ±.005	.06 ±.03	.989 ±.004	35.52 ±.22	87.12 ±5.03	4.2	18
68.	-10.24 ±1.20	.869 +.067 -.076	12.55 +.30 -.55	.902 ±.049	1.68 ±.12	.983 ±.010	-.02 ±.11	.986 ±.009	39.6 ±0.4	108.2 ±9.8	2.6	18
75.	-13.43 ± .69	.632 +.058 -.031	12.64 + .67 - .50	.994 +.0 -.028	2.07 ±.15	.934 ±.012	.08 ±.06	.979 ±.007	45.9 ± .3	126.4 ±7.5	2.7	18
98.	-14.58 + 3.05 - 5.71	.719 ±.075	19.51 + .68 -2.41	.692 ±.044	3.94 ±.30	.875 ±.022	.15 ±.27	.977 ±.013	61.3 ±2.5	199.5 ±22.9	3.3	20
120.	-38.66 +11.85 -19.84	.380 ±.105	24.22 +3.71 -5.09	.715 +.067 -.082	7.44 ±1.05	.739 ±.041	.22 ±.41	.897 ±.028	97.2 ±4.8	272.3 ±11.4	0.52	20
135.	-65.50 + 6.42 - 7.52	.380 +.039 -.044	20.37 ±2.48	0.526 ±.026	7.04 ±.93	.684 ±.012	1.95 ±.96	.775 ±.018	105.1 ± 3.1	321.1 ± 7.7	1.4	20
145.	-46.68 ± 3.09	.568 ±.040	16.12 ±2.27	.431 ±.025	9.64 ±.92	.563 ±.027	2.01 ±.56	.753 ±.020	103.4 ± 2.7	324.2 ± 6.9	2.8	20
156.	-53.98 +2.83 -3.02	.573 ±.040	21.37 +3.23 -4.06	.255 ±.026	8.69 ±.90	.552 ±.032	1.00 ±.58	.755 ±.026	114.1 ± 3.1	323.2 ± 7.2	2.5	20

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Ханхасаев М.Х., Никитиу Ф., Сапожников М.Г. E4-85-612

Эффекты нарушения зарядовой симметрии из фазового анализа упругого $\pi^{\pm 4}$He-рассеяния

При энергиях пиона 20-160 МэВ выполнен фазовый анализ упругого $\pi^{\pm 4}$He-рассеяния с целью определения чисто адронных фазовых сдвигов. Не обнаружено никакого статистически значимого различия между адронными фазовыми сдвигами из $\pi^+ \, ^4$He- и $\pi^- \, ^4$He-рассеяния. Представлен также фазовый анализ одновременно π^+- и $\pi^- \, ^4$He-рассеяния с учетом внешних кулоновских поправок. Получено хорошее описание параметра угловой асимметрии A(θ).

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Charge Symmetry Breaking Effects from Phase Shift Analysis of Elastic $\pi^{\pm 4}$He Scattering

A phase shift analysis of elastic $\pi^{\pm 4}$He scattering at energies 20-160 MeV was performed to determine pure hadronic phase shifts. No statistically significant difference between the hadronic phase shifts deduced from $\pi^+ \, ^4$He and $\pi^- \, ^4$He scattering was observed. The results are also presented of a simultaneous phase shift analysis of both π^+ and $\pi^- \, ^4$He scattering with account of external Coulomb corrections. A good description of the angular asymmetry parameter A(θ) is obtained.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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