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THE PROBABILITY OF MUON STICKING TO HELIUM IN THE MUON CATALYZED FUSION $dt\mu \to \mu^4He + n$

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1. Introduction

Beginning from 1977 the phenomenon of muon catalysis of nuclear fusion in deuterium-tritium mixture is being intensively investigated in many laboratories of the world, both theoretically and experimentally (see reviews[1-4]). The bottle neck of the chain of muon catalyzed fusion

$$\mu^- \xrightarrow{d\mu} t\mu \longrightarrow dt\mu \begin{cases} \xrightarrow{1-\omega_s} \alpha + n + \mu^- \\ \xrightarrow{\omega_s} \mu\alpha + n \end{cases} + Q = 17.6 \text{ MeV} \quad (1)$$

is the lost of muons in the ($\mu\alpha + n$) channel where the muon sticks to the α-particle. The problem of muon sticking was discussed by Zeldovich[5] and Jackson[6] in 1957 for the first time. More careful analysis has been performed a few years ago[7-8], still in a framework of the Born-Oppenheimer approximation and for the case of an infinite rate of nuclear fusion, the latter having been treated in a sudden approximation (BO+S method). The estimated accuracy of these calculations was about 10%[x)].

In recent paper[10] a muon wave function of $dt\mu$ mesic molecule calculated with Monte-Carlo method without approximations characteristic for BO method is involved. Thus obtained values of sticking coefficients are by 25% lower than those previously calculated in BO+S method.

Only recently some reliable experimental information on muon sticking in the reactions $dd\mu \to \mu^3He + n$[11] and $dt\mu \to \mu^4He + n$[12] has become available. The experimental values of muon sticking probabilities appeared somehow smaller than the theoretical estimates within BO+S method. In the $dd\mu$ case the achieved experimental accuracy is at the level of a few percent, and it is expected that in the near future experimental data of comparable or better accuracy will be available for muon sticking following other nuclear reactions (pd, dt, etc).

In this situation it seems reasonable and up-to-date to perform an analysis of the approximations used in the BO+S method and to develope a regular method of a more accurate calculation of muon stick-

x) Recent calculation[9] within BO method is irrelevant, since there has been used as the initial state muon wave function that of $dt\mu$ mesic molecule at the equilibrium internuclear distance $R=R_0$, instead of $R \to 0$ limit, as it is necessary.

ing probabilities in various reactions. These are the aims of the present paper.

The BO+S method involves some approximations, each of them having a corresponding smallness parameter such as the ratio of muon to nucleon mass, m_μ / m_N, the ratio of a characteristic range of nuclear interaction to a muon Bohr radius, R_N / a_μ, and so on.

We show in Sec. 2 that corrections due to the finite nucleus mass are most important, and, hence, a muon wave function more precise than the BO one is needed, first of all.

In Section 3 a calculation of the muon sticking probability based on the exact muon wave function is presented. The corrections to the sudden approximation are evaluated in Sections 4 and 5. Our results are summarized and discussed in the concluding Section 6.

Only the case of dt-fusion is considered here mainly due to the intense interest on this specific problem. The developed method, however, holds for nuclear reactions involving other hydrogen isotopes /13/. The comparison between theoretical and experimental determination of the sticking probability in all these cases (dd,pd,pt,etc.) is equally important for the solving of the sticking problem in the dt reaction and allows one to gain information which is, in a sence, complementary to that provided by studying the dt reaction.

2. Analysis of the BO+S Method Approximations

The sticking probability ω_s which is actually measured (effective sticking probability) can be expressed as

$$\omega_s = \omega_s^\circ (1-\gamma), \tag{2}$$

where ω_s° is the probability that the muon is bound to the α-particle just after fusion and γ is shaking-off, or reactivation, coefficient, i.e., the probability that during the slowing down of the produced $\mu\alpha$ atom the muon is shaked off as result of stripping reactions in collisions with the nuclei in the target. The BO+S method for the calculation of ω_s° relies on the following main assumptions:

A) The muon follows adiabatically the motion of the two nuclei until fusion occurs.

B) Fusion is an extremely sudden process, i.e., its time scale is very short in comparison with any other time relevant to the process.

C) Any effect associated with finite nuclear size and finite radius of nuclear forces is neglected: dt fusion occurs when the two nuclei are at zero distance between them, the muon moving in the Coulomb field of point-like nucleus with the charge and mass of their compound.

For the case of dt system this means that the initial muon wave function $\Psi_{in}(\vec{r})$ just before fusion is the wave function of μ^5He in the 1s-state with a point-like nucleus

$$\Psi_{in}^{BO}(\vec{r}) = \varphi_{1s}(\vec{r}). \tag{3}$$

The amplitude of muon sticking to the α-particle in the $(\mu\alpha)_{nl}$-state, F_{nl}, is simply expressed as the overlap between Ψ_{in} and the wave function Ψ_f of $\mu\alpha$ mesic atom in the nl-state, moving with velocity v, the value of which is defined by an energy release Q=17.6 MeV in reaction (1)$^{x)}$.

$$\Psi_f = e^{i m_\mu \vec{v}\vec{r}} \varphi_{nl}(\vec{r}), \quad \vec{q} = m_\mu \vec{v}. \tag{4}$$

$$F_{nl} = \int d^3 r\, \varphi_{nl}^*(\vec{r}) e^{-i\vec{q}\vec{r}} \Psi_{in}(\vec{r}) \tag{5}$$

In papers /7,8/ partial ω_{nl} and total ω_s° sticking probabilities were calculated with formulae:

$$\omega_{nl} = |F_{nl}|^2, \quad \omega_s^\circ = \sum \omega_{nl} \tag{6}$$

and function $\Psi_{in}(\vec{r}) = \Psi_{in}^{BO}(\vec{r})$ were used.

Concerning the three main approximations approved, clearly: A) is exact in the limit the relative motion of two nuclei is extremely slow in comparison with the muon's. The relevant smallness parameter equals the ratio of the muon mass m_μ to the reduced mass of two nuclei m_N,

$$\varkappa = m_\mu / m_N. \tag{7}$$

For dt system one has $\varkappa$ = 0.09, whereas for dd and pd, respectively 0.11 and 0.15. The relevant parameter $\tilde{\varkappa}$ can as well be defined as the ratio of nuclei velocity v_N to muon velocity $v_\mu = \alpha c$.

$$\tilde{\varkappa} = v_N / v_\mu. \tag{8}$$

Near the nuclei equilibrium position v_N can be estimated from the relation between the nuclei vibrational energy and the muon energy

x) In paper /8/ the final state wave function was taken as

$$\Psi_f = e^{i m \vec{v}\vec{r}} \varphi_{nl}(\vec{r}), \text{ where } m = m_\mu m_\alpha (m_\mu + m_\alpha)^{-1}$$

is the reduced mass of ($\mu\alpha$) atom. It is shown in Sect. 5 that a straightforward consideration leads to formula (5).

$$E_{vib} \approx m_N v_N^2 \simeq (m_\mu/m_N)^{1/2} m_\mu v_\mu^2 \qquad (9)$$

from where

$$\tilde{\varkappa} \simeq (m_\mu/m_N)^{3/4}. \qquad (10)$$

In this case one obtains $\tilde{\varkappa}_{dt}=0.15$, $\tilde{\varkappa}_{dd}=0.19$, $\tilde{\varkappa}_{pd}=0.26$. These and similar estimates enable one to evaluate the expected contribution from the corrections to the BO approximation at the 10÷20% level.

Approximation B) is based on the fact that the lifetime $\tau_N \approx 10^{-20}$ s of the compound nucleus $^5He^*$, which can be accepted as a time scale, is small compared to a characteristic time of muon motion in a mesic atom $\tau_\mu \approx 10^{-19}$s. The corresponding parameter

$$\eta = \tau_N/\tau_\mu \simeq 0.1. \qquad (11)$$

The effects due to the nuclei finite size depend on the parameter

$$\xi = R_N/a_\mu \simeq 0.03, \qquad (12)$$

where R_N and a_μ are characteristic nuclear scale and muon orbit radius. BO+S method corresponds to the case $\varkappa = \eta = \xi = 0$. Calculate now the corrections to BO+S due to finite values of these parameters.

3. The Calculation of the Muon Sticking Probability ω_s°

We calculate the muon sticking probability ω_s° without the Born-Oppenheimer approximation but using an exact dtμ mesic molecule wave function. We will remain in a framework of a sudden approximation and neglect the nuclei dimensions and range of nuclear forces in comparison with the mesic molecular dimension.

In a sudden approximation the sticking probabilities are still defined with formulae (5)-(6), where, however, the muon initial wave function $\Psi_{in}(\vec{z})$ at the moment of fusion is expressed via the dtμ mesic molecule wave function $\Psi^{J v}(\vec{z},\vec{R})$ in a rotational - vibrational state (Jv) in the following way

$$\Psi_{in}(\vec{z}) = \lim_{R\to 0} \frac{\Psi^{Jv}(\vec{z},\vec{R})}{\left\{\int d^3z\,|\Psi^{Jv}(\vec{z},\vec{R})|^2\right\}^{1/2}}. \qquad (13)$$

R being the distance between d and t in dtμ mesic molecule. The wave functions $\Psi^{Jv}(\vec{z},\vec{R})$ have been calculated in papers[14,15] with algorithms[16,17] for numerical solution of the Coulomb three-body problem in the adiabatic representation[18]. The R→0 asymptotics of the mesic molecule wave functions has been found in paper[19]. For J=0 states it is:

$$\Psi^{Jv}(\vec{z},\vec{R}) \underset{R\to 0}{=} A^{Jv} R^J \left\{ \sum_{N=1}^{\infty} a_N^{Jv}\phi_{N0}(\vec{z}) + \int_0^\infty dk\, a_k^{Jv}\phi_{k0}(\vec{z}) \right\}, \qquad (14)$$

i.e., in this limit the variables $\vec{z}$ and $\vec{R}$ which describe the motion of the muon and nuclei are factorized. Here $\phi_{N0}(\vec{z})$ and $\phi_{k0}(\vec{z})$ are discrete and continuous spectra wave functions of the mesic atom in ℓ=0 state with nucleus charge Z=2 and mass of tμ mesic atom[18]. Coefficients A^{Jv}, a_N^{Jv} and a_k^{Jv} are known from the numerical solution of the Coulomb three-body problem[18,19] and are normalized according to

$$\sum_{N=1}^{\infty} |a_N^{Jv}|^2 + \int_0^\infty dk\, |a_k^{Jv}|^2 = 1. \qquad (15)$$

From (13)-(15) it follows that the normalized muon initial state function has the form[x]:

$$\Psi_{in}(\vec{z}) \equiv \Psi_{in}^{Jv}(\vec{z}) = \sum_N a_N^{Jv}\phi_{N0}(\vec{z}) + \int dk\, a_k^{Jv}\phi_{k0}(\vec{z}). \qquad (16)$$

The muon wave functions at the moment of fusion, i.e., coefficients a_N^{Jv} and a_k^{Jv}, depend, though weakly, on the mesic molecular state (Jv) (see Table 1). The probabilities P_{Jv} of the nuclear reaction from the various rotational - vibrational states (Jv) are known from the cascade calculations[21]

$$P_{Jv} = \begin{cases} 0.84, & (Jv)=(01) \\ 0.16, & (Jv)=(00) \\ 0.00, & J\neq 0 \end{cases} \qquad (17)$$

The probability of fusion from rotational states with J≠0 being

x) Taking into account of nuclear dt interaction (see[20]) essentially influences coefficient A^{Jv} only, coefficients a_N^{Jv} are practically unchanged and coefficients a_k^{Jv} are changed noticeably for momenta $k \gtrsim R_N^{-1} \gg 1$ only (R_N being the range of nuclear dt interaction).

negligibly small x), the muon sticking should be considered for $J=0$ states only.

Sticking coefficients $\omega_{n\ell}^{J v}$ are calculated with formula analogous to (6)

$$\omega_{n\ell}^{J v} = \left| F_{n\ell}^{J v} \right|^2 , \tag{18}$$

where, according to (5) and (16)

$$\begin{aligned} F_{n\ell}^{J v} &= \sum_N a_N^{J v} F_N(n\ell) + \int dk\, a_k^{J v} F_k(n\ell) , \\ F_N(n\ell) &= \int d^3 r\, \Psi_{n\ell}^{*}(\vec r)\, e^{-i\vec q \vec r} \phi_{N0}(\vec r) , \\ F_k(n\ell) &= \int d^3 r\, \Psi_{n\ell}^{*}(\vec r)\, e^{-i\vec q \vec r} \phi_{k0}(\vec r) . \end{aligned} \tag{19}$$

Here $\vec q = m_\mu \vec v$, v =5.843 is the velocity of $\mu\alpha$ mesic atom calculated within relativistic kinematics for the process $dt\mu \to \mu\alpha + n$. (A more accurate definition/10/ of v is relevant, because the value ω_s is extremely sensitive to v.)

Coefficients $a_N^{J v}$ and $a_k^{J v}$ for the states $J=0$ are presented in Table 1 and shown in Fig. 1b.

Here and below, unless otherwise stated, the units are used $e = \hbar = m = 1$, $m = m_\mu m_t/(m_\mu + m_t)$ where m_μ = 105.66 MeV, m_t =2808.94 MeV are masses of muon and tritium, respectively. Formfactors $F_N(1s)$ and $F_k(1s)$ calculated with formulae (19) are displayed in Fig.1a. They satisfy the closure relation

$$\sum_N \left|F_N(1s)\right|^2 + \int_0 dk \left|F_k(1s)\right|^2 = \left\{1 + (q a)^2\right\}^{-1} , \tag{20}$$

where $a = (m_\mu + m_\alpha)/2 m_\mu m_\alpha$.

Although state N=1 dominates in the normalization condition (15) of the mesic molecule wave function ($|a_1|^2 = 0.98$), its contribution to (20) is about 10% only. Hence, despite the smallness of a_N and a_k their inclusion essentally decreases the value of ω_s^{BO} calculated in the BO approximation due to the destructive interference with N=1 term. Thus the probability ω_{1s} decreases from ω_s^{BO} =0.903·10^{-2} for the wave function $\varphi_{1s}(\vec r)$ of μ^5He atom to ω_{1s} =0.867·10^{-2} for the μ^3He atom wave function $\phi_{1s}(\vec r)$. Inclusion of descrete spectrum

x) Mesic molecule $dt\mu$ is formed in state $(J v)=(11)$/22/, but for all states with $J\neq 0$ its de-excitation rates significantly exceed the rates of nuclear fusion/21/.

Table 1

Formfactors $F_N(1s)$ and $F_k(1s)$ and coefficients a_N and a_k of decomposition (16) x)

N	$F_N(1s), 10^{-1}$	a_N (J=0,v=1)	a_N (J=v=0)	$a_N F_N/a_1 F_1$ (J=v=0)
1	0.9312	0.9831	0.9827	1
2	0.3514	-0.0733	-0.0747	-0.0287
3	0.1936	-0.0293	-0.0297	-0.0063
4	0.1263	-0.0173	-0.0175	-0.0024
5	0.0905	-0.0124	-0.0126	-0.0012
k	$F_k(1s), 10^{-1}$	$a_k, 10^{-1}$	$a_k, 10^{-1}$	$(a_k F_k/a_1 F_1), 10^{-1}$
0.2	0.2273	-0.2702	-0.2740	-0.0623
0.4	0.3223	-0.3610	-0.3659	-0.1179
0.6	0.3964	-0.4039	-0.4090	-0.1621
0.8	0.4605	-0.4140	-0.4190	-0.1929
1.0	0.5188	-0.4017	-0.4061	-0.2106
1.2	0.5737	-0.3755	-0.3793	-0.2176
1.4	0.6265	-0.3420	-0.3451	-0.2162
1.6	0.6784	-0.3060	-0.3086	-0.2094
1.8	0.7301	-0.2705	-0.2726	-0.1990
2.0	0.7822	-0.2372	-0.2389	-0.1869
2.5	0.9174	-0.1680	-0.1690	-0.1550
3.0	1.0636	-0.1187	-0.1193	-0.1268
3.5	1.2215	-0.0847	-0.0851	-0.1040
4.0	1.3831	-0.0615	-0.0618	-0.0854
5.0	1.5997	-0.0343	-0.0344	-0.0550
6.0	1.3295	-0.0205	-0.0205	-0.0273
7.0	0.7081	-0.0130	-0.0130	-0.0092
8.0	0.3143	-0.0087	-0.0087	-0.0028
9.0	0.1461	-0.0060	-0.0060	-0.0008
10	0.0748	-0.0043	-0.0043	-0.0003

x) The values are calculated at v=5.843, $q = (m_\mu/m) v$ =6.063, m/m_μ =0.963748, in units $e = \hbar = m = 1$.

states N>1 in decomposition (16) decreases ω_{1s} down to $0.770\cdot 10^{-2}$ and, at last, the continuum taken into account diminishes it to $\omega_{1s} = 0.653\cdot 10^{-2}$. About 23% of muons are captured to excited nl states.

The probability of muon sticking to excited states nl with $n \geq 5$ was calculated with formula [13]:

$$\omega_n = \sum_{l=0}^{n-1} \omega_{nl} = \left(4\zeta/n\right)^3 \cdot \frac{\left[(1-\zeta/n)^2 + (qa)^2\right]^{n-3}}{\left[(1+\zeta/n)^2 + (qa)^2\right]^{n+3}} \times$$

$$\times \left\{\left[(1-\zeta)(1-\zeta^2/n^2) + (1+\zeta)(qa)^2\right]^2 + \frac{4\zeta^2}{3}\left(1-\frac{1}{n^2}\right)(qa)^2\right\}, \quad (21)$$

where

$$\zeta = \frac{m_\alpha (m_\mu + m_{^5He})}{m_{^5He}(m_\mu + m_\alpha)}.$$

The resulting initial sticking probability

$$\omega_s^\circ = \sum_{J v} \sum_{nl} P_{Jv}\, \omega_{nl}^{Jv} = 0.848\cdot 10^{-2} \qquad (22)$$

is by 27% smaller than the value $\omega_s^{BO} = 1.164\cdot 10^{-2}$ calculated within the Born-Oppenheimer approximation[x)]. Calculated here ω_{nl}^{00} reasonably coincide with ω_{nl}^{AC} from paper [10] obtained with Monte-Carlo method (they are listed in the third coloumn of Table 2). Such an agreement, first of all, evidences the correctness and high accuracy of both methods. From Table 2 one can also see that ω_{nl}^{ov} practically do not depend on the vibrational state v of $dt\mu$ mesic molecule.

4. Corrections to the Sudden Approximation

In this section we perform a more detailed, as compared to Sect. 2, analysis of the sudden approximation. We'll continue neglecting the nuclear finite size effects (see Sect. 5). However, start from a more accurate definition for the sticking probability of muon to helium.

Expressions (5), (6) and (18) for ω_{nl}^{Jv} are simplified versions of the general definition

$$\omega_{nl}^{Jv} = \lambda_{nl}^{Jv} / \lambda^{Jv}, \qquad (23)$$

x) The difference between this value and those presented in papers [7, 8] is due to the contribution from the states nl with $n \geq 4$.

Table 2

Sticking probabilities ω_{nl}^{Jv}, 10^{-2}, for the reaction $(dt\mu)^{Jv} \to (\mu\alpha)_{nl} + n$ from the states $(Jv) = (01)$ and (00)

nl	ω_{nl}^{01}	ω_{nl}^{00}	$\omega_{nl}^{AC\,x)}$	$\omega_{nl}^{BO\,xx)}$
1s	0.6526	0.6502	0.689	0.9030
2s	0.0937	0.0934	0.099	0.1287
2p	0.0239	0.0238	0.024	0.0321
3s	0.0285	0.0284	0.030	0.0391
3p	0.0086	0.0086	0.009	0.0115
3d	0.0003	0.0003	-	0.0003
4s	0.0122	0.0121	0.013	0.0166
4p	0.0038	0.0037	-	0.0051
4d+4f	0.0003	0.0003	-	0.0003
$\sum_{n=5}^{\infty}\sum_{l=0}^{n-1}$	0.0242	0.0241	0.031	0.0278
Total	0.848	0.845	0.895±0.004	1.164

$\omega_s^\circ = 0.848$

x) Data from paper [10]. Values ω_{4p} and ω_{3d} are included in the sum over the higher states.

xx) The listed values are calculated with formulae (5)-(6) with functions (3). The contribution from $n \geq 5$ states was found with formula (21).

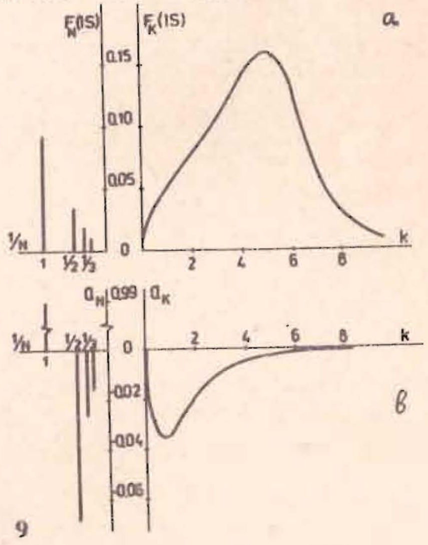


Fig.1. Form factors $F_N(1s)$ and $F_k(1s)$ calculated with formula (19) (a) and coefficients a_N and a_k of the mesic molecule $dt\mu$ wave functions $\Psi^{00}(\vec{r}, \vec{R})$ decomposition (14)(b).

where $\lambda_{n\ell}^{J\upsilon}$ is a partial rate of nuclear reaction

$$(dt\mu)^{J\upsilon} \longrightarrow (\mu d)_\beta + n \qquad (24)$$

with $(\mu d)_\beta$ system in discrete spectrum state $\beta=(n\ell)$ and

$$\lambda^{J\upsilon} = \sum_{\beta=\{n\ell,\vec k\}} \lambda_\beta^{J\upsilon} \qquad (25)$$

is a total rate of nuclear fusion (1) from mesic molecular states $(J\upsilon)$. Estimate the uncertainties appearing due to transition from (23) to (18).

In the first order of the perturbation theory in fusion reaction $dt \to d+n$ amplitude T partial rate $\lambda_\beta^{J\upsilon}$ is defined with formula

$$\lambda_\beta^{J\upsilon} = |T|^2 p_\beta \left| \int d^3r\, \psi_\beta^*(\vec r)\, e^{-i\vec q_\beta \vec r}\, \Psi^{J\upsilon}(\vec r,0)\right|^2 \qquad (26)$$

(in what follows J=0, and indices $J\upsilon$ are omitted). Here p_β is the momentum of $(\mu d)_\beta$ system in c.m.s.:

$$p_\beta = \left[\frac{2(Q-\varepsilon_\beta)\, m_n(m_\mu+m_d)}{m_n+m_\mu+m_d}\right]^{1/2} \qquad (27)$$

Q=17.6 MeV is the energy release in reaction $dt\mu \to d+n+\mu^-$, ε_β is the energy of $(\mu d)_\beta$,

$$q_\beta = \frac{m_\mu}{m_\mu+m_d}\, p_\beta . \qquad (28)$$

From (23), (25) and (26) follows standard expression (18) for the sticking probability $\omega_{n\ell}$, provided the dependence of q and p on final state β in formula (26) is neglected and closure relation

$$\sum_\beta \psi_\beta^*(\vec r\,')\, \psi_\beta(\vec r) = \delta(\vec r - \vec r\,')$$

applied.

In order to estimate the accuracy of this approximation put $p_\beta = p_0 + \delta p_\beta$ then

$$\omega_\beta(p_\beta) = \omega_\beta(p_0)+\delta\omega_\beta,\quad \lambda_\beta = \lambda_\beta(p_0)+\delta\lambda_\beta,\quad \lambda = \lambda(p_0)+\delta\lambda$$
$$\delta\omega_\beta/\omega_\beta = \delta\lambda_\beta/\lambda_\beta - \delta\lambda/\lambda . \qquad (29)$$

The main contribution to the total rate λ comes from final states β, where muon is a spectator, i.e.,at $\beta = k$, $|k-q|\lesssim a_\mu^{-1}$. The average energy carried away by muon is about $\varepsilon_c \approx 10$ keV, the total

rate λ being proportional to the phase space volume of relative nd motion

$$\lambda = |T|^2 p \int |\Psi^{J\upsilon}(\vec r,0)|^2 d^3r . \qquad (30)$$

Here momentum p is defined up to terms $\sim \varepsilon_c/Q$ with formula

$$p = \left(\frac{2Q\, m_n m_d}{m_n+m_d}\right)^{1/2} . \qquad (31)$$

In contrast to λ, partial rates λ_β quickly vary with momentum p_β. For instance, the partial rate for $\beta=1s$ is

$$\lambda_{1s} \sim \frac{p_{1s}}{\left[1+(q_{1s}/2)^2\right]^4} \underset{p_{1s}\gg 1}{\sim} p_{1s}^{-7} . \qquad (32)$$

Let $p_0 = p_{1s}$, then relative errors of partial fusion rates are minimal and can be estimated with formula

$$\left|\frac{\delta\lambda_\beta}{\lambda_\beta}\right| \simeq \lambda_\beta^{-1}\left|\frac{d\lambda_\beta}{dp_\beta}\right|\delta p_\beta \lesssim 7\,\frac{\delta p_\beta}{p_0} \simeq 7\,\frac{\varepsilon_c}{2Q} \lesssim 2\cdot 10^{-3} . \qquad (33)$$

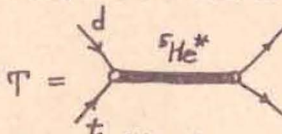
The relative error in total rate, according to (30), is

$$\frac{\delta\lambda}{\lambda} \simeq \frac{|p-p_0|}{p} \approx \frac{m_\mu m_n}{2m_d(m_d+m_\mu)} \simeq 3\cdot 10^{-3} . \qquad (34)$$

Thus, the uncertainties in ω_s^0 caused by neglected dependence of mesic atom $(\mu d)_{n\ell}$ momentum on the state $n\ell$ in which it is produced, do not exceed 0,5%.

Return now to the effects of finite rate of nuclear fusion. In order to take them into account the energy dependence of the nuclear reaction amplitude has to be considered. By using a simple resonance model to describe the nuclear interaction in the coupled dt and nd channels, the T-matrix is given by

$T =$



$$= \frac{V_1|{}^5He\rangle\langle{}^5He|V_2}{E-E_R-i\frac{\Gamma}{2}} .$$

Fig. 2

The description of the nuclear reaction $(dt\mu)^{J\upsilon}\to(\mu d)_{n\ell}+n$ results in a three coupled channels problem $\mu+d+t\to\mu+d+n$ and $\mu+{}^5He^*$ x). The amplitude $M_{n\ell}^{J\upsilon}$ for this reaction can be represented by a graph shown in Fig. 3.

x) We are using here a simplified version of a model developed in [20, 23] which fits all the experimental data on dt reaction in the resonance ^{5}He $(3/2^+)$ region.

The diagram involves the propagator of the muon G_c in the Coulomb field of a point-like nucleus with charge Z=2 and mass m_d+m_t, which can be conveniently written by using the spectral representation

$$\langle \vec{r}'|G_c|\vec{r}\rangle = \sum_i \frac{\varphi_i^*(\vec{r}')\,\varphi_i(\vec{r})}{E-\varepsilon_i} . \tag{35}$$

By neglecting effects connected with finite fusion radius, the $dt\mu$ mesic molecule wave function in $R\to 0$ limit is decomposed over the basis of the Coulomb functions involved in spectral representation (34) of

$$\Psi^{J\upsilon}(\vec{r},\vec{R}) \underset{R\to 0}{\simeq} \chi^{J\upsilon}(\vec{R})\left\{\sum_N b_N^{J\upsilon}\varphi_{N0}(\vec{r}) + \int dk\, b_k^{J\upsilon}\varphi_{k0}(\vec{r})\right\}, \tag{36}$$

i.e., over functions $\varphi_{N0}(\vec{r})$ and $\varphi_{k0}(\vec{r})$ of discrete and continuous spectra of μ^5He mesic atom. Note, that coefficients $b_N^{J\upsilon}$ and $b_k^{J\upsilon}$ in decomposition (36) slightly differ from coefficients $a_N^{J\upsilon}$ and $a_k^{J\upsilon}$ in (16). From (24)-(26) we obtain (an integral over k is included in a sum over N):

$$M_\beta^{J\upsilon} = \chi^{J\upsilon}(0)\,T(E_1)\sum_N \tilde{b}_N^{J\upsilon}\int d^3r\, \varphi^*_{n\ell}(\vec{r})\, e^{-i\vec{q}_\beta\vec{r}}\,\varphi_{N0}(\vec{r}) \tag{37}$$

where

$$T(E) = \langle \alpha, n; k_2 | T | d,t; k_1\rangle , \qquad E_N = E-\varepsilon_N ,$$

$$\tilde{b}_N^{J\upsilon} = b_N^{J\upsilon}\cdot\frac{T(E_N)}{T(E_1)} , \qquad E = \frac{k_1^2}{2M_1} = \frac{k_2^2}{2M_2} - Q , \tag{38}$$

N=1 corresponds to 1s state.

Should the energy dependence of the T-matrix be neglected, expression (27) reduces to the result of the sudden approximation (5), (16), the only difference, however, being that functions $\varphi_{n\ell}(\vec{r})$ of μ^5He mesic atom are involved instead of μ^3He ones $\phi_{n\ell}(\vec{r})$. It is the first term of decomposition (27) that corresponds to the Born-Oppenheimer approximation.

Note that the energy dependence of the T-matrix influences only the non-adiabatic corrections to the muon wave function. In other words, corrections to the value ω_s^0 calculated in a sudden approximation, are rather small, because they change only the contribution from non-adiabatic corrections to the BO approximation.

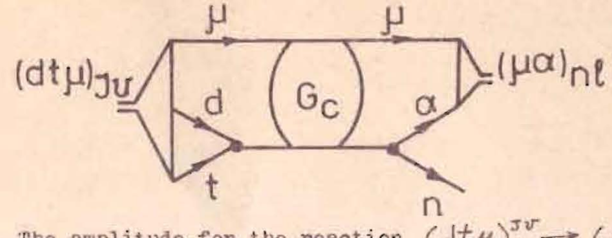


Fig. 3. The amplitude for the reaction $(dt\mu)^{J\upsilon} \to (\mu\alpha)_{n\ell} + n$ in the three coupled channels problem.

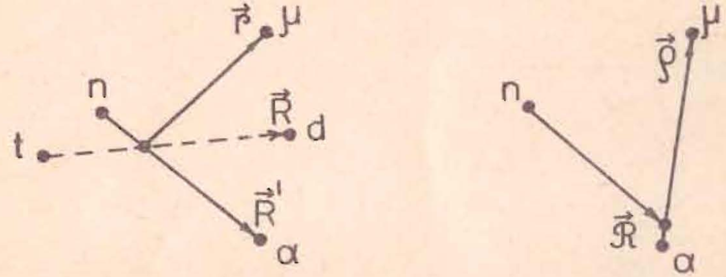


Fig. 4. The relative coordinates of the particles involved in the reaction (1).

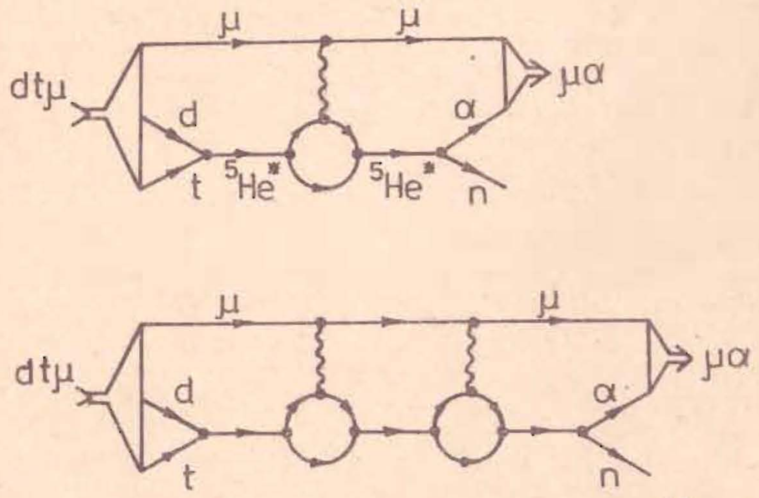


Fig. 5. The examples of the diagrams for the reaction $dt\mu \to \mu\alpha + n$ with the internuclear interactions sandviches by the Coulomb interaction of muon with nuclei.

From (38) it follows that

$$\tilde{b}_N = b_N \left\{ 1 + \frac{(\varepsilon_1 - \varepsilon_N)(E_R - i\Gamma/2)}{E_R^2 + \Gamma^2/4} \right\}. \tag{39}$$

All the non-adiabatic corrections $\delta\omega_s^\circ$ are linear in the real parts of coefficients $\tilde{b}_N$ (see Sect 3), hence, corrections $\Delta\omega_s^\circ$ due to the energy dependence of T-matrix are

$$\Delta\omega_s^\circ \simeq \delta\omega_s^\circ \frac{\varepsilon_c E_R}{E_R^2 + \Gamma^2/4} \approx 0.1\, \delta\omega_s^\circ. \tag{40}$$

Since $\delta\omega_s^\circ/\omega_s^\circ$ =0.23 corrections $\Delta\omega_s^\circ$ to the sudden approximation value do not exceed 3%, i.e., are essentially smaller than one could expect basing on the naive arguments of Sect. 2.

5. Other Corrections

In this section we consider corrections to $\omega_{n\ell}^{J v}$ arising from the finite nuclear size and finite range of nuclear forces, as well as some other corrections, in particular, these, due to nuclear final and initial state interaction influence on the muon wave function.

First of all, take into account the fact that α-particle is produced in fusion reaction not exactly in c.m.s. of nuclei (as it has been assumed in formula (6)), but is somehow spread. To do this, calculate amplitude $M_{n\ell}^{Jv}$ of nuclear reaction (24), which corresponds to the diagram in Fig. 3. Vertex form factors for $^5He^* \to d+t$ and $^5He^* \to n+\alpha$ in the coordinate representation

$$\begin{aligned} \xi_1(\vec{R}) &= \langle d t \,|\, V_1 \,|\, ^5He^* \rangle \\ \xi_2(\vec{R}) &= \langle \alpha n \,|\, V_2 \,|\, ^5He^* \rangle \end{aligned} \tag{41}$$

are localized inside $R \lesssim R_N$, $R_N \approx 4$ fm being the characteristic size of $^5He^*$ resonance. For the sake of simplicity write amplitude $M_{n\ell}^{Jv}$ for the mesic molecule wave function in the Born-Oppenheimer approximation: $\Psi^{BO}(\vec{z}, \vec{R}) \underset{R \to 0}{\simeq} \chi(\vec{R})\varphi_{1s}(\vec{z})$ (it will be clear from the following that using of the exact wave function does not influence the results):

$$\begin{aligned} M_{n\ell}^{Jv} = {} & \left(E - E_R - i\frac{\Gamma}{2}\right)^{-1} \int d^3R\, d^3R'\, d^3z\, \chi^{Jv}(\vec{R})\, \xi_1(\vec{R}) \times \\ & \times \psi_{n\ell}^*(\vec{\rho})\, e^{-i\vec{Q}\vec{\mathcal{R}}}\, \xi_2(\vec{R}')\, \varphi_{10}(\vec{z}). \end{aligned} \tag{42}$$

Here $\vec{R}$ and $\vec{R}'$ are relative coordinates of nuclei in initial and final states, ρ is muon coordinate with respect to α-particle, $\vec{\mathcal{R}}$ is $\mu\alpha$ atom coordinate with respect to neutron, $\vec{Q} = (m_\mu + m_\alpha)\vec{v}$ is $\mu\alpha$ atom momentum in c.m.s. Using relations (see Fig. 4)

$$\begin{aligned} \vec{\rho} &= m_n \vec{R}'/(m_n + m_\alpha) + \vec{z} \\ \vec{\mathcal{R}} &= \vec{\rho}\, m_\mu/(m_\mu + m_\alpha) - \vec{R}' \end{aligned} \tag{43}$$

transform (42)

$$\begin{aligned} M_{n\ell} &= B \int d^3R'\, \xi_2(\vec{R}')\, e^{i\vec{Q}\vec{R}'} \int d^3z\, \psi_{n\ell}^*(\vec{z})\, e^{-i\vec{q}\vec{z}}\, \varphi_{10}(\vec{z} - \vec{x}) \\ \vec{x} &= \frac{m_n}{m_n + m_\alpha}\vec{R}', \quad B = (E - E_R - i\Gamma/2)^{-1} \int d^3R\, \chi(\vec{R})\, \xi_1(\vec{R}). \end{aligned} \tag{44}$$

Expanding $\varphi_{10}(\vec{z} - \vec{x})$ in small parameter $\vec{x}$, obtain

$$M_{n\ell} = B \cdot g(Q) \left(F_{n\ell} + F_{n\ell}^{(1)} + F_{n\ell}^{(2)} + \cdots \right), \tag{45}$$

where $g(Q) = \int d^3R'\, \xi_2(\vec{R}')\, e^{i\vec{Q}\vec{R}'}$. The leading term $F_{n\ell}$ in decomposition (45) coincides with the sticking amplitude defined with formula (5), and corrections are

$$\begin{aligned} F_{n\ell}^{(1)}/F_{n\ell} &\simeq i(R_N/5a_\mu) \\ F_{n\ell}^{(2)}/F_{n\ell} &\simeq (R_N/5a_\mu)^2 \end{aligned} \tag{46}$$

hence,

$$\begin{aligned} \omega_{n\ell} &= |F_{n\ell} + F_{n\ell}^{(1)} + F_{n\ell}^{(2)}|^2 \approx |F_{n\ell} + F_{n\ell}^{(2)}|^2 + |F_{n\ell}^{(1)}|^2 = \\ &= |F_{n\ell}|^2 \left(1 + O\{(R_N/5a_\mu)^2\}\right). \end{aligned} \tag{47}$$

Thus, the corrections to $\omega_{n\ell}$ due to the final range of nuclear fusion are rather small[x)]

$$\delta\omega_{n\ell}/\omega_{n\ell} \simeq \left(\frac{R_N}{5a_\mu}\right)^2 \lesssim 3\cdot 10^{-5}$$

---

x) Using of the exact wave function of $dt\mu$ mesic molecule instead of $\Psi^{BO}(\vec{z}, \vec{R})$ influences only non-adiabatic corrections from the far away part of the continuum $k \gtrsim R_N^{-1} \approx 60$. This, however, is nonessential, because, according to Sect. 3, the main contribution to these corrections comes from the smaller momenta $k \lesssim q \approx 6$, $qR_N \simeq 10^{-1} \ll 1$.

Consider now corrections caused by strong interaction influence between nuclei in initial and final states on the muon wave function. This means that amplitude of nuclear reaction (24) should be calculated not in the first order in the amplitude of nuclear interaction (as it has been done when obtaining formula (26)), but exactly. The diagrams should be taken into account where the internuclear interaction is sandwiched by the Coulomb interaction of muon with nuclei (Examples of such graphs are shown in Fig. 5). As it was shown in[20, 23], such diagrams taken into account when calculating the total rate $\lambda^{J v}$ of nuclear reaction lead to about 5% renormalization of nuclear amplitude T in formula (26)[x)]. The corrections to the sticking probability $\omega_{n\ell}$ are essentially smaller, since, according to (23), $\omega_{n\ell}$ do not involve the nuclear reaction amplitude. They can be estimated as follows. First, neglect the structure of ^5He resonance (i.e., consider it interacting with muon as a point-like charge), then sum of all the diagrams of the type in Fig. 5 will coincide with diagram in Fig. 3, corresponding amplitude having been calculated in Sect. 4.

To take into account the structure of $^5He^*$, one should include the $^5He^*$ charge form factor dependence on the transferred momentum q. The characteristic value of $\vec{q} \simeq m_\mu \vec{v}$ is small compared to the inverse radius of $^5He^*$: $qR_N \approx 10^{-1}$. Corrections to the sticking probability due to the influence of ^5He resonance structure on muon wave function are defined by monopole and quadrupole form factors and are about $(qR_N)^2 \simeq 10^{-2}$, i.e., $\delta\omega_s^\circ/\omega_s^\circ \lesssim 10^{-2}$.

Of the same order of smallness should be the corrections due to finite sizes of nuclei d,t and α. At last, note the correction to $\omega_{n\ell}$ due to vacuum polarization

$$\frac{\delta\omega_{n\ell}}{\omega_{n\ell}} \approx 5\cdot 10^{-3} \qquad (48)$$

---

x) As it is clear from the previous discussion and the form of the diagrams 5, the influence of the nuclear dt interaction on the muon wave function $\Psi_{in}(\vec{r})$ is negligible. In this connection the statements of papers[27,28] about the importance of such influence seem strange.

6. Concluding Remarks

In the present paper for the probability of muon sticking to helium in reaction (1) the value is obtained $\omega_s^\circ = 0.848\cdot 10^{-2}$ in the framework of the sudden approximation. The uncertainty of this calculation due to uncertainties in the used numerical wave function of dtμ mesic molecule $\Psi^{Jv}(\vec{r},\vec{R})$ is noticeably smaller than the corrections to the sudden approximation, which are, according to our estimations, about 3%.

Our result is in good agreement with this of paper[10], where the sticking probability for the state $(Jv)=(00)$ of dtμ mesic molecule has been calculated for the mesic molecule wave function obtained with the Monte-Karlo method. As an argument to the correctness of the calculation can also be a coincidence between the calculated with the same technique sticking coefficient $\omega_d = 0.12$[13] for the reaction $dd\mu \to \mu^3He + n$ with the experimentally measured one $\omega_d = 0.126 \pm 0.004$[11].

The observed value $\omega_s = \omega_s^\circ(1-\gamma)$ is smaller than the calculated ω_s° by $\Delta\omega_s^\circ = \gamma\omega_s^\circ$ due to possible stripping processes during $\mu\alpha$ mesic atom slowing down in the medium: ionization from 1s-state of $\mu\alpha$ [7,8], step-by-step ionization[8,25,26], muons shaking off the excited states[24], etc. According to paper[24] the resulting sticking coefficient is

$$\omega_s = \omega_s^\circ \left\{ 0.96 - 0.03\,\frac{\varphi}{\varphi+2} \right\} \exp\left\{ -0.26 - 0.07\varphi \right\}. \qquad (49)$$

At $\varphi \approx 1$

$$\omega_s = 0.58\cdot 10^{-2}.$$

This value is almost twice less than the first estimates[5,6] within the Born-Oppenheimer approximation, but still approximately twice exceeds the experimental values reported in recent experiments of Idaho-Los-Alamos group[12] at $\varphi \approx 1$. The reason for such a drastic discrepancy between theory and experiment is still to be understood.

Emphasize, however, that up-to-date uncertainties in ω_s are due to unsufficiently well known reactivation coefficient γ, while initial sticking probability ω_s° is now established quite reliably.

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References

1. L.I.Ponomarev, Proc. of the X European Conf. on Controlled Fusion and Plasma Physics, Moscow, September 14-19, 1981.
2. J.Bracci and G.Fiorentini, Phys. Rep. 1982, 86, p. 169.
3. Proc. of the Third International Conf. on Emerging Nuclear Energy Systems, 6-9 June, 1983, Helsinki, Finland. Atomkernenergie-Kerntechnik, 1983, vol. 43, pp. 175-221.
4. Contribution to the Muon-Catalyzed Fusion Workshop, 7-8 June, 1984, Jackson Hole, Wyoming, USA, S.E.Jones organizer, Idaho National Engineering Laboratory EGG-PPD-6655.
5. Ya.B.Zeldovich, Zh. Eksp. Teor. Fiz., 1957, 33, p. 310. English transl. Sov. Phys.- JETP, 1958, 6, p. 242.
6. J.D.Jackson, Phys. Rev., 1957, 106, p. 330.
7. S.S.Gerstein, Yu.V.Petrov, L.I.Ponomarev, N.P.Popov, L.P.Presnjakov and L.N.Somov, Zh. Eksp. Teor. Fiz., 1981, 80, p. 1990; English transl. Sov. Phys.- JETP, 1981, 53, p. 872.
8. L.Bracci and G.Fiorentini, Nucl. Phys., 1981, A364, p. 383.
9. S.K.Kaufmann, B.Müller and J.Rafelski, Preprint UCT-TP 16/1984, Cape Town, 1984.
10. D.Ceperley and B.J.Alder, Phys.Rev., 1985, A31, 1999.
11. D.V.Balin, A.A.Vorobyov et al. Phys. Lett. 141B, 174, 1984; Preprint LINP-964, Leningrad 1984.
12. S.Jones, Proc. of the IX Int. Conf. on Atomic Physics, Seattle 1984.
13. L.N.Bogdanova, V.E.Markushin, V.S.Melezhik, L.I.Menshikov and L.I.Ponomarev, Preprint ITEP-37, Moscow 1985. (Submetted to Phys. Lett. B).
14. S.I.Vinitsky, V.S.Melezhik, L.I.Ponomarev, I.V.Pusynin, T.P.Puzynina, L.N.Somov and N.F.Truskova, Zh. Eksp. Teor. Fiz., 1980, 79, p. 698; English transl. Sov. Phys.- JETP, 1981, 52, p. 353.
15. A.D.Gocheva, V.V.Gusev, V.S.Melezhik, I.V.Puzynin, T.P.Puzynina, L.I.Ponomarev, L.N.Somov and S.I.Vinitsky, Phys. Lett., 1985, 153B, p. 349.
16. V.S.Melezhik, I.V.Puzynin, T.P.Puzynina and L.N.Somov. J. Comp. Phys., 1984, 54, p. 221.
17. V.S.Melezhik and L.N.Somov, Preprint JINR P11-81-856, Dubna 1981.
18. S.I.Vinitsky and L.I.Ponomarev, Physics of Elementary Particles and Atomic Nuclei, 1982, 13, p. 1336.

19. S.I.Vinitsky, V.S.Melezhik and L.I.Ponomarev, Yad. Fiz., 36, 465, 1982; J. Sov. Nucl. Phys., 1982, 36, p. 272.
20. L.N.Bogdanova, V.E.Markushin, V.S.Melezhik and L.I.Ponomarev, Yad. Phys., 1981, 34, p. 1191. Sov. J. Nucl. Phys., 1981, 34, p. 662.
21. L.N.Bogdanova, V.E.Markushin, V.S.Melezhik and L.I.Ponomarev. Zh. Eksp. Teor. Fiz., 1982, 83, p. 1615; Sov. Phys.- JETP, 1982, 56, p. 931.
22. S.I.Vinitsky, L.I.Ponomarev, I.V.Pusynin, T.P.Puzynina, L.N.Somov, and M.P.Faifman, Zh. Eksp. Teor. Phys. 4, 1978, 849; Sov. Phys.- JETP, 1979, 47, p. 4444.
23. L.N.Bogdanova, V.E.Markushin and V.S.Melezhik. Zh. Eksp. Teor. Fiz., 1981, 81, p. 829; Sov. Phys. - JETP, 1981, 54, p. 174.
24. L.I.Menshikov. Preprint IAE-4263/12, Moscow, 1985.
25. U.Takahashi, The contribution to the Muon-Catalyzed Fusion Workshop, see Ref. 3.
26. L.I.Menshikov and L.I.Ponomarev, Preprint IAE-4237/12, Moscow, 1985.
27. V.B.Belyaev, S.E.Brener, E.M.Gandyl, A.L.Zubarev and B.F.Irgaziev. J. of Phys., 1982, G8, p. 903.
28. J.Rafelski, S.K.Kaufmann and B.Müller, Preprint UST-TP 28/1985, Cape Town, 1985.

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СООБЩЕНИЯ, КРАТКИЕ СООБЩЕНИЯ, ПРЕПРИНТЫ И СБОРНИКИ ТРУДОВ КОНФЕРЕНЦИЙ, ИЗДАВАЕМЫЕ ОБЪЕДИНЕННЫМ ИНСТИТУТОМ ЯДЕРНЫХ ИССЛЕДОВАНИЙ, ЯВЛЯЮТСЯ ОФИЦИАЛЬНЫМИ ПУБЛИКАЦИЯМИ.

Ссылки на СООБЩЕНИЯ и ПРЕПРИНТЫ ОИЯИ должны содержать следующие элементы:

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Колпаков И.Ф. В кн. XI Международный симпозиум по ядерной электронике, ОИЯИ, Д13-84-53, Дубна, 1984, с.26.

Савин И.А., Смирнов Г.И. В сб."Краткие сообщения ОИЯИ", № 2-84, Дубна,1984,с.3.

Богданова Л.Н. и др. Е4-85-425

Вероятность прилипания мюона к гелию в реакции мюонного катализа $dt\mu \to \mu^4He + n$

Вычислен коэффициент прилипания мюона к гелию в реакции $dt\mu \to \mu\alpha + n$, $\omega_s^o = 0{,}848\cdot 10^{-2}$. Обсуждаются различные источники возможных погрешностей и показано, что в рамках принятых приближений погрешность вычисленного значения ω_s^o не превышает 3%. При вычислении использованы точные волновые функции мезомолекулы $dt\mu$, найденные в адиабатическом представлении задачи трех тел. С учетом последующего стряхивания мюонов при торможении $\mu\alpha$ результирующая вероятность прилипания ω_s равна $0{,}58\cdot 10^{-2}$.

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Препринт Объединенного института ядерных исследований. Дубна 1985

Bogdanova L.N. et al. E4-85-425

The Probability of Muon Sticking to Helium in the Muon Catalyzed Fusion $dt\mu \to \mu^4He + n$

In the sudden perturbation approximation the probability of sticking of the muon to helium ω_s^o is found to equal $0.848\cdot 10^{-2}$ in the reaction $dt\mu \to \mu\alpha + n$. In calculations we have used the accurate wave functions of the mesic molecule $dt\mu$ obtained in the adiabatic representation of the three-body problem. Corrections to the sudden approximation do not exceed 3%. In view of a subsequent shaking-off muons during deceleration of $\mu\alpha$ the resulting sticking probability ω_s equals $0.58\cdot 10^{-2}$.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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