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CALCULATION OF THE MATRIX ELEMENTS OF THE HAMILTONIAN OF THE INTERACTING VECTOR BOSON MODEL USING COMPUTER ALGEBRA.

Matrix Elements of the Hamiltonian and Some U(6)-Clebsch-Gordon Coefficients

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1. INTRODUCTION

In our previous paper [9] hereafter referres to as I, we discussed an algorithm, realized on Computer Algebra Systems (CAS), for the investigation of algebraic nuclear model problems. In this study we are going to use this algorithm for the calculation of the matrix elements of the Hamiltonian of the Interacting Vector Boson Model (IVBM) (the basic aspects of IVBM are given in I) and their relation to the U(6)-Clebsch-Gordon coefficients.

2. MATRIX ELEMENTS OF THE IVBM HAMILTONIAN

The tensorial structure of the Hamiltonian (formulae (2.2.4) of I) has been investigated in [1,2]. It has been shown there that the first three terms in the r.h.s. of the expression of the Hamiltonian can be expressed as linear combinations of the first and second order invariant operators of the group U(6) and its subgroups according to the chain

$$\begin{array}{l} U(6) \supset SU(3) \times SU(2) \\ \qquad\qquad \cup \\ \qquad\qquad SO(3)\,, \end{array} \tag{2.1}$$

i.e., these terms are diagonal in the basis of Bargmann and Moshinsky (BM) (see formulae (2.3.6) of I). The off-diagonal terms of the Hamiltonian (the last two terms of (2.24) of I) can be characterized by a set of quantum numbers, which define their transformational properties along decomposition (2.1). As it is shown in Section 2.3 of I, the same quantum numbers characterize the state of the BM-basis. Hence, the action of the different terms of the Hamiltonian on the states of the BM-basis is given by the general rules of coupling of the labels, that define the different irreducible representation (IRs) along chain (2.1); i.e., by the generalized Clebsch-Gordon coefficients (CGCs) (or the corresponding isoscalar factors (IFs) of (2.1)). Here we use an approach identical to the one used for the calculation of the matrix elements of the SU(3)-quadrupole operator in I. This approach is based on the fact that by definition the off-diagonal terms of the Hamiltonian are the SO(3)-scalars, i.e., their action on the states (2.3.6) of I does not change the angular momentum L. On the other hand these terms have definite tensorial properties in the "pseudospin" space (the corresponding values of the "pseudospin" T and its third projection T are given in the Table).

Table

"Pseudospin" classification of the IVBM Hamiltonian

Type of interaction	(t, t_0)
$A^L(p,n)\,A^L(p,n)$	(2,2)
$A^L(n,p)\,A^L(n,p)$	(2,-2)
$A^L(p,n)\,A^L(p,p)$	(2,1) (1,1)
$A^L(n,n)\,A^L(p,n)$	(2,1) (1,1)
$A^L(n,p)\,A^L(n,n)$	(2,-1) (1,-1)
$A^L(p,p)\,A^L(n,p)$	(2,-1) (1,-1)

As has been pointed out in Section 2.3 of I (see (2.3.11) of I) the states of Bargmann and Moshinsky (BM) are also a complete set of highest-weight vectors for the IR (N,T) of the "pseudospin" group U(2). For this reason, along with the notation (2.3.6) of I we shall also use the following notation for the BM-states

$$\left|\begin{matrix}(\lambda,\mu)\\ \alpha,T,T\end{matrix}\right\rangle . \qquad (2.2)$$

Hence, the action of $H^t_{t_0}$ (any of the operators listed in the table) brings about states with $T'=T+t$, $T+t-1$, ..., $T-t$ compatible with the third projection rule $T'=T+t$.

Thus taking into account the "pseudospin" structure of $H^t_{t_0}$, through a direct application of $H^t_{t_0}$ on the states of BM we obtain

$$H^t_{t_0}\left|\begin{matrix}(\lambda,\mu)\\ \alpha,T,T\end{matrix}\right\rangle = \sum_{s,k} C_s^{(k)}\, T^{k-t_0}_{-1}\left|\begin{matrix}(\lambda+2k,\mu-k)\\ \alpha+s,T+k,T+k\end{matrix}\right\rangle \qquad (2.3)$$

$t \geq k \geq t_0\,;\quad \lambda+2k\geq 0;\quad \mu-k\geq 0;\quad k$ – integer

and the integer s varies in ranges compatible with $(\lambda+2k,\mu-k)$. The matrix elements $C_s^{(k)}$ (physical isoscalar factors (PIFs)) are closely related to the reduced matrix elements of $H^t_{t_0}$ and, as it will be shown in the following Section, to the usual isoscalar factors (IFs) of chain (2.1) as well. The application of the Wigner-Eckart theorem in regard to both SU(3) and SU(2) results in the following expression for the reduced matrix elements of $H^t_{t_0}$

$$\left\langle\begin{matrix}(\lambda+2k,\mu-k)\\ \alpha',T+k\end{matrix}\right\|H^t_{t_0}\left\|\begin{matrix}(\lambda,\mu)\\ \alpha,T\end{matrix}\right\rangle = \left(\frac{(k-t_0)!\,(2T+2k)!}{2^{k-t_0}(2T+k+t_0)!}\right)^{1/2}\times$$

$$\begin{pmatrix}T+k & t & T\\ -T-t_0 & t_0 & T\end{pmatrix}^{-1}\sum_s C_s^{(k)}\, f^{(\lambda+2k,\mu-k)}_{\alpha',\alpha+s}, \qquad (2.4)$$

where $\begin{pmatrix}T+k, & t, & T\\ -T-t_0 & t_0 & T\end{pmatrix}$ is a 3j-symbol /3/ and $f^{(\lambda+2k,\mu-k)}_{\alpha',\alpha+s}$ is an overlap integral (see formulae (2.3.12) of I).

The PIFs $C_s^{(k)}$ in (2.3) can be calculated directly by calculating seperately the left and right h.s. of eq.(2.3).

Thus in the case of the operator $A^L(p,n)\,A^L(p,n)$ characterized by $t=2$ and $t_0=2$ (see the Table) one has

$$A^L(p,n)\,A^L(p,n)\left|\begin{matrix}(\lambda,\mu)\\ \alpha,T,T\end{matrix}\right\rangle = \sum_s C_s^{(2)}(L)\left|\begin{matrix}(\lambda+4,\mu-2)\\ \alpha+s,T+2,T+2\end{matrix}\right\rangle \qquad (2.5)$$

The direct action of the operator $A^L(p,n)\,A^L(p,n)$ on the BM-states and the expressions for the PIFs $C_s^{(2)}(L)$ are given in /10/. It should be mentioned here, that the PIFs $C_s^{(2)}(0)$ are equal to zero because by definition the operator $A^0(p,n)$ is the raising component of the "pseudospin" operator (see (2.2.8a) of I) and the sums $C_s^{(2)}(1)+C_s^{(2)}(2)$ are equal to zero because of the relation /3/

$$\sum_{L,M}(-1)^M C^{LM}_{1\mu\,1\nu}\, C^{L\,-M}_{1\rho\,1\sigma} = (-1)^{\rho+\sigma}\,\delta_{\mu,-\sigma}\,\delta_{\nu,-\rho} \qquad (2.6)$$

which, in our case, leads to

$$A^1(p,n)\,A^1(p,n) + A^2(p,n)\,A^2(p,n) = 2A^0(p,n)\,A^0(p,n). \qquad (2.7)$$

The operator $A^L(n,p)\,A^L(n,p)$ is a more complicated one. First of all (see the Table) this operator is characterized by the quantum numbers $t = 2$ and $t_0 = -2$ and its action on the BM-states is expressed in the following way (in the notations (2.3.13a,b) of I)

$\lambda + \mu - L$ - even

$$A^L(n,p)\,A^L(n,p)\,|\ell_1,\ell_2,\tau,\alpha> =$$

$$\sum_s C_s^{(2)}(L)\,T_{-1}^4\,|\ell_1+2s+2,\ \ell_2-2s-2,\tau-s+1,\ \alpha+s>+$$

$$\sum_s C_s^{(1)}(L)\,T_{-1}^3\,z\,|\ell_1+2s+1,\ \ell_2-2s-2,\ \tau-s,\ \alpha+s>+$$

$$\sum_s C_s^{(0)}(L)\,T_{-1}^2\,|\ell_1+2s,\ \ell_2-2s,\tau-s,\ \alpha+s>+ \qquad (2.8a)$$

$$\sum_s C_s^{(-1)}(L)\,T_{-1}\,z\,|\ell_1+2s-1,\ \ell_2-2s,\ \tau-s-1,\ \alpha+s>+$$

$$\sum_s C_s^{(-2)}(L)\,|\ell_1+2s-2,\ \ell_2-2s+2,\ \tau-s-1,\alpha+s>;$$

$\lambda + \mu - L$ -odd

$$A^L(n,p)\,A^L(n,p)\,z\,|\ell_1,\ell_2,\tau,\alpha> =$$

$$\sum_s C_s^{(2)}(L)\,T_{-1}^4\,z\,|\ell_1+2s+2,\ \ell_2-2s-2,\tau-s+1,\alpha+s>+$$

$$\sum_s C_s^{(1)}(L)\,T_{-1}^3\,|\ell_1+2s+1,\ell_2-2s,\ \tau-s+1,\ \alpha+s>+$$

$$\sum_s C_s^{(0)}(L)\,T_{-1}^2\,z\,|\ell_1+2s,\ell_2-2s,\tau-s,\ \alpha+s>+ \qquad (2.8b)$$

$$\sum_s C_s^{(-1)}(L)\,T_{-1}\,|\ell_1+2s-1,\ \ell_2-2s+2,\ \tau-s,\alpha+s>+$$

$$\sum_s C_s^{(-2)}(L)\,z\,|\ell_1+2s-2,\ \ell_2-2s+2,\ \tau-s-1,\alpha+s>;$$

The operators $A^L(n,p)\,A^L(n,p)$ satisfy an equation similar to (2.7), which results in the following relation for the PIFs in (2.8)

$$C_s^{(k)}(1) + C_s^{(k)}(2) = 2C_s^{(k)}(0). \qquad (2.9)$$

By definition the operator $A^0(n,p)\,A^0(n,p)$ is proportional to T_{-1}^2 and in this case only the PIFs $C_0^{(0)}(0)$ are not equal to zero (see ref. /10/).

The operator $A^1(n,p)$ is of the following type (expressed by means of the p- and n-bosons (2.2.1a,b) of I)

$$A_M^1(n,p) = \sum_{\mu,\nu} C_{1\mu\,1\nu}^{1\ M}\, n_\mu p_\nu^* = \sum_{\mu,\nu} (-1)^\nu C_{1\mu\,1\nu}^{1\ M}\, n_\mu \frac{\partial}{\partial p_{-\nu}}. \qquad (2.10)$$

The direct action of the operator A (n,p) A (n,p) on the BM-states (both for $\lambda+\mu-L$ -even and odd) is given in /10/.

The operator T_{-1} is given by

$$T_{-1} = -(\tfrac{3}{2})^{1/2}\, A^0(n,p). \qquad (2.11)$$

With the help of (3.4) and (3.5) of I one obtains

$$T_{-1}|\ell_1,\ell_2,\tau,\alpha> = \frac{(\ell_1+2\tau)}{\sqrt{2}}\,n_1\,|\ell_1-1,\ell_2,\tau,\alpha> - \frac{2\tau}{\sqrt{2}}\,z\,|\ell_1-1,\ell_2,\tau-1,\alpha>;$$

$$T_{-1}\,z\,|\ell_1,\ell_2,\tau,\alpha> = -\frac{2\tau+1}{\sqrt{2}}\,|\ell_1-1,\ \ell_2+2,\tau,\ \alpha> \qquad (2.12)$$

$$+\frac{\ell_1+2\tau+1}{\sqrt{2}}\,n_1\,z\,|\ell_1-1,\ell_2,\tau,\alpha> + \frac{2\tau}{\sqrt{2}}\,|\ell_1+1,\ell_2,\tau-1,\alpha+1>;$$

The comparison between the results obtained by the direct action of $A^L(n,p)\,A^L(n,p)$ and the results obtained by the successive action of T_{-1} on the states of the BM-basis gives expressions for the PIFs in (2.8a,b). The results for $\lambda+\mu-L$ -even are given in ref. /10/, Sec.1, while the corresponding results for $\lambda+\mu-L$ -odd are given in ref. /10/, Sec.2,

The last four terms of the Hamiltonian (formulae (2.2.4) of I) can be divided into two types (see the Table):

i) the operators $A^L(p,n)\,A^L(p,p)$ and $A^L(n,n)\,A^L(p,n)$ which are a mixture of "pseudospin" tensor operators with $(t,t_0) = (2,1)$ and $(t,t_0) = (1,1)$. The action of these operators on the BM-states is given in the following way

$\lambda+\mu-L$ -even

$$\sum_s C_s^{(2)}(1)\, T_{-1}\,|\ell_1+2s+2,\ \ell_2-2s-2,\ r-s+1,\ \alpha+s\rangle\ +$$
$$+\sum C_s^{(1)}(1)\, z\,|\ell\ +2s+1,\ \ell\ -2s-2,\ r-s,\ \alpha+s\rangle\ ; \tag{2.13a}$$

$\lambda+\mu-L$ -odd

$$\sum_s C_s^{(2)}(L)\, T_{-1} z\,|\ell_1+2s+2,\ \ell_2-2s\ -2,\ r-s+1,\ \alpha+s\rangle +$$
$$+\sum_s C_s^{(1)}(L)\,|\ell_1+2s+1,\ \ell_2-2s,\ r-s+1,\ \alpha+s\rangle\ ; \tag{2.13b}$$

ii) the operators $A^L(n,p)\, A^L(n,n)$ and $A^L(p,p)\, A^L(n,p)$ which are a mixture of "pseudospin" tensor operators with $(t,t_0) = (2,-1)$ and $(t,t_0) = (1,-1)$. In this case the action of these operators on the states of the BM-basis is given by

$\lambda+\mu-L$ -even

$$\sum_s C_s^{(2)}(L)\, T_{-1}^3\,|\ell_1+2s+2,\ \ell_2-2s-2,\ r-s+1,\ \alpha+s\rangle\ +$$
$$\sum_s C_s^{(1)}(L)\, T_{-1}^2\, z\,|\ell_1+2s+1,\ \ell_2-2s-2,\ r-s,\ \alpha+s\rangle\ +$$
$$\sum_s C_s^{(0)}(L)\, T_{-1}\,|\ell_1+2s,\ \ell_2-2s,\ r-s,\ \alpha+s\rangle\ + \tag{2.14a}$$
$$\sum_s C_s^{(-1)}(L)\, z\,|\ell_1+2s-1,\ \ell_2-2s,\ r-s-1,\ \alpha+s\rangle\ ;$$

$\lambda+\mu-L$ -odd

$$\sum_s C_s^{(2)}(L)\, T_{-1}^3\, z\,|\ell_1+2s+2,\ \ell_2-2s-2,\ r-s+1,\ \alpha+s\rangle +$$
$$\sum_s C_s^{(1)}(L)\, T_{-1}^2\,|\ell_1+2s+1,\ \ell_2-2s,\ r-s+1,\ \alpha+s\rangle +$$
$$\sum_s C_s^{(0)}(L)\, T_{-1}\, z\,|\ell_1+2s,\ \ell_2-2s,\ r-s,\ \alpha+s\rangle \tag{2.14b}$$
$$\sum_s C_s^{(-1)}(L)\,|\ell_1+2s-1,\ \ell_2-2s+2,\ r-s,\ \alpha+s\rangle\ ;$$

The direct action of all these operators on the basis of BM is given in [10]. As in the previous cases, the comparison between the results obtained by the successive action of T_{-1} in (2.14a,b) and the results obtained by the direct action of the operators i) and ii) gives expressions for the PIFs in (2.14a,b). The corresponding results are given in [10].

3. RELATIONS BETWEEN THE PHYSICAL ISOSCALAR FACTORS AND THE USUAL ISOSCALAR FACTORS OF DECOMPOSITION $U(6) \supset SU(3)\times SU(2) \cup SO(3)$

In the previous Section we obtained the PIFs of the different terms of the Hamiltonian of IVBM. These PIFs are given in ref. [10], Sec.1 if $\lambda+\mu-L$ -even and in ref. [10], Sec.2 if $\lambda+\mu-L$ -odd. The PIFs are obtained through a direct action of the terms of the Hamiltonian on the states of the BM-basis. Hence, $C_s^{(k)}(L)$ contain all information of the tensor structure of the Hamiltonian. On the other hand, as already mensioned, these PIFs are closely related to the usual IFs of decomposition (2.1). It has been shown in [1,2] that the terms of the Hamiltonian of IVBM can be expressed as linear combinations of tensor operators along chain (2.1). These operators are of the following type:

$$T([\chi]_6\ ;\ (\lambda,\mu)(2T)\ ;\ L=0, T_0\ ), \tag{3.1}$$

where $[\chi]_6$ is the IR of $U(6)$, (λ,μ) and $(2T)$ of $SU(3)$ and $SU(2)$ respectively and T is the third projection of the "pseudospin" T.

Taking into account the $U(6)$-structure of (3.1) its action on the BM-states can be expressed in the following way:

$$\sum_{\lambda_2,\mu_2,\alpha_2,T_2} C^{[\chi]_6\ \ [N]_6\ \ [N]_6}_{(\lambda,\mu)(2T)\ (\lambda_2,\mu_1)(2T_1)\ (\lambda_2,\mu_2)(2T_2)}\ C^{(\lambda,\mu)\ \ (\lambda_1,\mu_1)\ \ (\lambda_2,\mu_2)}_{0\ \ \alpha_1 L_1\ \ \alpha_2 L_1}$$
$$C^{T\,T_1\,T_2}_{T_0\,T_1\,T_1+T_0} \left| \begin{matrix} [N]_6 & (\lambda_2\mu_2) \\ \alpha_2 L_1, & T_2, T_1+T_0 \end{matrix} \right\rangle = \tag{3.2}$$
$$\sum_{k,s} C_s^{(k;\,[\chi]_6\,(\lambda,\mu))}(L_1)\, T_{-1}^{k-T_0} \left| \begin{matrix} [N]_6 & (\lambda_1+2k,\ \ \mu-k) \\ \alpha_1+s,\ L_1, & T_1+k,\ \ T_1+k \end{matrix} \right\rangle .$$

where $C^{[\chi]\ [N]\ [N]}_{(\lambda,\mu)(2T)\ (\lambda_1,\mu_1)(2T_1)\ (\lambda_2,\mu_2)(2T_2)}$ is the IF of the chain $U6(6) \supset SU(3) X SU(2)$, $C^{(\lambda,\mu)\ (\lambda_1,\mu_1)\ (\lambda_2,\mu_2)}_{0\quad \alpha_1 L_1\quad \alpha_2 L_2}$ is the IF of the decomposition $SU(3) \supset SO(3)$ and the PIFs $C_s^{(k,[\chi][\lambda,\mu])}(L_1)$ in fact coincide with the PIFs in (2.3).

The problem of calculation of the IFs of the decomposition $SU(3) \supset SO(3)$ is discussed in a number of papers (see for example /4,5,6/). Hence, through a direct application of the generalized Wigner-Eckart theorem one obtains the following expression for the CGCs of decomposition (2.1).

$$C^{[\chi]\quad [N]\quad [N]}_{(\lambda,\mu)(2T)\quad (\lambda_1,\mu_1)(2T_1)\quad (\lambda_2,\mu_2)(2T_2)}\ C^{(\lambda,\mu)\quad (\lambda_1,\mu_1)\quad (\lambda_2,\mu_2)}_{0\quad \alpha_1,L_1\quad \alpha' L_1}\ C^{T\ T_1\ T_2}_{T_0\ T_1 T_1+T_0} =$$

$$= [\langle N || T^{[\chi]} || N \rangle]^{-1} \sum_{k,s} \delta_{T_2, T_1+k}\ \delta_{\lambda_2, \lambda_1+2k}\ \delta_{\mu_2, \mu_1-k} \times \tag{3.3}$$

$$\left(\frac{(k-T_0)!(2T_1+k+T_0)!}{2^{k-T_0}(2T_1+2k)!} \right)^{1/2} C^{(k;[\chi],(\lambda,\mu))}(L_1)\, f^{(\lambda+2k,\mu-k)}_{\alpha',\alpha_1+s} \ ,$$

where $\langle N || T^{[\chi]} || N \rangle$ is the reduced matrix element of (3.1) along chain (2.1). Thus, the last expression (3.3) gives an explicit formulae for the calculation of some of the U(6)-CGCs (or the corresponding IFs) and their relation to the PIFs of the Hamiltonian.

4. GENERAL CAS ALGORITHM AND ALGEBRAIC NUCLEAR PROBLEMS

The calculation of the matrix elements and the diagonalization of different algebraic model Hamiltonians, as described in the previous Sections, leads to a great number of similar, but very cumbersome operations. For this reason, in a number of cases, it is more convenient to use programs for analytical calculations on a computer (CAS). There are a lot of CAS, which are

characterized by different possibilities. The algorithm, presented in I and in this paper, is of an universal character and can be realized on different CAS. In our case the algorithm is realized on CAS-REDUCE-2. Its realization on CAS-REDUCE-3, SCHOONSCHIP and AMP is in process.

In this Section we are going to discuss the general scheme of calculation and the problems that appear in this approach.

1/ Construction of Differential Operators

The operators of the physical observables are expressed as polynomials in some type of boson creation and annihilation operators. In Fock's representation the annihilation operators are treated as differential operators. The latter is very convenient for the application of some types of CAS, for example REDUCE-2, REDUCE-3 and AMP. For other CAS, for example, SCHOONSCHIP, where the ordering of the operators is of importance, the problem is reduced to the calculation of multiple commutators. The comparison between both these methods in regard to the computational time is of great importance.

2/ Tensorial Structure

The operators of the physical observables are characterized by specific tensorial properties along a given chain of subgroups (in our case the chain of subgroups is (2.1); see also (3.1)).

3/ Polynomial Bases

In Section 2.3 of I we discussed in detail the polynomial basis of BM. Polynomial bases of this type are very convenient for calculations realized on CAS. Other polynomial bases will be discussed in forthcoming papers.

4/ Reduction to Irreducible Fundamental Blocks

The reduction is realized through the application of a number of substitutions (see formula (3.5) of I). This operation is a very cumbersome one and time consuming.

5/ Generation of Linear Systems of Equations for the PIFs

This procedure is carried out by a comparison between the results obtained by the direct action of the operators (see /10/ and the results obtained in the expansion formula of the type (2.13a,b) or (2.14a,b). This leads to the generation of solvable linear systems of equations for the PIFs, which is due to the linear independence of the irreducible fundamental blocks.

6/ Derivation of the PIFs

The systems of equations discussed above are always self-consistent and can be solved by the usual Gauss algorithm.

7/ Derivation of the Usual CGCs

The PIFs are closely related to the usual CGCs of chain (2.1). The derivation of the CGCs is discussed in Section 3. In general, the procedure given there is not yet very well understood and investigations in this field are in progress. However, we are convinced, that this procedure will give an algorithm for solving the very complicated problem of the calculation of CGCs of different chains of subgroups.

8/ Calculation of Overlap Integrals

This problem is discussed in detail in [7].

9/ Calculation of Matrix Elements

Following the procedure given in 6/ and 8/ through formula similar to (2.4), one can derive expressions for the matrix elements of the different physical observables.

10/ Eigenvalue Problem

Through a diagonalization of the Hamiltonian one obtains explicit expressions for the energies of the system as functions of the parameters of the Hamiltonian. It must be pointed out that the degree of the characteristic equations depends on the IR of the group U(6).

11/ Numerical Calculations

The parameters of the model Hamiltonian are fitted to the experimental data of the energy levels of the system. For example, numerical calculations in the rotational limit of IVBM are performed in [8].

The running time depends heavily on the type of the operator and it cannot be estimated in advance. The computing of the direct action of the terms of the IVBM Hamiltonian on the BM-basis and the reduction to fundamental irreducible blocks,which are characteristic for the algorithm given above, runs from 2 to 30 min on a computer ES-1060, operating system OS, programming language - REDUCE-2, high speed storage 1M bytes in JINR, Dubna. A part of the calculations were held in TSS-IBM in Sofia on a computer IBM 370/148, operating system OS/VS1, high speed storage 1M bytes, running time in comparison with ES-1060 - 2-3 times slower.

5. CONCLUSIONS

In paper [9] and present paper we discussed an approach,which can be widely used in algebraic nuclear models.The general technique of this approach is illustrated on the examples of the SU(3)-quadrupole operator and the off-diagonal terms of the IVBM Hamiltonian.

In our opinion, this algorithm, realized on CAS, is of great interest not only for theoretical nuclear physics (IVBM in particular) but also for group theory in general, because it can find its application in the calculation of different combinations of CGCs (or IFs) along certain chains of subgroups.

In the particular case of IVBM, the results of the present paper clarify the complicated algebraic structure of the Hamiltonian and show an explicit way for its diagonalization. The diagonalization of the IVBM Hamiltonian in the BM-basis will be discussed in future publications.

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Вычисление матричных элементов гамильтониана взаимодействующих векторных базонов с использованием компьютерной алгебры. Матричные элементы гамильтониана и некоторые U(6)-коэффициенты Клебша-Гордана

Дается алгоритм вычисления матричных элементов гамильтониана МВВБ и некоторых U(6) - коэффициентов Клебша-Гордана. Этот алгоритм реализован на языке Системы Аналитических Вычислений REDUCE-2.

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Calculation of the Matrix Elements of the Hamiltonian of the Interacting Vector Boson Model Using Computer Algebra. Matrix Elements of the Hamiltonian and Some U(6)-Clebsch-Gordon Coefficients

An algorithm for the calculation of the matrix elements of the IVBM Hamiltonian and some U(6) - Clebsch-Gordon coefficients is presented. The algorithm is realized on a Computer Algebra System REDUCE-2.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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