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COLLECTIVE EFFECTS IN THE RESONANCE FLUORESCENCE SPECTRUM

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I. INTRODUCTION

The problem of collective resonance fluorescence has been extensively discussed in the literature [1-10]. Using a classical approach, Senitzky [8] has shown that the consideration of cooperative atomic behaviour of resonance fluorescence leads to the prediction of an additional sideband without the well-known triplet [11]. The simple classical model of Senitzky does not use any mechanism for line broadening, so the spectrum is infinitely sharp. In the recent works, the form of the additional sideband is found for the two [3] or few atoms case [5].

In the present paper, using the methos of boson representation [13,14] we investigate the collective spectrum of resonance fluorescence for the N atom case. The analytic forms for the well-known triplet and additional sideband are given here.

II. THE DRESSED ATOMIC EQUATION OF MOTION

We consider the beam of N two-level atoms interacting with resonant incident and emitted fields. The atoms are assumed to have one velocity and the influence of the photon impulse on the atomic velocity is ignored. In the boson representation [13,14] when each atomic level is compared with a boson variable, the Hamiltonian of the system in dipole and rotating wave approximation is of the form:

$$H = \frac{1}{2}\omega(a_2^+ a_2 - a_1^+ a_1) + \omega_0 \beta^+ \beta + \sum_\lambda \omega_\lambda b_\lambda^+ b_\lambda + \sum_\lambda (g_\lambda a_2^+ a_1 b_\lambda + h.c.) + (g_0 a_2^+ a_1 \beta + h.c.), \quad (1)$$

where ω is the energy gap between two atomic levels (the system of $\hbar = 1$), ω_0 is the frequency of an incident field, ω_0 is assumed to be equal to ω, a_2^+, a_2 and a_1^+, a_1 are the creation and annihilation operators for the atoms on the upper and lower levels, respectively. They satisfy the commutation relation

$$[a_i, a_j^+] = \delta_{ij} \quad (i, j = 1,2). \quad (2)$$

b^+_λ and b_λ are the creation and annihilation operators for the emitted photons with the frequency ω_λ. The λ specifies the wave-vector and polarization indices, and β^+ and β are the creation and annihilation operators of the incident photons. The factors g_0 and g_λ are the usual coupling constant

$$g_0 = \sqrt{\frac{\omega}{2\epsilon_0 V}}\cdot d(\vec{e}\cdot\vec{u}), \qquad g_\lambda = \omega\sqrt{\frac{1}{2\epsilon_0\omega_\lambda V}}\cdot d(\vec{e}_\lambda\cdot\vec{u}_d),$$

where d is the matrix element of the atomic transition (d is assumed to be real), $\vec{u}_d$ is the unit vector of the atomic dipole, $\vec{e}$ and $\vec{e}_\lambda$ are the unit vectors of the polarization of incident and emitted fields.

Further, the incident field is assumed to be strong, so that one can write it in the form /15/

$$\beta \to \sqrt{m}\, e^{-i\phi}, \qquad \beta^+ \to \sqrt{m}\, e^{i\phi}, \qquad \beta^+\beta \to m - i\frac{\partial}{\partial\phi}. \tag{3}$$

where m and ϕ are the number of photons and phase of the incident field (m is assumed to be large).

In phase representation (3) the Hamiltonian (1) is of the form

$$H = \frac{1}{2}\omega\,(a_2^+a_2 - a_1^+a_1) + \omega_0\,(m - i\frac{\partial}{\partial\phi}) + \sum_\lambda \omega_\lambda b_\lambda^+ b_\lambda + \sum_\lambda g_\lambda (a_2^+ a_1 b_\lambda + b_\lambda^+ a_1^+ a_2) + \frac{\Omega}{2}(a_2^+ a_1 e^{-i\phi} + a_1^+ a_2 e^{i\phi}), \tag{4}$$

where $\Omega = 2\sqrt{m}\, g_0$ is the Rabi frequency.

Making the unitary transformation

$$U = e^{-i\frac{\phi}{2}(a_2^+a_2 - a_1^+a_1 + 2\sum_\lambda b_\lambda^+ b_\lambda)}$$

we have

$$H = \sum_\lambda (\omega_\lambda - \omega_0) b_\lambda^+ b_\lambda + \sum_\lambda g_\lambda (a_2^+ a_1 b_\lambda + \text{h.c.}) + \frac{\Omega}{2}(a_2^+ a_1 + a_1^+ a_2) + \omega_0\,(m - i\frac{\partial}{\partial\phi}). \tag{5}$$

Further we use the canonical transformation

$$a_2 = \frac{1}{\sqrt{2}} c_1 + \frac{1}{\sqrt{2}} c_2, \qquad a_1 = -\frac{1}{\sqrt{2}} c_1 + \frac{1}{\sqrt{2}} c_2. \tag{6}$$

The new operators

$$c_2 = \frac{1}{\sqrt{2}} a_1 + \frac{1}{\sqrt{2}} a_2, \qquad c_1 = -\frac{1}{\sqrt{2}} a_1 + \frac{1}{\sqrt{2}} a_2 \tag{7}$$

also are boson operators. They satisfy the commutation relations

$$[c_i, c_j^+] = \delta_{ij} \quad (i, j = 1, 2). \tag{8}$$

After making the canonical transformation (6) the Hamiltonian (5) is of the form

$$H = H_0 + H_E, \tag{9}$$

where

$$H_0 = \omega_0\,(m - i\frac{\partial}{\partial\phi}) + \frac{\Omega}{2}(c_2^+ c_2 - c_1^+ c_1),$$

$$H_E = \sum_\lambda (\omega_\lambda - \omega_0) b_\lambda^+ b_\lambda + \frac{1}{2}\sum_\lambda g_\lambda [(c_2^+ c_2 - c_1^+ c_1) b_\lambda + b_\lambda^+ (c_2^+ c_2 - c_1^+ c_1)] + \frac{1}{2}\sum_\lambda g_\lambda (b_\lambda^+ c_2^+ c_1 + c_1^+ c_2 b_\lambda) - \frac{1}{2}\sum_\lambda g_\lambda (b_\lambda^+ c_1^+ c_2 + c_2^+ c_1 b_\lambda),$$

where H_0 is the part of the Hamiltonian including only the interaction of atoms with the incident field.

The eigenstates of the Hamiltonian H_0 are the dressed atomic states /16/ and consequently c_1, c_1^+ and c_2, c_2^+ are the annihilation and creation operators for the dressed atoms on the level $\epsilon_1 = -\Omega/2$ and $\epsilon_2 = +\Omega/2$, respectively.

We shall denote the dressed atomic operators by subscrip R $R_{ij} = c_i^+ c_j$ ($i, j = 1, 2$). It is easy to see that $R_{ij} = [R_{ji}]^+$.

Using the commutation relation (8) one can find the equation of motions

$$\dot{R}_3(t) = -i\sum_\lambda g_\lambda (b_\lambda^+(t)\, R_{12}(t) - R_{21}(t)\, b_\lambda(t)) - i\sum_\lambda g_\lambda (b_\lambda^+(t)\, R_{21}(t) - R_{12}(t)\, b_\lambda(t)), \tag{10}$$

$$\dot{R}_{12}(t) = -i\Omega R_{12}(t) + \frac{i}{2}\sum_\lambda g_\lambda (b_\lambda^+(t)\, R_3(t) - R_3(t)\, b_\lambda(t)) + i\sum_\lambda g_\lambda (b_\lambda^+(t)\, R_{12}(t) + R_{12}(t)\, b_\lambda(t))$$

$$\dot{b}_\lambda(t) = -i\,(\omega_\lambda - \omega_0)\, b_\lambda(t) - \frac{i}{2} g_\lambda [R_3(t) + R_{21}(t) - R_{12}(t)]. \tag{11}$$

Making the harmonic approximation [17], which assumes the atomic operators to evolve as they would in the absence of any coupling to the field, we have the solution of equation (11)

$$b_\lambda(t) = \tilde{b}_\lambda(t) - i\,\mathfrak{U}_\lambda(t) \tag{12}$$

where

$$\tilde{b}_\lambda(t) = e^{-i(\omega_\lambda - \omega_0)t} \cdot b_\lambda ,$$

$$\mathfrak{U}_\lambda(t) = \frac{g_\lambda}{2} \cdot S_-(t) \cdot I(\omega_\lambda - \omega_0 \,, t),$$

$$S_-(t) = R_3(t) + R_{21}(t) - R_{12}(t), \quad I(x,t) = \lim_{\epsilon \to 0} \frac{e^{-i(x - i\epsilon)t} - 1}{-(x - i\epsilon)} .$$

The validity of this approximation is discussed in paper [17]. It will be used only when $b_\lambda(t)$ appears inside a summation over λ. Substituting (12) into equation (10), one can find

$$\dot{R}_3(t) = -\gamma R_3(t) + \gamma R_3(t)(R_{21}(t) + R_{12}(t)) + \gamma(R_{12}(t) - R_{21}(t)) + F_1^{(v)}(t), \tag{13}$$

$$\dot{R}_{12}(t) = -i\Omega R_{12}(t) - \frac{3}{2}\gamma R_{12}(t) - \frac{\gamma}{2} R_{21}(t) - \frac{\gamma}{2} R_3^2(t) + F_2^{(v)}(t), \tag{14}$$

where

$$\gamma = \pi \sum_\lambda g_\lambda^2 \delta(\omega_\lambda - \omega_0),$$

$$F_1^{(v)}(t) = i\sum_\lambda g_\lambda (\tilde{b}_\lambda^+(t)\, R_{12}(t) - R_{21}(t)\, \tilde{b}_\lambda(t)) - i\sum_\lambda g_\lambda (\tilde{b}_\lambda(t)\, R_{21}(t) - R_{12}(t)\, \tilde{b}_\lambda(t)),$$

$$F_2^{(v)}(t) = \frac{i}{2}\sum_\lambda g_\lambda (\tilde{b}_\lambda^+(t)\, R_3(t) - R_3(t)\, \tilde{b}_\lambda(t)) - i\sum_\lambda g_\lambda (\tilde{b}_\lambda(t)\, R_{12}(t) + R_{21}(t)\, \tilde{b}_\lambda(t)).$$

The $F_1^{(v)}(t)$ and $F_2^{(v)}(t)$ are the Langevin forces. Their expectation values over the vacuum state of the radiation field are zero.

One can show that the Langevin forces $F_1^{(v)}(t)$ and $F_2^{(v)}(t)$ do not influence the following calculation of a steady state spectrum and these components are ignored. In the limit of large Ω the solutions of equations (13)-(14) are found in the form

$$R_3^{(1)}(t) = e^{-\gamma t}\,[R_3(0) - \frac{\gamma}{i\Omega} R_3(0)(R_{21}(0) - R_{12}(0)) + \frac{\gamma}{i\Omega}(R_{12}(0) - R_{21}(0))] + \frac{\gamma}{i\Omega}(R_3(0) - 1) \cdot R_{21}(0) \cdot e^{i\Omega t - \frac{3}{2}\gamma t} - \frac{\gamma}{i\Omega}(R_3(0) + 1)\, R_{12}(0)\, e^{-i\Omega t - \frac{3}{2}\gamma t}, \tag{15}$$

Substituting (15) into equation (14) we have

$$R_{12}^{(2)}(t) = e^{-i\Omega t - \frac{3}{2}\gamma t}[R_{12}(0) + \frac{\gamma R_{21}(0)}{2i\Omega}] - \frac{\gamma R_{21}(0)}{2i\Omega} e^{i\Omega t - \frac{3}{2}\gamma t} - \frac{\gamma R_3^2(0)}{2i\Omega} e^{-2\gamma t} + \frac{\gamma}{2}\frac{R_3^2(0)}{i\Omega} - \frac{i\gamma^3}{6\Omega^3}[(R_3(0) - 1)\, R_{21}(0)]^2 \cdot e^{2i\Omega t - 3\gamma t} + \frac{i\gamma^3}{2\Omega^3}[(R_3(0) + 1)\, R_{12}(0)]^2 \cdot e^{-2i\Omega t - 3\gamma t}. \tag{16}$$

In the solutions (15)-(16) the components at frequencies 0, $\pm\Omega$ are correct up to the terms $O(\frac{N\gamma}{\Omega})$ and the components at frequencies $\pm 2\Omega$ are correct up to the terms $O((\frac{N\gamma}{\Omega})^3)$, where $\frac{N\gamma}{\Omega}$ is assumed to be a small parameter.

III. SPECTRUM OF SCATTERED LIGHT

The steady-state spectrum of resonance fluorescence is defined by [10,18]

$$P(\omega_\lambda - \omega_0) = \lim_{t \to \infty} \frac{d}{dt} < b_\lambda^+(t)\, b_\lambda(t) > .$$

Using equation (11) one can write spectrum $P(\omega_\lambda - \omega_0)$ in the form

$$P(\omega_\lambda - \omega_0) = \lim \frac{g_\lambda}{2} \mathrm{Re} \int_0^\infty < S_+(t+\tau) S_-(t) > \cdot$$

$$\cdot e^{i(\omega_\lambda - \omega_0)\tau} \cdot d\tau.$$

In the long time limit the correlator $< S_+(t+\tau) S_-(t) >$ is independent of t and the steady-state spectrum becomes [10]

$$P(\omega_\lambda - \omega_0) = \frac{g_\lambda^2}{2} \mathrm{Re} \int_0^\infty < S_+(\tau) S_-(0) >_S \cdot e^{i(\omega_\lambda - \omega_0)\tau} d\tau. \tag{17}$$

Here we denote by $<\ldots>_S$ the expectation value over the atomic steady-state.

In the Markovian and secular approximation [7] the master equation describing the collective decay of the atomic system in the presence of the incident field is given by

$$\frac{\partial \rho(t)}{\partial t} = -i\frac{\Omega}{2} [R_3(t), \rho(t)] - \frac{\gamma}{4} \{R_3(t) + R_{21}(t) R_{12}(t) + R_{12}(t) R_{21}(t)] \rho(t) - R_3(t) \rho(t) R_3(t) - R_{21}(t) \rho(t) R_{12}(t) - R_{12}(t) \rho(t) R_{21}(t) + \text{h.c.}\}, \tag{18}$$

where $\rho(t)$ is the atomic density matrix.

The stationary solution of equation (18) is given by

$$\rho = \frac{1}{N+1} \sum_{M=-N,-N+2,\ldots}^{N} |M><M|, \tag{19}$$

where $|M>$ is an eigenstate of the operators $\hat{N}$ and $a_2^+ a_2 - a_1^+ a_1$. Substituting (15) and (16) into (17) and taking the expectation value over the steady-state density operators (19), one can write the steady-state spectrum (17) in the form

$$P(\omega_\lambda - \omega_0) = \frac{1}{3} g^2 N \left(\frac{N}{2} + 1\right) \{ \frac{\gamma}{(\omega_\lambda - \omega_0)^2 + \gamma^2} + \frac{3}{4} \frac{\gamma}{(\omega_\lambda - \omega_0 - \Omega)^2 + \frac{9}{4}\gamma^2} + \frac{3}{4} \frac{\gamma}{(\omega_\lambda - \omega_0 + \Omega)^2 + \frac{9}{4}\gamma^2} + \frac{\gamma}{2\Omega} \frac{\omega_\lambda - \omega_0 + \Omega}{(\omega_\lambda - \omega_0 + \Omega)^2 + \frac{9}{4}\gamma^2} + \frac{\gamma}{2\Omega} \frac{\omega_\lambda - \omega_0 - \Omega}{(\omega_\lambda - \omega_0 - \Omega)^2 + \frac{9}{4}\gamma^2} +$$

$$\frac{\gamma^3}{30\Omega^3} (N+3)(N-1) [\frac{\omega_\lambda - \omega_0 + 2\Omega}{(\omega_\lambda - \omega_0 + 2\Omega)^2 + 9\gamma^2} + \frac{\omega_\lambda - \omega_0 - 2\Omega}{(\omega_\lambda - \omega_0 - 2\Omega)^2 + 9\gamma^2}]\}. \tag{20}$$

The spectrum (20) consists of the usual triplet from works [7,10]. The peak intensity of the triplet is proportional to N^2 and the triplet structure is analogous to the one-atom case. In addition to the usual triplet, the collective spectrum (20) also consists of sidebands at the harmonics of the Rabi frequency, which have been predicted by Senitzky [8] using the quasiclassical approach.

The peak intensity of the additional spectra is proportional to $\frac{N^4}{\Omega^3}$, and their spectrum structure is analogous to a "dispersion curve". In the one-atom case (N = 1) the additional spectra of the spectrum (20) vanish.

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REFERENCES

1. Agarwal G.S. et al. Phys.Rev., 1977, A15, p.1613.
2. Amin A.S., Cordes J.G. Phys.Rev., 1978, A18, p.1298.
3. Agarwal G.S. et al. Phys.Rev., 1980, A21, p.257.
4. Narducci L.M. et al. Phys.Rev., 1978, A18, p.1571.
5. Carmichael H.J. Phys.Rev.Lett., 1979, 43, p.1106.
6. Drummond P.D., Casmichael H.J. Opt.common., 1978, 27,p.160.
7. Agarwal G.S. et al. Phys.Rev.Lett., 1979, 42, p.1260.
8. Senitzky I.R. Phys.Rev.Lett., 1978, 40, p.1334.
9. Drummond P.D., Hassan S.S. Phys.Rev., 1980, A22, p.662.
10. Compagno G., Persico F. Phys.Rev., 1982, A25, p.3138.
11. Mollow B.R. Phys.Rev., 1969, 188, p.1969.
12. Чельцов В.Ф. Квантовая электроника, 1984, 11, с.1014.
13. Боголюбов Н.Н./мл./, Чан Куанг, Шумовский А.С. ОИЯИ, Р4-85-130, Дубна, 1985.
14. Kumar S., Mehta C.L. Phys.Rev., 1980, A21, p.1573.
15. Wo I. Biealynicki Birula, Zofia Bialynicki Birula Phys.Rev., 1976, A14, p.1101.
16. Cohen-Tannoudji C., Reynaud S. J.Phys., 1977, B10,p.345.
17. Witley R.M., Strond C.R.(Jr.). Phys.Rev.,1976,A14,p.1498.

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Боголюбов Н.Н./мл./, Шумовский А.С., Чан Куанг E4-85-360
Коллективные эффекты в спектре резонансной флуоресценции

В работе опубликованы результаты по исследованию формы коллективного спектра резонансной флуоресценции, в том числе и формы дополнительных спектров, предсказанных Зенитским. Метод бозонного представления атомов был использован для случая N двухуровненых атомов, взаимодействующих с резонансным внешним полем и с полем излучения. Получены аналитические формулы для обычного триплета и дополнительных пиков. Пиковая интенсивность дополнительных спектров пропорциональна N^4. В случае одного атома дополнительные спектры исчезают.

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Bogolubov N.N.(Jr.), Shumovsky A.S., Tran Quang E4-85-360
Collective Effects in the Resonance Fluorescence Spectrum

The results on the investigations of the form collective resonance fluorescence spectrum including the additional sideband predicted by Senitsky are presented. The method of boson representation for atoms is used for the case of N two-level atoms interacting with a resonance driving field and emitted field. The analytic formulas for the usual triplet and additional sideband are obtained. The peak intensity of the additional spectra is proportional to N^4. In the one-atom case the additional spectra vanish.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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