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EXCITATION-ENERGY DEPENDENCE OF NUCLEAR PROPERTIES IN THE HYPERSPHERICAL-FUNCTION METHOD

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1. INTRODUCTION

In the last years many experiments have been performed with heavy ions where the nuclei get very highly excited in deep-inelastic collisions. One of the results of these investigations is the fact that macroscopic nuclear properties change with varying excitation energy and that this modification influences the reaction process (fission). On the other hand, various theoretical attempts have been undertaken to describe the temperature dependence of such nuclear characteristics like density distribution, size and shape of the nucleus, going out from the nucleon-nucleon interaction where the temperature is equivalent to the excitation energy of the nucleus. This change of the nuclear properties is assumed to be a consequence of the washing-out of shell-effects with increasing heating. In highly excited fragments the number of available single-particle states and their width become very large. Hence, it is not possible to characterize the properties of separate nuclear states. But, as it was pointed out by Bozzolo and Vary recently [1], a realistic nuclear equation of state for finite nuclei, which has been derived mictoscopically, would be of great importance for interpreting physical phenomena in heavy-ion scattering and high-energy particle-nucleus collisions. A first step in this direction has been done by Bloch and di Dominics [2], who have introduced a theoretical framework to explain the thermal behaviour of finite nuclei in the mean-field theory. This model has been extended to include the effects of superconductivity by Mosel et al. [3]. On the basis of this refined formalism Sauer et al. [4] calculated average properties self-consistently using a phenomenological Hamiltonian with a Skyrme force for the nucleon-nucleon interaction. In a very recent paper Bozzolo and Vary [1] employed a realistic microscopic effective Hamiltonian with the spherical finite-temperature Hartree-Fock approximation (FTHF) to evaluate the thermodynamic behaviour of the nuclei ^{16}O and ^{40}Ca. They found that the thermal response of these nuclei is substantially greater than that obtained with zero-range phenomenological forces, and that this effect is much more important for lighter than for heavier ions. Generalizing the results of the numerical calculations they gave a parametrization of the temperature dependence of the excitation energy and the rms-radius for each nucleus considered.

In the present work we propose a quite other way to explore the dependence of the density distribution and the nuclear radius on excitation energy without explicit introduction of the temperature parameter T into the calculational method. A collective Hamiltonian with a realistic nucleon-nucleon interaction describes the nucleus as a many-particle system consisting of A nucleons. The solution of the corresponding Schrödinger equation can be utilized to investigate the variation of collective nuclear properties with increasing excitation energy. Of course, it is necessary to point out that in such a type of calculation all the energy of the nucleus is concentrated on one degree of freedom, especially the nuclear radius, while in the Thomas-Fermi-Model, by which the temperature has been introduced into nuclear physics, and in the Hartree-Fock approximation the excitation energy is distributed over a lot of single-particle degrees of freedom. Consequently, one has to expect a stronger energy dependence of the nuclear size than it is given by the mean-field theory. This distinction will be discussed when the results obtained by the two approaches are compared in Sec.3. Besides, there will be presented an example for the construction of the interaction potential of two excited nuclei and for the application of this formalism to the interpretation of inelastic scattering of ions leading to high excitation of one of them. The main results of this paper are summarized in Sec.4. But before, the theoretical method used for the calculations is described briefly in Sec.2.

2. THE CALCULATIONAL METHOD

In this work the investigation is restricted to light nuclei up to $A = 16$. In this range of nuclear masses it is possible to describe the nuclear properties in the framework of the hyperspherical-function method /5/. Utilizing this approach the wave function Ψ of a nucleus with mass number A can be expanded in standard hyperspherical polynomials according to

$$\Psi = \rho^{-(3A-4)/2} \sum_{K\gamma} \chi_{K\gamma}(\rho) Y_{K\gamma}(\Theta_i) \qquad (1)$$

with the normalization condition $\int \chi^2_{K\gamma}(\rho) d\rho = 1$, where K is an analogue of the angular momentum called the global momentum. The value of K characterizes the chosen approximation used for the numerical calculation. In the present paper only the lowest order $K = K_{min} = A - 4$ is taken into account. The subscript $\gamma = [f] \epsilon LST$ denotes all other quantum numbers of the degenerate single-particle states. The collective variable ρ is the hyperradius and the Θ_i refer to the hyperspherical angles. The hyperspherical harmonics $Y_{K\gamma}(\Theta_i)$ are the eigen-

functions of the angular part of the Laplacian for the many-particle system. They obey the equation $\Delta\Omega Y_{K\gamma}(\Theta_i) = -K(K+1) Y_{K\gamma}(\Theta_i)$. The radial part of the wave function (1) is given by the eigenfunctions of the following operator equation

$$\left\{ \frac{d^2}{d\rho^2} - \frac{L_K(L_K+1)}{\rho^2} - \frac{2m}{h^2}\left(E + W^{K\gamma}_{K\gamma}(\rho)\right)\right\} \chi_{K\gamma}(\rho) = \frac{2m}{h^2} \sum_{K'\gamma' \neq K\gamma} W^{K'\gamma'}_{K\gamma}(\rho)\chi_{K'\gamma'}(\rho), \qquad (2)$$

where the eigenvalues E are the corresponding energy levels of the nucleus. In eq.(2) the matrix elements of the potential energy of the nucleon-nucleon interaction are denoted by $W^{K'\gamma'}_{K\gamma}(\rho)$, the value of the angular momentum L_K is determined by the relation $L_K = K + \frac{1}{2}(3A - 6)$. Then, the radial wave functions $\chi_{K\gamma}(\rho)$ are used to calculate the matrix elements of the density operator. In the hyperspherical-function method the radial density distribution takes the form

$$\rho_{ij}(R) = \frac{16}{\sqrt{\pi}} \frac{\Gamma\left(\frac{5A-11}{2}\right)}{\Gamma\left(\frac{5A-14}{2}\right)} \int_R^\infty \frac{(\rho^2 - R^2)^{\frac{5A-16}{2}}}{\rho^{5A-13}} \chi_i(\rho)\chi_j(\rho) d\rho$$
$$+ \frac{8}{3} \frac{(A-4)}{\sqrt{\pi}} \frac{\Gamma\left(\frac{5A-11}{2}\right)}{\Gamma\left(\frac{5A-16}{2}\right)} \int_R^\infty \frac{R^2(\rho^2 - R^2)^{\frac{5A-18}{2}}}{\rho^{5A-13}} \chi_i(\rho)\chi_j(\rho) d\rho, \qquad (3)$$

where the density of nuclear states (diagonal matrix elements of the density operator) is normalized according to

$$4\pi \int \rho_{ii}(R) R^2 dR = A \qquad (4)$$

and the rms-radius of each state can be obtained as follows

$$\overline{R^2_{ii}} = \frac{\int \rho_{ii}(R) R^4 dR}{\int \rho_{ii}(R) R^2 dR}. \qquad (5)$$

In this way one gets the nuclear size as a function of the excitation energy of the nucleus, and it is possible to test the validity of the calculation by comparing the results for the ground state with experimental data. Figure 1 shows the effective potential

$$V_{eff}(\rho) = \frac{h^2}{2m} \frac{L_K(L_K+1)}{\rho^2} + W^{K\gamma}_{K\gamma}(\rho),$$

the energy levels, and the first three radial wave functions found by solution of eq.(2) for the configuration $|s^4p^{12}\rangle$ of

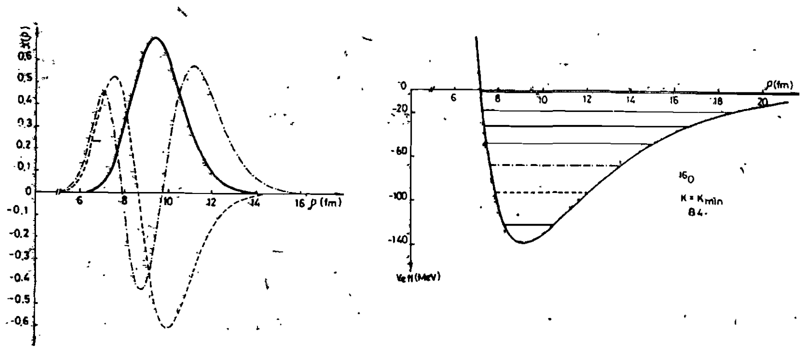

Fig.1. Effective potential from eq.(2) for the nucleus ^{16}O and the corresponding radial wave functions of the first three energy levels. The horizontal lines indicate the calculated energy states.

the nucleus ^{16}O employing the realistic nucleon-nucleon interaction potential B4 proposed by Brink and Boeker [6]. It is seen that the shape of the potential gets broader automatically for higher-lying energy states. In the mean-field theory where oscillator potentials are used for an effective description of the interaction this behaviour is simulated by performing a set of numerical calculations for increasing parameter T which corresponds to a higher excitation of the nucleus. Besides, this formalism gives the correct shape of the radial wave function related to the various excited states.

3. RESULTS OF CALCULATION

In this section the numerical results obtained for the nucleus ^{16}O are presented and compared with that of ref. [1]. At first it is of interest to have a look at the change of the density distribution when the excitation energy varies. Figure 2 shows that this distribution gets broader and broader when E^* increases. Due to the normalization condition (4) its maximum value decreases simultaneously. In the case of the highest excitation the density in the central region ($R \lesssim R_{nuc}$) is so small that the nucleus no longer exists. Its matter is dissolved in space what is characterized by a very large tail of the density distribution function. Because the method described above does not contain the parameter T explicitly it is necessary to convert the excitation energy E^* into the temperature T for the aim of comparison with the results of other authors. This conversion was done in two ways using the phenomenological formula resulting from the Thomas-Fermi-Model [7]

(T in MeV)

$$E^* = 0.1\ \mathrm{MeV}^{-1} A T^2 \tag{6}$$

and the parametrization given by Bozzolo and Vary [1]

$$E^* = 0.185\ \mathrm{MeV}^{-1} A(T-T_0)^2 \text{ for } T > T_0 \ (T_0 = 1\ \mathrm{MeV}). \tag{7}$$

The relations between E^* and T following from eqs.(6),(7) are represented graphically in fig.3.

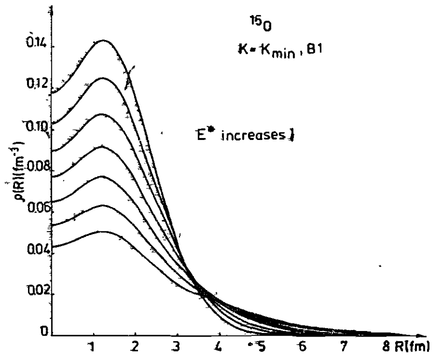

Fig.2. Radial density distribution of ^{16}O computed with the nucleon-nucleon interaction B1 at various excitation energies.

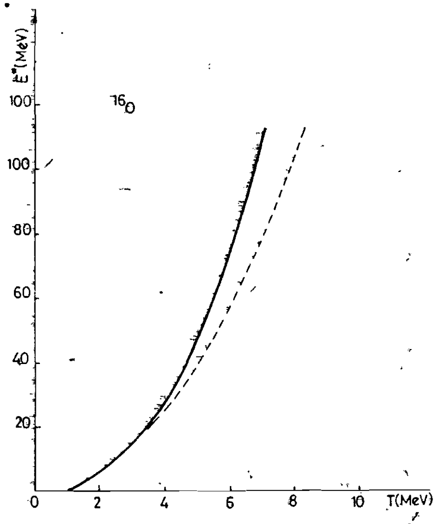

Fig.3. Excitation energy as a function of nuclear temperature for the nucleus ^{16}O. The dashed line refers to eq.(6), the solid one to formula (7).

Then, the temperature dependence of the rms-radius is considered for two parameter sets (B1 and B4) of the nucleon-nucleon interaction and it is compared with that described by the equation

$$\mathrm{rms}\,(T) = 2.74\ \mathrm{fm}\ (1 + 5.6\cdot 10^{-3}\ \mathrm{MeV}^{-2}\ T^2), \tag{8}$$

which results from the finite temperature Hartree-Fock method

In either case the calculation performed in the framework of the hyperspherical-function method gives a more rapid increase of the rms-radius for increasing temperature than it is predicted by FTHF. This is demonstrated in fig.4. The distinction between our two calculations consists in different ground

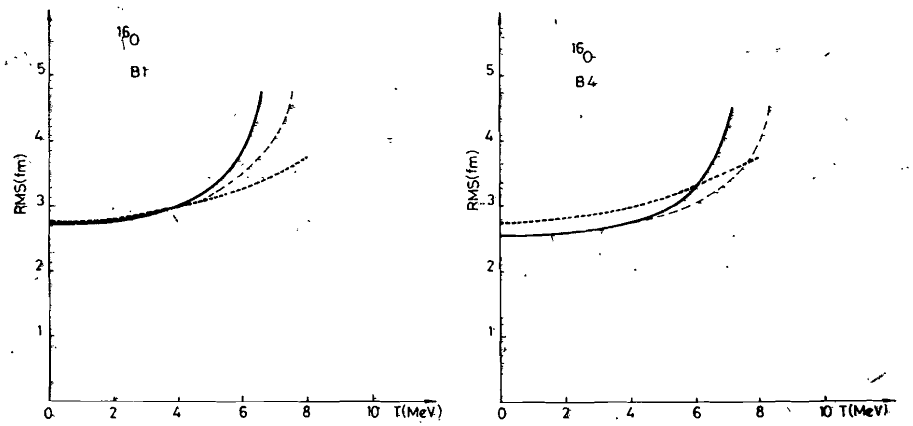

Fig.4. Temperature dependence of the rms-radius of ^{16}O for two types of nucleon-nucleon interaction. The dashed and solid curves show the results according to formulas (6) and (7), respectively (comp.fig.3), in comparison with that of ref.[1] (thick dashed line).

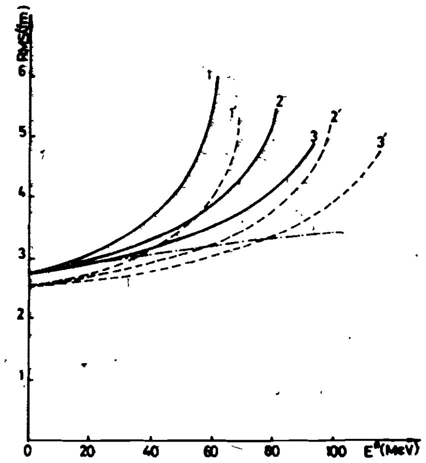

Fig.5. Dependence of the rms-radii on excitation energy for the nuclei ^{12}C(1,1′), ^{15}O(2,2′), and ^{16}O(3,3′). The dash-dotted line represents the result of Bonzolo and Vary[1].

state radii (for comparison: rms_{exp} = 2.54 fm) and in the strength of the energy dependence of the nuclear size. In particular, the potential B1 causes a stronger increase of the radius with energy than B4.

In principle, the same result is obtained if the phenomenological formula (4) is used to convert E* into T. The most important outcome which can be reported here is the fact that for the temperature corresponding to the binding energy of the nucleus eq.(8) determines a finite value of the rms-radius, while according to the calculation presented here the nuclear size goes to infinity. This is equivalent to the dissolving of the nucleus mentioned at the beginning of this section and reflects the physical situation in a more realistic manner.

The reason for this behaviour is the treatment of the problem so that all excitation energy is assumed to be concentrated only on one degree of freedom – the nuclear radius. Although this assumption means a very strong restriction on the lot of possible excitations of single-particle degrees of freedom in the chaotic process of nuclear excitation, the final result is in good agreement with the reality. Finally, fig.5 shows the dependence of the rms-radius on the excitation energy for the nuclei ^{12}C, ^{15}O and ^{16}O in comparison with the result for ^{16}O following from the relation

$$rms(E^*) = 2.74\ \mathrm{fm}\ (1.0056 + 6.51\ 10^{-3}\ \mathrm{MeV}^{-1/2}\sqrt{E^*} + 1.89\ 10^{-3}\ \mathrm{MeV}^{-1} E^*)^{/1/}. \tag{9}$$

The curves 1(B1) and 1′(B4) correspond to the nucleus ^{12}C, the functions 2(B1) and 2′(B4) refer to ^{15}O, and the curves 3(B1) and 3′(B4) represent the behaviour of the nucleus ^{16}O. The brackets contain the chosen parameter set of the nucleon-nucleon interaction. In either case the thermal response of the nuclear radius gets substantially greater when the mass number A decreases. This is in agreement with the prediction following from the calculation with a realistic microscopic effective Hamiltonian in the mean-field theory.

As a next step it is interesting to consider the influence of the change of the nuclear properties due to excitation on the interaction potential of two heavy nuclei and on the theoretical cross sections for inelastic nucleus-nucleus scattering. First of all this effect is expected to play an important role for the description of such processes in which giant resonances at about 20 MeV get excited in the nucleus-nucleus collision. To investigate the modification of the interaction potential of ^{16}O + ^{16}O at various total excitation energies of the system and for different distributions of this energy between the fragments, the folding potential with Skyrme forces[8,9] was constructed using the calculated radial density distributions of ^{16}O. The result is presented in Fig.6. It is seen that the interaction potential $V_{0,0}$ for both the nuclei being in the ground state is the deepest one. In principle, it gets flater with increasing excitation energy of the system, but the potential $V_{01^+,01^+}$ for both the fragments being in the first excited state is deeper than $V_{0,0_2^+}$ for the combination in which one nucleus is in the ground state and the other is in the second excited state, although the total excitation energy of the system is higher in the first case. This behaviour may be explained by the different structures of the wave functions characterizing several nuclear states which determine the radial density dis-

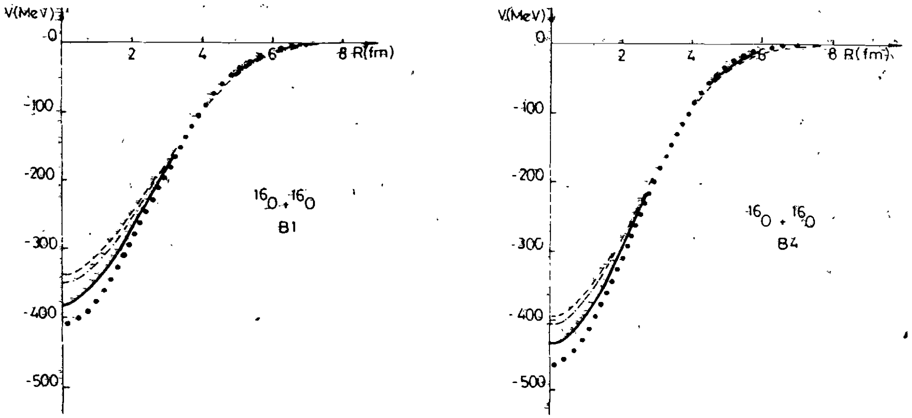

Fig.6. Folding potential with Skyrme forces for the system $^{16}O + {}^{16}O$. Dotted line: $V_{0,0}$, solid line: $V_{0,0_1^+}$, dash-dotted line: $V_{0_1^+,0_1^+}$, dashed line: $V_{0,0_2^+}$.

tribution of the nucleus and consequently the folding potential. These interaction potentials were calculated for two types of nucleon-nucleon interaction (B1, B4) where in either case the same dependence on excitation energy was obtained.

A first attempt to describe microscopically the angular distribution of inelastic nucleus-nucleus scattering leading to the high excitation of one nucleus was undertaken for the systems $^{12}C(^3He, {}^3He)^{12}C^{0^+}$, $E_{^{12}C\,0^+}$ = 20.3 MeV at the incident energy $E_{^3He}$ = 108 MeV, and $^{12}C(^4He, {}^4He^{0^+})^{12}C$, $E_{^4He\,0^+}$ =20.1 MeV at $E_{^4He}$ = 65 MeV[10]. The results shown in figs.7 and 8 are in good accordance with the experimental data. This means that it is necessary to take into account the effect of nuclear excitation in order to calculate cross sections of inelastic nucleus-nucleus scattering.

4. CONCLUSIONS

The predictions for the nuclear behaviour at excitation given by the approach described above differ from results based on mean-field theories. Especially, the present calculation leads to a dependence of the rms-radius on excitation energy so that the nuclear size goes to infinity when the excitation energy of the nucleus reaches its binding energy, while the mean-field theory predicts a finite value. In agreement with FTHF using a realistic microscopic effective Hamiltonian this

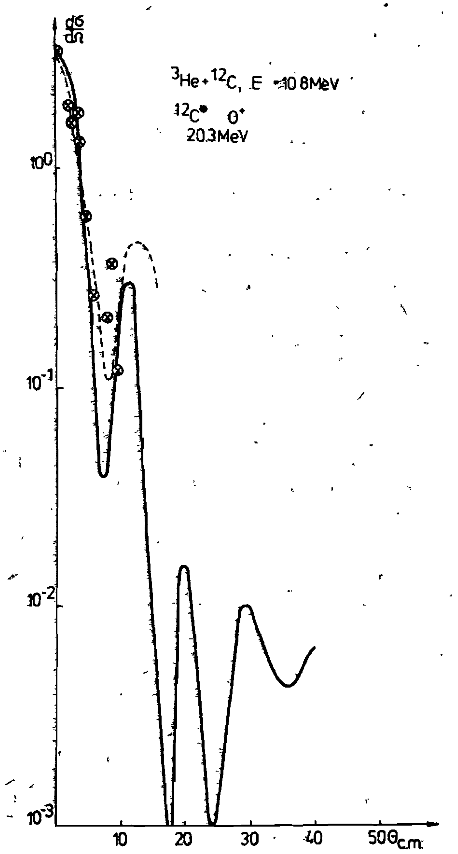

Fig.7. Angular distribution of the inelastic scattering of $^3He + {}^{12}C$ at $E_{^3He}$ = 108 MeV. The solid line refers to the present calculation, the dashed line shows the result of a phenomenological description[11]. The points are the experimental data.

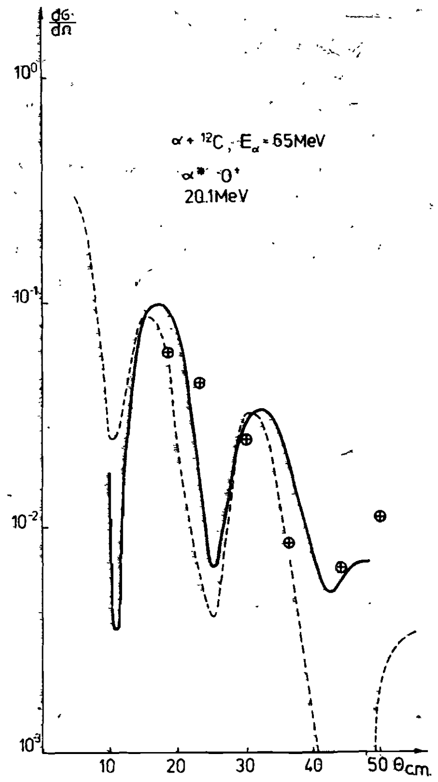

Fig.8. Cross sections of the inelastic scattering of $^4He + {}^{12}C$ at $E_{^4He}$ = 65 MeV[12]. The curves have the same meaning as in fig.7.

method gives a greater thermal response of lighter than for heavier nuclei. The consideration of the modification of nuclear properties for excited nuclei allows us to describe inelastic nucleus-nucleus scattering processes microscopically in good accordance with experimental data.

The authors are indebted to J.P.Vary for stimulating discussion at the beginning of this work. We wish to thank R.V.Jolos and Nguyen Dinh Dang for useful comments.

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Саупе Г., Шитикова К.В. E4-85-44

Зависимость свойств ядер от энергии возбуждения в методе гиперсферических функций

Целью работы является изучение зависимости распределения плотности и средних квадратичных радиусов легких ядер от энергии возбуждения в методе гиперсферических функций. Температурная зависимость этих свойств для ^{16}O существенно больше чем те результаты, которые получены в теории среднего поля. В частности, размеры ядер стремятся к бесконечности, когда энергия возбуждения близка к энергии связи, в то время как в других теориях эта величина имеет в этом случае конечное значение. Ядерно-ядерный потенциал взаимодействия, полученный с учетом изменения свойств возбужденных ядер, позволяет описать угловые распределения неупругого рассеяния ионов в разумном согласии с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Saupe G., Shitikova K.V. E4-85-44

Excitation-Energy Dependence of Nuclear Properties in the Hyperspherical-Function Method

The dependence of the density distribution and the rms-radius of light nuclei on excitation energy is investigated in the framework of the hyperspherical-function method. The thermal response predicted for ^{16}O is essentially greater than the results from mean-field theories. Especially, the nuclear size goes to infinity when the excitation energy of the nucleus reaches its binding energy, while other theories give a finite value in this case. The construction of nucleus-nucleus interaction potentials taking into account the modification of the properties of excited nuclei allows us to describe the angular distributions of inelastic nucleus-nucleus scattering in reasonable agreement with experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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