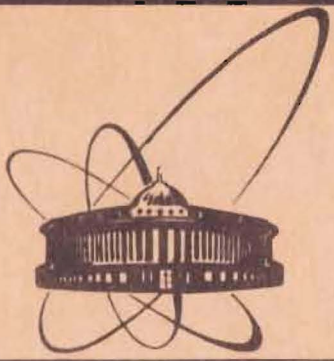

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Z. Árvay

DESCRIPTION OF FEW QUARK-LEPTON SYSTEMS IN A GAUSSIAN APPROXIMATION

1985

Nonrelativistic Hamiltonian approximation of the many-body problem equations has been used for the description of few quark-lepton systems. As a further approximation the orbital wave functions were constructed from Gaussians. A test of the model has been performed for quarks bounded in nuclear and separately for leptons bounded in molecular systems.

DESCRIPTION OF A SIX-QUARK CLUSTER

This time Quantum Chromodynamics (QCD) is the widely accepted candidate as a theory of strong interactions. In this theory coloured quarks interact with each other through a gluon-exchange potential that obeys the rules of infrared confinement and ultraviolet freedom.

Nuclei consist of nucleons and nucleons are supposed to consist of three valence quarks. In nuclear theory the NN interaction plays a central role and in QCD the NN (two nucleon) system is a six-quark cluster. By means of description of the six-quark cluster one can study the nuclear forces too.

Following Liberman[1] the quark-quark interaction as a sum of the color confinement and gluon-exchange terms were taken:

$$V_{ij} = \lambda_i \lambda_j \left[v(r_{ij}) + \frac{1}{6} \left(\frac{\hbar}{Mc}\right)^2 \sigma_i \sigma_j \nabla^2 v(r_{ij}) \right]. \qquad (1)$$

where $v(r_{ij}) = \frac{1}{2} K r_{ij}^2$.

In this formula λ_i and λ_j are the SU(3) color-matrices, M is the quark mass and K is the coupling constant. We are restricted to up and down quark flavours.

The antisymmetric wave function is a product of three parts

$$\Psi = \Psi^x \cdot \Psi^{SF} \cdot \Psi^c, \qquad (2)$$

where Ψ^c is the antisymmetric color, Ψ^{SF} is the symmetric spin-flavour and Ψ^x is the symmetric space wave function. The space part is constructed from one-particle Gaussians:

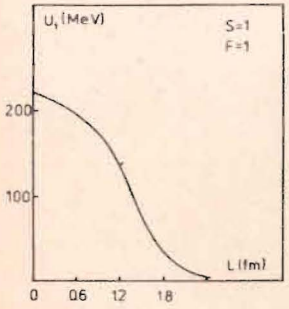
$$\Psi = \left(\frac{2\rho}{\pi}\right)^{3/4} \exp\left[-\rho r^2\right]. \qquad (3)$$

The energy of the few quark cluster is given by the expectation value of the nonrelativistic Hamiltonian with the quark-quark interaction potential given in (1). Varying the ρ nucleon radius parameter and fitting the nucleon and Λ particle masses one can determine the M and K parameters.

In the case of a six-quark cluster one can separate the external part of the whole NN interaction:

$$U_1 = \langle\Psi|\Psi\rangle^{-1} \langle\Psi| \sum_{1}^{3}\sum_{4}^{6} V_{ij} |\Psi\rangle. \qquad (4)$$

At the values S = 1, F = 1 (S - spin, F - flavour) and with the parameters Mc^2 = 151 MeV, K = 21.8 MeV fm^{-2} and $\langle r^2\rangle^{1/2}$ = = 0.9 fm we have computed the U_1 part of the NN force which is shown in due Figure. The main features of the nuclear forces - the short range repulsive core and the long range saturation - were reproduced.



The U_1 component of the NN potential as a function of the internucleon separation distance L.

GROUND STATE PROPERTIES OF SMALL MOLECULES

As an example for lepton systems we have investigated the 2n-electron small molecules. For the description of molecula electrons we applied the ab initio FSGO (Floating Spherical Gaussian Orbital) molecular orbital method. In this model an antiparallel spin electron pair is represented with a spherical Gaussian and the orbital radii and coordinates of the orbital centres are considered as variational parameters [2]. The antisymmetric wave function of the 2n-electron system is taken as a determinant. The total energy of the molecula is given in the next form:

$$E = 2\sum_{j,k} (j|k)\, T_{jk} + \sum_{k,\ell,p,q} (k\ell|pq)[\,2\,T_{k\ell}\,T_{pq} - T_{kq}\,T_{\ell p}\,] + \sum_{\mu<\nu} \frac{Z_\mu Z_\nu}{r_{\mu\nu}}, \qquad (5)$$

where

$$(j|k) = \int \phi_j^* \left(-\frac{1}{2}\nabla^2\right) \phi_k\, dv - \sum_{\nu}^{m} \int \phi_j^* \frac{Z_\nu}{r_\nu}\, \phi_k dv, \qquad (6)$$

Table

Calculated results of molecular parameters (all quantities in atomic units)

System	L		R		C		E	
	I	II	I	II	I	II	I	II
HeH^+	1.287	1.445[3]	1.125		0.211		-2.3713	-2.9774[4]
LiH	3.226	3.0145[5]	0.707		-0.0076		-6.5727	-8.0703[6]
			2.435		2.8829			
LiH_2^+	3.713	3.436[7]+	0.709	0.7095[7]+	0.000	0.1372[7]+	-6.920	-6.920[7]+
	1.505	1.507[7]+	1.779	1.7778[7]+	3.260	3.3541[7]+		

L - internuclear distance; R - orbital radius;
C - orbital centre; E - total energy;
I - this work; II - other work; + - calculation.

$$(k\ell|pq) = \int \phi_k^*(1)\,\phi_\ell(1)\,\phi_p^*(2)\,\phi_q(2)\,r_{12}^{-1}\,dv_1 dv_2, \qquad (7)$$

$$T_{k\ell} = S_{k\ell}^{-1},\ S_{k\ell} = \int \phi_k^*\,\phi_\ell\,dv,\ \phi_k = \left(\frac{2\rho}{\pi}\right)^{3/4} \exp\{-\gamma(\vec{r} - \vec{R})^2\}. \qquad (8)$$

For calculation of integrals we used the Gaussian integral transformed expression of the Coulomb potential:

$$\frac{1}{r} = \int_0^\infty s^{-1/2}\, e^{-sr^2}\, ds, \quad (r > 0). \qquad (9)$$

As an application of the model we have computed some ground state properties of the HeH^+, LiH and LiH_2^+ molecules and/or molecula ions. The results and comparisons with other theoretical and experimental data are summarized in the Table.

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Арвай З. E4-85-295

Описание системы нескольких тел кварков или лептонов в гауссовском приближении

Проведены расчеты энергии шестикварковой системы и изучены свойства основных состояний молекулярных систем малого размера. Для описания низкоэнергетических кварк-лептонных систем использовался единый нерелятивистский гамильтонов подход. Пространственная часть волновой функции конструировалась с помощью функции Гаусса. Хорошее согласие достигнуто с другими экспериментальными данными и теоретическими расчетами.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Árvay Z. E4- 85-295

Description of Few Quark-Lepton Systems in a Gaussian Approximation

Computations of the energy of a six-quark cluster and studies of the ground state properties of small molecules have been performed. A unique nonrelativistic Hamiltonian model was used for description of the low energy quark-lepton system. The space part of the wave function was constructed from one-particle Gaussians. Satisfactory agreement with experimental data and other theoretical results has been obtained.

The investigation has been performed at the the Laboratory of Nuclear Problems, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985