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V.P.Gerdt, N.A.Kostov, P.P.Raychev*,
R.P.Roussev*

CALCULATION OF THE MATRIX ELEMENTS OF THE HAMILTONIAN OF THE INTERACTING VECTOR BOSON MODEL USING COMPUTER ALGEBRA.

Basic Concepts of the Interacting Vector Boson Model and Matrix Elements of the SU(3)-Quadrupole Operator

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* Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia, Bulgaria

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1. INTRODUCTION

Advances in the recently developed algebraic nuclear models[1-6] have introduced different semi-simple Lie groups for the description of the collective nuclear properties. Although the general properties of these groups are well understood there always exist some computational problems:

1) the calculation of the direct action of the generators of the group and of the physical tensor operators (angular momentum, quadrupole operator, Hamiltonian, etc.) on a given polynomial basis;

2) an overlap integral computational problem;

3) matrix elements and Clebsch-Gordon coefficients (CGCs) of simple Lie groups via certain canonical or noncanonical chains of subgroups;

4) eigenvalue problem for different model Hamiltonians (Interacting Boson Model[3], Interacting Vector Boson Model (IVBM)[5,6] and other symplectic models).

All these problems are closely related to the diagonalization of the model Hamiltonian and its application to the investigation of certain types of nuclear properties.

In this paper we present an algorithm for the calculation of the matrix elements of the IVBM Hamiltonian in the polynomial basis of Bargmann and Moshinsky (BM)[7,8]. This algorithm is realized on a Computer Algebra System (CAS)[9,10].

In Section 2 of this paper we present the basic concepts of IVBM and the BM basis. This Section also deals with a brief argumentation of the advantages of the application of CAS to algebraic nuclear problems.

All problems that appear in the presented algorithm are illustrated in Section 3 on the comparatively simple example of the SU(3)-quadrupole operator.

2. BASIC CONCEPTS

2.1. Some Algebraic Nuclear Problems

The calculation of the matrix elements of the operators of the physical observables (angular momentum, quadrupole momentum, Hamiltonian, etc.) plays a main role in the algebraic nuclear models. These operators are constructed by means of boson creation b_m and annihilation b^*_m operators. The basic states of the system are expressed as

$$|[\Gamma]\rangle = P^{[\Gamma]}(b)|0\rangle, \qquad (2.1.1)$$

where $P^{[\Gamma]}(b)$ is a polynomial in the creation operators b_m, $|0\rangle$ is the vacuum state of the system ($b^*_m|0\rangle = 0$) and $[\Gamma]$ is a complete set of quantum numbers defining the transformational properties of the system. On the other hand the set $[\Gamma]$ is determined by the irreducible representations (IRs) of the dynamical group of the system G and its subgroups via the decomposition

$$\begin{array}{ccccccc} G & \supset G_1 & \supset G_2 & \supset \dots & \supset SO(3) & \supset O(2) & \\ \Gamma & \gamma_1\Gamma_1 & \gamma_2\Gamma_2 & & \gamma\ L & M, & \end{array} \qquad (2.1.2)$$

i.e., the set $[\Gamma]$ includes the angular momentum L and its third projection M. The extra labels γ, γ_1 ... appear if chain(2.1.2) is not a canonical one.

The operators of the physical observables $T^{[\beta]}(b,b^*)$ transform according to the decomposition

$$G \supset G'_1 \supset G'_2 \supset \dots \qquad (2.1.3)$$

where in the common case the subgroups in (2.1.3) do not coincide with the subgroups of (2.1.2).

In this way the problem is reduced to the calculation of the matrix elements

$$\langle[\Gamma']|T^{[\beta]}|[\Gamma]\rangle = \langle 0|P^{[\Gamma']}(b^*)\,T^{[\beta]}(b,b^*)\,P^{[\Gamma]}(b)|0\rangle. \qquad (2.1.4)$$

The calculation of (2.1.4) can be carried out through a direct application of the lemma of Racah[11] and the generalized Wigner-Eckart theorem. However, in the common case, there are no explicit analytical expressions for the CGSs (or the corresponding IFs), which appear in this approach. For this reason, in a number of cases, it is much more expedient to calculate (2.1.4) in a direct way. The latter leads to a great number of similar but very cumbersome operations. That is why, in this case, it is more convenient to use the powerful instrument of CAS, and in particular the universal CAS REDUCE-2[12].

2.2. Interacting Vector Boson Model

IVBM[5,6] assumes that the collective nuclear motions can be described by means of two types of vector bosons - p- and n-bosons. The corresponding creation and annihilation operators satisfy the usual commutation relations

$$[p^{*m}, p_n] = [n^{*m}, n_n] = \delta_{m,n}$$

$$[p^{*m}, n_n] = [n^{*m}, p_n] = 0, \qquad (2.2.1a)$$

where

$$(p_m)^* = p^{*m} = (-1)^m p^*_{-m} \;;\quad (p^{*m})^* = p_m$$

$$(n_m)^* = n^{*m} = (-1)^m n^*_{-m} \;;\quad (n^{*m})^* = n_m \,. \qquad (2.2.1b)$$

The operators $\vec{p}$ and $\vec{n}$ are SO(3)-vectors. Moreover they transform according to two independent IR $[1]_3$ of the group U(3) ($\vec{p}^*$ and $\vec{n}^*$ transform according to the conjugate IR of U(3) $[1]^*_3 = [0,0,-1]_3 = [1,1]_3$)*. Thus it can be assumed that p- and n-bosons belong to a "pseudospin" doublet differing in an additional quantum number $\sigma = \pm 1/2$ (a "pseudospin" projection). Hence, instead, of p_m and n_m one can introduce the operators

$$u_m(\sigma = \tfrac{1}{2}) = p_m \;;\quad u_m(\sigma = -\tfrac{1}{2}) = n_m;\quad [u^{*m}(\sigma), u_n(\rho)] = \delta(\sigma,\rho)\,\delta_{m,n} \,. \qquad (2.2.2)$$

It can be shown that $u_m(\sigma)$ transform according to the IR $[1]_6$ of the group U(6), while the quantum numbers m and σ define their transformational properties via the chain

$$\begin{array}{ll} U(6) \supset U(3)\times U(2) & \\ \quad\;\; SO(3) \cup \quad SU(2) \cup & (2.2.3) \\ \quad\;\; O(2) \cup & \end{array}$$

It has been shown in [5] that the most general one- and two-body Hamiltonian, which conserves the number of bosons can be expressed in the following way

$$H = \sum_{\sigma} h(\sigma)\, A^0(\sigma,\sigma) + \sum_{L,J,M} (2L+1) \begin{Bmatrix} 1 & 1 & L \\ 1 & 1 & J \end{Bmatrix} (-1)^M$$

$$\times \{ V^L(p,p;p,p)\,[A^J_M(p,p)\,A^J_{-M}(p,p) + A^J_M(n,n)\,A^J_{-M}(n,n)]$$

$$+ 4V^L(p,n;p,n)\,A^J_M(p,p)\,A^J_{-M}(n,n)$$

$$+ V^L(p,p;n,n)\,[A^J_M(p,n)\,A^J_{-M}(p,n) + A^J_M(n,p)\,A^J_{-M}(n,p)]$$

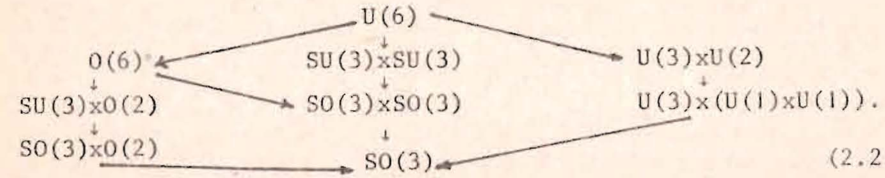
$$+ 2V^L(p,p;p,n)\,[A^J_M(p,p)\,A^J_{-M}(n,p) + A^J_M(n,p)\,A^J_{-M}(n,n) \qquad (2.2.4)$$

$$+ A^J_M(n,n)\,A^J_{-M}(p,n) + A^J_M(p,n)\,A^J_{-M}(p,p)]\},$$

where $h(\sigma)$ and $V^L(\rho,\sigma;\kappa,\nu)$ are phenomenological constants and

$$A^L_M(\rho,\sigma) = \sum_{m,n} C^{LM}_{1m1n}\, u_m(\rho)\, u^*_n(\sigma) \qquad (2.2.5)$$

(C^{LM}_{1m1n} are the usual Clebsch-Gordon coefficients [14]). The operators $A^L_M(\rho,\sigma)$ (36 in number) are SO(3)-tensors (L = 0,1,2) and generate the group U(6), which is a group of dynamical symmetry for the Hamiltonian (2.2.4). The group U(6) includes the rotational group SO(3) and the following chains of subgroups are possible in IVBM



(2.2.6)

On the other hand the operators of the other physical observables (angular and quadrupole momenta, transition operators and so on) can also be expressed by means of (2.2.5). Thus [5]

$$L_M = -\sqrt{2} \sum_{\sigma} A^1_M(\sigma,\sigma)\,; \qquad Q_M = \sqrt{6} \sum_{\sigma} A^2_M(\sigma,\sigma)\,; \qquad (2.2.7)$$

are the operators of the angular and quadrupole momenta (in fact Q_M is only a part of the total operator Q^{tot}_M but the matrix elements of Q_M and Q^{tot}_M coincide between states with an equal number of bosons).

Further, one can introduce the "pseudospin" operators

$$T_1 = \sqrt{\tfrac{3}{2}}\, A^0(p,n)\,;\quad T_{-1} = -\sqrt{\tfrac{3}{2}}\, A^0(n,p)\,;\quad T_0 = -\tfrac{\sqrt{3}}{2}\,(A^0(p,p) - A^0(n,n)) \qquad (2.2.8a)$$

and the number of boson operator

$$N = -\sqrt{3}\,(A^0(p,p) + A^0(n,n)). \qquad (2.2.8b)$$

The remaining operators $A^L(p,n)$ and $A^L(n,p)$ (L = 1,2) can be treated as vector and quadrupole transition operators between states that differ in the number of p- and n-bosons, but not in the total number of bosons.

The operators L_M and Q_M generate the subgroup SU(3) / generate SO(3) ⊂ SU(3) /, while T_M generate the sungroup SU(2) (the addi-

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* In this paper we use the notations of Vanages [13] for the IR of U(r) and SU(r).

tion of N extends SU(3) and SU(2) to U(3) and U(2) respectively). In other words, the operators (2.2.7, 2.2.8a,b) define chain (2.2.3), which is equivalent to the right side of the general scheme (2.2.6). This equivalence leads us to the conclusion that the calculation of the matrix elements of $A^L_M(\sigma,\rho)$ and H can be carried out in the basis of BM [7,8].

2.3. Bargmann-Moshinsky Basis

The basis of BM is constructed with the help of two types of vector bosons transforming according to two independent IR of SU(3) with $(\lambda,\mu)=(1,0)$ and corresponds to chain (2.2.3). The states of the basis can be written as

$$\left|\begin{matrix}(\lambda,\mu)\\ \alpha LM\end{matrix}\right\rangle = P^{(\lambda,\mu)}_{\alpha LM}(p,n)\,|0\rangle, \qquad (2.3.1)$$

where $|0\rangle$ is the vacuum state and $P^{(\lambda,\mu)}_{\alpha LM}$ is a polynomial in the creation operators p_m and n_m. The labels (λ,μ) determine the IR of SU(3), while L and M are the angular momentum and its third projection, which determine the transformational properties of the states along the decomposition SU(3) $\supset$ SO(3) $\supset$ O(2). This decomposition, however, is not a canonical one, i.e., in a given IR(λ,μ) of SU(3) there can be more than one state characterized by the quantum numbers (L,M). The quantum number α in (2.3.1) is the missing label that differs states with equal (L,M).

In the case of the most symmetrical IR of SU(3) $(\lambda,0)$ the label α can be neglected; states with given L and M appear only once. The corresponding normalized states can be constructed only by the operators p_m (or n_m)

$$\left|\begin{matrix}(\lambda,0)\\ LM\end{matrix}\right\rangle = \left[\frac{(L+M)!(L-M)!(\lambda+L)!!(2L+1)}{2^{L-M}(\lambda+L+1)!(\lambda-L)!!}\right]^{1/2}, \times (\vec{p}^2)^{\frac{1}{2}(\lambda-L)} \sum_{\ell} \frac{p_1^{\ell}\, p_0^{L+M-2\ell}\, p_{-1}^{\ell-M}}{\ell!(L+M-2\ell)!(\ell-M)!}, \qquad (2.3.2)$$

where $\vec{p}^{\,2} = p_0^2 - 2p_1p_{-1}$ and the range of ℓ is determined by the fact that the exponents in (2.3.2) are all nonnegative integers.

Using (2.2.8a) one can test directly that the states (2.3.2) have a "pseudospin" $T=\lambda/2$ and $T_0=T$. The states with $T_0=-T$ can be obtained by the substitution $\vec{p}\to\vec{n}$.

The state $\left|\begin{matrix}(0,\mu)\\ L\;M\end{matrix}\right\rangle$ can be constructed by an SO(3)-vector transforming according to (0,1). This vector is given by the vector product

$$A_m = (\vec{p}\times\vec{n})_m = \left|\begin{matrix}(0,1)\\ 1\;m\end{matrix}\right\rangle , \qquad (2.3.3a)$$

that is,

$$A_1 = p_1n_0 - p_0n_1;\quad A_0 = p_1n_{-1} - p_{-1}n_1;\quad A_{-1} = p_0n_{-1} - p_{-1}n_0 . \qquad (2.3.3b)$$

The normalized states $|{(0,\mu)\atop LM}\rangle$ can be obtained from (2.3.2) by substituting $p_m\to A_m$, $\lambda\to\mu$ and multiplying by $[(\mu+1)!]^{-1/2}$. These states correspond to $T=T_0=0$.

The general state $\left|\begin{matrix}(\lambda,\mu)\\ \alpha LL\end{matrix}\right\rangle$ (a highest-weight SO(3)-vector with $M=L$) can be constructed with the help of five elementary permissible diagrams (EPDs), which are the highest-weight states of SO(3)-multiplets belonging to the low-lying IRs of SU(3). These EPDs are of the following type:

$$p_1 = \left|\begin{matrix}(1,0)\\ 1\;1\end{matrix}\right\rangle;\quad A_1 = \left|\begin{matrix}(0,1)\\ 1\;1\end{matrix}\right\rangle;\quad \vec{p}^{\,2} = \left|\begin{matrix}(2,0)\\ 0\;0\end{matrix}\right\rangle$$
$$A = -\left|\begin{matrix}(0,2)\\ 0\;0\end{matrix}\right\rangle;\quad z = \left|\begin{matrix}(1,1)\\ 1\;1\end{matrix}\right\rangle . \qquad (2.3.4)$$

We have already given explicit expressions for the EPDs $\vec{p}^{\,2}$ and A_1. The remaining EPDs are expressed as

$$A = -A_0^2 + 2A_1A_{-1} = \vec{p}^{\,2}\vec{n}^{\,2} - (\vec{p}\cdot\vec{n})^2$$
$$z = (\vec{p}\times(\vec{p}\times\vec{n}))_1 = p_1A_0 - p_0A_1 \qquad (2.3.5)$$

Then the general BM states can be written as

$$\left|\begin{matrix}(\lambda,\mu)\\ \alpha LL\end{matrix}\right\rangle = z^{\beta}(p_1)^{L-\mu+2\alpha}(A_1)^{\mu-2\alpha-\beta}(\vec{p}^{\,2})^{\frac{1}{2}(\lambda+\mu-1-2\alpha-\beta)}A^{\alpha}|0\rangle, \qquad (2.3.6)$$

where*

$$\beta = \begin{cases} 0 & \text{if } \lambda+\mu-L \text{ even} \\ 1 & \text{if } \lambda+\mu-L \text{ odd.} \end{cases} \qquad (2.3.7)$$

It should be noted that because of the relation

$$z^2 = \vec{p}^{\,2}A_1 - p_1^2A \qquad (2.3.8)$$

the EPD z in (2.3.6) appears at most linearly. The ranges of α and L in (2.3.6) are determined by the fact that the exponents in (2.3.6) are all nonnegative integers. Thus α runs the values in the range

$$\max\{0,\tfrac{1}{2}(\mu-L)\} \le \alpha \le \min\{\tfrac{1}{2}(\mu-\beta),\ \tfrac{1}{2}(\lambda+\mu-L-\beta)\}. \qquad (2.3.9)$$

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* The states (2.3.6) differ from the states (3.8) from [8] in the definition of the label α and coincide upto a phase factor $(-1)^{\alpha}$.

The BM-states with an arbitrary third projection of the angular momentum can be obtained from (2.3.6) with the help of the well-known formulae

$$\left|\begin{matrix}(\lambda,\mu)\\ \alpha L M\end{matrix}\right\rangle = \left[\frac{2^{L-M}(L+M)!}{(2L)!(L-M)!}\right]^{1/2} L_{-1}^{L-M} \left|\begin{matrix}(\lambda,\mu)\\ \alpha L L\end{matrix}\right\rangle \qquad (2.3.10)$$

It can be shown that the BM-states are also a complete set for the IR(N,T) of the "pseudospin" group SU(2) generated by the operators (2.2.8a). This is due to the fact that in chain (2.2.3) the groups SU(3) and SU(2) are mutually complementary; the IR (λ,μ) of SU(3) determines the IR (N,T) of SU(2) by means of the following relations

$$N = \lambda + 2\mu; \qquad T = \frac{\lambda}{2}, \qquad (2.3.11)$$

where N is the number of bosons and T is the "pseudospin" of the state.

At last, it should be pointed out, that the states (2.3.6) are neither nornalized nor orthogonal in the additional label α, i.e., one has

$$\left\langle\begin{matrix}(\lambda,\mu)\\ \alpha L L\end{matrix}\right|\left.\begin{matrix}(\lambda,\mu)\\ \alpha' L L\end{matrix}\right\rangle = f^{(\lambda,\mu)}_{\alpha\alpha'}(L) \qquad (2.3.12)$$

where $f^{(\lambda,\mu)}_{\alpha\alpha'}(L)$ are the so-called overlap integrals. An explicit expression for the overlap integrals in the case of the BM-basis is given by formulae (2.4) of [15].

Further, when convenient, we shall also use the following notations for the BM-states (2.3.6)

i) $\lambda+\mu-L$ -even

$$\left|\begin{matrix}(\lambda,\mu)\\ \alpha L L\end{matrix}\right\rangle \equiv |\ell_1,\ell_2,r_1\,\alpha\rangle \qquad (2.3.13a)$$

$$L = \ell_1 + \ell_2; \quad 2T = \ell_1 + 2r + 1$$

ii) $\lambda+\mu-L$ -odd

$$\left|\begin{matrix}(\lambda,\mu)\\ \alpha L L\end{matrix}\right\rangle = z\,|\ell_1,\ell_2,r_1\,\alpha\rangle \qquad (2.3.13b)$$

$$L = \ell_1 + \ell_2 + 1; \quad 2T = \ell_1 + 2r + 1$$

The integers ℓ_1, ℓ_2, r and α coincide with the corresponding exponents of the different EPDs in (2.3.6).

3. MATRIX ELEMENTS OF THE SU(3)-QUADRUPOLE OPERATOR - AN EXAMPLE OF THE GENERAL TECHNIQUE

In this Section we are going to discuss in detail the general technique of calculating the matrix elements of the SU(3)-quadrupole operator Q (2.2.7) between the states of the BM-basis. First of all it should be noted, that, by definition, the operator Q_0 is an U(2)-scalar and transforms as a tensor of second rank in the space of the angular momentum. Thus, taking into account the tensorial structure of Q_0, one can write directly

$$Q_0\left|\begin{matrix}(\lambda,\mu)\\ \alpha L L\end{matrix}\right\rangle = \sum_{\substack{s=0,\pm1\\ k=0,1,2}} a_s^{(k)} L_{-1}^{k} \left|\begin{matrix}(\lambda,\mu)\\ \alpha+s \quad L+k \quad L+k\end{matrix}\right\rangle \qquad (3.1)$$

and the problem is reduced to the calculation of the coefficient $a_s^{(k)}$.

The operator Q_0 can be expressed in terms of the p- and n-boson operators (2.2.1a,b) in the following way

$$Q_0 = 2p_0p_0^* + p_1p_{-1}^* + p_{-1}p_1^* + 2n_0n_0^* + n_1n_{-1}^* + n_{-1}n_1^*. \qquad (3.2)$$

For further calculations it is more convenient to employ Fock's representation, where the creation operators are treated as independent variables and the corresponding annihilation operators are presented as differential operators, i.e.,

$$p_m^* = (-1)\,p^{*-m} = (-1)^m\frac{\partial}{\partial p_{-m}}; \quad n_m^* = (-1)^m n^{*-m} = (-1)^m\frac{\partial}{\partial n_{-m}}. \qquad (3.3a)$$

The vacuum state of the system is determined by

$$p_m^*|0\rangle = (-1)^m\frac{\partial}{\partial p_{-m}}|0\rangle = n_m^* = (-1)^m\frac{\partial}{\partial n_{-m}}|0\rangle = 0. \qquad (3.3b)$$

The action of the annihilation operators on the EPDs (2.3.4) is given by:

$$p_m^*p_1 = -\delta_{m,-1};$$

$$p_m^*A_1 = -\delta_{m,0}n_1 - \delta_{m,-1}n_0;$$

$$p_m^*\vec{p}^{\,2} = 2\delta_{m,1}p_1 + 2\delta_{m,0}p_0 + 2\delta_{m,-1}p_{-1};$$

$$p_m^* A = 2\delta_{m,1}(n_0 A_1 - n_1 A_0) + 2\delta_{m,0}(n_{-1} A_1 - n_1 A_{-1}) + 2\delta_{m,-1}(n_{-1} A_0 - n_0 A_{-1});$$

$$p_m^* z = \delta_{m,1} p_1 n_1 + \delta_{m,0}(p_0 n_1 - A_1) + \delta_{m,-1}(p_0 n_0 - p_1 n_{-1} - A_0);$$

$$n_m^* p_1 = 0;$$

$$n_m^* A_1 = \delta_{m,0}\, p_1 + \delta_{m,-1}\, p_0;$$

$$n_m^* \vec{p}^{\,2} = 0; \qquad (3.4)$$

$$n_m^* A = 2\delta_{m,1} z + 2\delta_{m,0}(p_1 A_{-1} - p_{-1} A_1) + 2\delta_{m,-1}(p_0 A_{-1} - p_{-1} A_0);$$

$$n_m^* z = -\delta_{m,1} p_1^2 - \delta_{m,0}\, p_1 p_0 + \delta_{m,-1}(p_1 p_{-1} - p_0^2).$$

The next problem is that, in the general case, when the operator (3.2) acts on the basis of BM (2.3.6) some of the terms that appear in the r.h.s. of the corresponding equations are linearly dependent and must be transformed into linearly independent fundamental blocks by means of the following substitutions:

$$z^2 = \vec{p}^{\,2} A_1^2 - p_1^2 A;$$

$$A_1 A_{-1} = \frac{1}{2}(A + A_0^2);$$

$$A_0 z = \frac{1}{2} p_1 A_0^2 - \frac{1}{2} p_1 A - p_{-1} A_1^2;$$

$$p_1 p_0 n_0 = p_1 A_0 - z + \vec{p}_1^{\,2} n_1 + 2 p_1 p_{-1} n_1;$$

$$p_1 (\vec{p}\cdot\vec{n}) = \vec{p}^{\,2} n_1 - z;$$

$$p_0 A_1 = p_1 A_0 - z;$$

$$p_{-1} A_1^2 z = -\vec{p}^{\,2} A_1^2 A_0 - p_1 p_{-1} A_1^2 A_0 - \frac{1}{2} p_1 A z + \frac{3}{4} p_1^2 A_0 A + \frac{1}{4} p_1^2 A_0^3;$$

$$p_1 A_0^4 = 4\vec{p}^{\,2} A_0^2 A_1^2 - 2p_1^2 A_0^2 A - p_1^2 A^2 - 4p_{-1}^2 A_1^4 + 4p_1 p_{-1} A_0^2 A_1^2 - 4p_1 p_{-1} A_1^2 A;$$

$$p_0^2 = \vec{p}^{\,2} + 2 p_1 p_{-1};$$

$$n_0 p_1 A_1 = A_1^2 + p_1 n_1 A_0 - n_1 z; \qquad (3.5)$$

$$p_1 n_{-1} = A_0 + p_{-1} n_1;$$

where the fundamental blocks appear in the r.h.s. of (3.5).

In this way the general technique of calculating the matrix elements of any operator in Fock's representation leads to the calculation of the commutator

$$<0\,|\,P(u^*)S(u)\,|0> = <0|[\,P(u^*),S(u)\,]|0> = <0\,|[\,P(\frac{\partial}{\partial u}),S(u)\,]|\,0>, \qquad (3.6)$$

where $S(u)$ and $P(u^*)$ are some polynomials in the creation and annihilation operators (2.2.2).

In the case of the Q_0 -operator (3.1), using (3.2) and (3.4) and (3.5), in the notations (2.3.13a,b) one obtains:

$\lambda + \mu - L$ -even

$$Q_0 |\ell_1,\ell_2,\tau,a> = (4\tau + 2a + \ell_2 - \ell_1)|\ell_1,\ell_2,\tau,a>$$

$$+ 12\tau p_{-1} |\ell_1+1,\ell_2,\tau-1,a> + 6a A_0^2 |\ell_1,\ell_2,\tau,a-1> \qquad (3.7a)$$

$\lambda + \mu - L$ -odd

$$Q_0 z |\ell_1,\ell_2,\tau,a> = -(\ell_1 - \ell_2 - 4\tau - 2a - 3) z |\ell_1,\ell_2,\tau,a> -$$

$$-3(4\tau + a + 2) A_0 |\ell_1+1,\ell_2,\tau,a>$$

$$-6\tau p_{-1} A_0 |\ell_1+2,\ell_2,\tau-1,a> - 6\tau z |\ell_1+2,\ell_2-2,\tau-1,a+1> \qquad (3.7b)$$

$$+ 9\tau A_0 |\ell_1+3,\ell_2-2,\tau-1,a+1> + 3\tau A_0^3 |\ell_1+3,\ell_2-2,\tau-1,a>$$

$$-6a p_{-1} A_0 |\ell_1,\ell_2+2,\tau,a-1> + 3a A_0^3 |\ell_1+3,\ell_2,\tau,a-1>.$$

On the other hand, in order to obtain the coefficients in (3.1) one has to calculate the action of the operators L_{-1} and L_{-1}^2 on the states (2.3.13a,b). The operator L_{-1} is of the following type (2.2.7).

$$L_{-1} = -p_0 p^*_{-1} + p_{-1} p^*_0 - n_0 n^*_{-1} + n_{-1} n^*_0 \, . \tag{3.8}$$

Then with the help of (3.4) and (3.5) one obtains:

$\lambda + \mu - L$ -even

$$L_1 \, |\ell_1, \ell_2, \tau, \alpha\rangle = -\ell_1 z |\ell_1 - 1, \ell_2 - 1, \tau, \alpha\rangle + (\ell_1 + \ell_2) A_0 |\ell_1, \ell_2 - 1, \tau, \alpha\rangle ;$$

$$\begin{aligned} L^2_{-1} |\ell_1, \ell_2, \tau, \alpha\rangle &= \frac{\ell_2 (2\ell_1 + 1)}{2} |\ell_1, \ell_2 - 2, \tau, \alpha + 1\rangle \\ &+ \frac{1}{2} \ell_2 (2\ell_1 + 2\ell_2 - 1) A_0^2 |\ell_1, \ell_2 - 2, \tau, \alpha\rangle + \ell_1 (2\ell_1 + 2\ell_2 - 1) p_{-1} |\ell_1 - 1, \ell_2, \tau, \alpha\rangle \\ &+ \ell_1 (\ell_1 - 1) |\ell_1 - 2, \ell_2, \tau + 1, \alpha\rangle ; \end{aligned} \tag{3.9a}$$

$\lambda + \mu - L$ -odd

$$\begin{aligned} L_{-1} z |\ell_1, \ell_2, \tau, \alpha\rangle &= \frac{1}{2} (\ell_1 - \ell_2 + 1) |\ell_1 + 1, \ell_2 - 1, \tau, \alpha + 1\rangle \\ &+ \frac{1}{2} (\ell_1 + \ell_2 + 1) A_0^2 |\ell_1 + 1, \ell_2 - 1, \tau, \alpha\rangle - (\ell_1 + \ell_2 + 1) p_{-1} |\ell_1, \ell_2 + 1, \tau, \alpha\rangle \\ &- \ell_1 |\ell_1 - 1, \ell_2 + 1, \tau + 1, \alpha\rangle ; \end{aligned}$$

$$\begin{aligned} L^2_{-1} z |\ell_1, \ell_2, \tau, \alpha\rangle &= \left\{ \frac{1}{2} (\ell_1 - \ell_2 + 1) + \ell_1 (\ell_1 + 1) \right\} z |\ell_1, \ell_2 - 2, \tau, \alpha + 1\rangle \\ &+ \left\{ \frac{1}{4} (\ell_1 + \ell_2 + 1)(4\ell_1 + 3) + \frac{1}{2} (\ell_1 + \ell_2)(\ell_1 - \ell_2 + 1) \right\} A_0 |\ell_1 + 1, \ell_2 - 2, \tau, \alpha + 1\rangle \\ &+ \frac{1}{4} (\ell_1 + \ell_2 + 1)(2\ell_1 + 2\ell_2 + 1) A_0^3 |\ell_1 + 1, \ell_2 - 2, \tau, \alpha\rangle + \ell_1 (\ell_1 - 1) z |\ell_1 - 2, \ell_2, \tau + 1, \alpha\rangle \\ &- \frac{1}{2} (\ell_1 + \ell_2 + 1)(2\ell_1 + 2\ell_2 + 1) p_{-1} A_0 |\ell_1, \ell_2, \tau, \alpha\rangle \\ &- \ell_1 (2\ell_1 + 2\ell_2 + 1) A_0 |\ell_1 - 1, \ell_2, \tau + 1, \alpha\rangle ; \end{aligned} \tag{3.9b}$$

Hence the comparison between the l.h.s. of eq. (3.1) (or (3.7a,b)) and the r.h.s. of eq.(3.1) (or (3.9a,b)) gives the expressions, given in the Appendix, for the matrix elements in (3.1) of the SU(3)-quadrupole operator /2,16/.

4. CONCLUSIONS

The general algorithm of the application of CAS to some group theoretical nuclear problems is illustrated on the example of the SU(3)-quadrupole operator. All basic characteristics of this approach appear in more complicated problems, for example, the calculation of the matrix elements of the IVBM Hamiltonian in the BM-basis. The latter will be discussed in /17/.

APPENDIX

The matrix elements of the SU(3)-quadrupole operator are of the following type:

$$a_1^{(2)} = 0; \quad a_0^{(2)} = \frac{6(\lambda + \mu - L - 2\alpha - \beta)}{(L+2)(2L+3)}; \quad a_{-1}^{(2)} = \frac{12\alpha}{(L+2)(2L+3)};$$

$$a_1^{(1)} = \frac{6\beta(\lambda + \mu - L - 2\alpha - \beta)(\mu - 2\alpha - \beta)}{(L+1)(L+2)}$$

$$a_0^{(1)} = -\frac{12\alpha\beta(L - \mu + 2\alpha + 1)}{(L+1)(L+2)} - \frac{6\beta}{L+1} - \frac{6(\lambda + \mu - L - 2\alpha - \beta)(\mu - 2\alpha)}{(L+1)(L+2)};$$

$$a_{-1}^{(1)} = \frac{12\alpha(L - \mu + 2\alpha)}{(L+1)(L+2)};$$

$$a_1^{(-1)} = -\frac{6(\lambda + \mu - L - 2\alpha - \beta)(\mu - 2\alpha - \beta)(\mu - 2\alpha - \beta - 1)}{(L+1)(2L+3)};$$

$$a_0^{(-1)} = 4\alpha \frac{L(L+1) - 3(L - \mu + 2\alpha + \beta)^2}{(L+1)(2L+3)}$$

$$-2(\lambda + \mu - L - 2\alpha - \beta) \frac{L(L+1) - 3(\mu - 2\alpha)^2}{(L+1)(2L+3)} - L + 2\mu - 4\alpha + 2\beta;$$

$$a_{-1}^{(-1)} = \frac{12\alpha(L - \mu + 2\alpha)}{(L+1)(2L+3)},$$

where, according to (2.3.7), β is equal to 0 or 1.

REFERENCES

1. Vanagas V., Nadjakov E., Raychev P. Preprint ICTP, Trieste IC/75/40, 1975; Bulg.J.Phys., 1975, 2, p.558.

2. Raychev P., Roussev R. Sov.J.Nucl.Phys., 1978, 27, p.1501.
3. Arima A., Iachello F. Ann.Phys. (N.Y.), 1978, 111, p.201.
4. Castanos O. et al. J.Math.Phys., 1979, 20, p.35.
5. Georgieva A., Raychev P., Roussev R. J.Phys.G: Nucl.Phys., 1982, 8, p.1377.
6. Georgieva A., Raychev P., Roussev R. J.Phys.G: Nucl.Phys., 1983, 9, p.521.
7. Bargmann V., Moshinsky M. Nucl.Phys., 1963, 23, p.177.
8. Moshinsky M. et al. Ann.Phys.(N.Y.), 1975, 95, p.139.
9. Gerdt V.P., Tarasov O.V., Shirkov D.V. Sov.Fiz.Usp., 1980, 23, p.59.
10. van Hulzen J.A., Calmet J. In: Computer Algebra Symbolic and Algebraic Computation. Computing Suppl., Springer-Verlag, 1982, 4, p.221.
11. Racah G. Phys.Rev., 1949, 76, p.1352.
12. Hearn A.C. Reduce User's Manual. Second Ed., Univ. of Utah, 1973.
13. Vanagas V. Algebraic Methods in Nuclear Theory. Vilnuis, Mintis, 1971.
14. Varshalovich D.A., Moskalev A,N., Hersonsky V. Quantum Theory of the Angular Momentum. "Nauka", Leningrad, 1975.
15. Alisauskas S., Raychev P., Roussev R. J.Phys.G: Nucl.Phys., 1982, 7, p.1213.
16. Afanasjev G.N., Avramov S.A., Raychev P.P. Sov.J.Nucl. Phys., 1972, 16, p.53.
17. Gerdt V.P. et al. JINR, E4-85-263, Dubna, 1985.

Гердт В.П. и др. E4-85-262

Вычисление матричных элементов гамильтониана модели взаимодействующих векторных бозонов с использованием компьютерной алгебры.

Основные положения модели взаимодействующих векторных бозонов и матричные элементы SU(3)-квадрупольного оператора

Исследована групповая структура гамильтониана взаимодействующих векторных бозонов. Представлен алгоритм вычисления матричных элементов операторов физических наблюдаемых в базисе Баргманна-Мошинского. Применение этого алгоритма показано на примере SU(3)-квадрупольного оператора. Алгоритм реализован на языке системы аналитических вычислений REDUCE-2.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Gerdt V.P. et al. E4-85-262

Calculation of the Matrix Elements of the Hamiltonian of the Interacting Vector Boson Model Using Computer Algebra.

Basic Concepts of the Interacting Vector Boson Model and Matrix Elements of the SU(3)-Quadrupole Operator

The algebraic structure of the IVBM Hamiltonian is discussed. An algorithm for calculation of the matrix elements of the operators of the physical observables in the basis of Bargmann-Moshinsky is presented on the example of the SU(3)-quadrupole operator. The algorithm is realized on the Computer Algebra System REDUCE-2.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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