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INDUCED ANAPOLE (TOROID) MOMENTS, A NEW TYPE OF POLARIZABILITY AND SOME RELATED EFFECTS

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A long time ago Zeldovich[1] considered a new kind of electromagnetic interaction (invariant under time reversal but odd under parity) and introduced a new static characteristic for spin particles, the anapole or pseudocharge. For spin 1/2 it is given by the following structure in the matrix element of the current

$$<p+q|\, j_\mu^{(\mathrm{anapole})}(0)\,|p> = i A(q^2)\bar{u}\,[q^2\gamma_\mu - (q\cdot\gamma)q_\mu]\,\gamma_5\, u\,, \tag{1}$$

which has been interpreted in ref.[1] in terms of a toroid current (i.e., a closed current circulating through a toroidal soleniod). Such a particular current distribution is neither an electric nor a magnetic mutipole, but represents a new type of dipole moment (d.m.) which, as subsequently shown[2], is only the first element of a whole (independent) family of toroid multipoles. A general configuration of charges and currents can be fully described only by considering all three families of electric, magnetic and toroid multipoles. Alongside the usual d.m.-s

$$Q_i(t) = \int x_i j_0(\vec{x},t)\, d^3x\,, \quad M_i(t) = \frac{1}{2}\int [\vec{x}\times\vec{j}(\vec{x},t)]\, d^3x\,,$$

the third (toroid) dipole is[2,3]

$$T_i(t) = \frac{1}{10}\int \{ x_i[\vec{x}\cdot\vec{j}(\vec{x},t)] - 2\vec{x}^2 j_i(\vec{x},t)\}\, d^3x\,, \tag{2}$$

and a simple calculation in the non-relativistic limit shows that indeed

$$A(q^2=0) = <\vec{p}=0, S_z=+\tfrac{1}{2}\,|T_3|\,\vec{p}=0, S_z=+\tfrac{1}{2}>/V, \quad V=(2\pi)^3\delta^3(\vec{p}=0)\,.$$

While various consequences of the anapole interaction have been investigated[4] soon after its proposal, there is now renewed interest in it stemming from the electro-weak theory (P-violation in atomic physics[5], anapole moments of leptons, quarks[6] or nuclei[7]) as well as from supersymmetry, grand unified models and neutrino physics (which have led to a more detailed analysis of Majorana fermions; strangely enough, the only electromagnetic structure such a (spin 1/2) particle may have[8,9] is just the one given by Eq.(1)).

In this context, the purpose of this note is: 1°. To emphasize that even if P, T are separately conserved, quantum particles which then cannot have intrinsic toroid d.m.-s may still get induced ones in the presence of an external conduction and/or displacement current, just as, e.g., protons or pions in an external electric field $\vec{E}^{ext}$ do acquire induced electric d.m.-s through their polarizabilities. 2°. To analyze the new type of polarizability which so emerges, its implications in Compton scattering and the new van der Waals (v.d.W.) forces which appear on its account. In this respect we hope to clarify some points in the study [10] of the higher multipole contributions to the v.d.W. potential. Other related questions regarding some P, T violating effects will also be touched upon.

To make things clear, we start from the multipole decomposition /2/ of a classical current density $\vec{j}(\vec{x},t)$ up to terms including the toroid d.m.

$$j_i(\vec{x},t) = \dot{Q}_i \delta^3(\vec{x}) - [\dot{Q}_{ik} - \epsilon_{ik\ell} M_\ell + \frac{1}{6}\delta_{ik} \dot{\overline{r^2}}]\, \partial_k \delta^3(\vec{x}) +$$

$$+ \frac{1}{2}[2\dot{Q}_{ijk} + \frac{1}{10}(\delta_{ij}\dot{\overline{r_k^2}} + \delta_{ik}\dot{\overline{r_j^2}}) - \frac{1}{3}(\epsilon_{ij\ell} M_{k\ell} + \epsilon_{ik\ell} M_{j\ell}) + \qquad (3)$$

$$+ (\delta_{ik} T_j + \delta_{jk} T_k - 2\delta_{jk} T_i)\, \partial_j \partial_k \delta^3(\vec{x}) + \dots$$

$\overline{r^2}$, $\overline{r_i^2}$, Q_{ij}, M_{ij}, Q_{ijk} are respectively the mean squared radius, squared radius of the electric dipole distribution, electric and magnetic quadrupole and electric octupole; the dot means differentiation with respect to t (summation over repeated indices is understood). When a system of currents specified by $\vec{j}(\vec{x},t)$ interacts with an external field $\vec{A}^{ext}(\vec{x},t)$, the contribution of T_i to the interaction energy appears to be /1,2/

$$W(t) = -\vec{T}(t)\cdot[\nabla\times\nabla\times\vec{A}^{ext}]_{\vec{x}=o,t} = -\vec{T}(t)\cdot[\nabla\times\vec{H}^{ext}]_{\vec{x}=o,t} =$$

$$= -\vec{T}(t)\cdot[4\pi\vec{J}^{ext} + \dot{\vec{D}}^{ext}]_{\vec{x}=o,t},$$

where $\vec{J}^{ext}$ and $(4\pi)^{-1}\dot{\vec{D}}^{ext}$ are the external conduction and displacement currents. The response of a quantum system to this contact interaction (as seen, $\vec{T}$ interacts with the source of $\vec{H}^{ext}$) will be described by the following dynamic (i.e., frequency dependent) "ana"-polarizability

$$\gamma_{ij}(\omega) = i\int e^{i\omega t}\theta(t) \langle [T_i(t), T_j(0)] \rangle\, dt =$$

$$= \sum_n \left[\frac{\langle 0|T_i|n\rangle\langle n|T_j|0\rangle}{\omega_{no} - \omega - i\epsilon} + \frac{\langle 0|T_j|n\rangle\langle n|T_i|0\rangle}{\omega_{no} + \omega + i\epsilon} \right], \qquad (4)$$

($\omega_{no} = E_n - E_o$, E_o and E_n are the energies of the ground and excited states), exactly in the same way as the response to the perturbations $-\vec{Q}\vec{E}^{ext}$ or $-\vec{M}\vec{H}^{ext}$ is given by the familiar electric and magnetic dipole polarizabilities $\alpha_{ij}(\omega)$, $\beta_{ij}(\omega)$ (whose definitions differ from that of $\gamma_{ij}(\omega)$ only by the replacements $T_i \to Q_i$, $T_i \to M_i$). So, irrespective of whether or not the system has a non-zero intrinsic toroid d.m., it will acquire, in general, an induced one /11/ of Fourier components $T_i^{ind}(\omega) = \gamma_{ij}(\omega)[\nabla\times\vec{H}^{ext}(\omega)]$ whenever $\gamma_{ij}(\omega) \neq 0$, just as it gets induced d.m.-s

$$Q_i^{ind}(\omega) = \alpha_{ij}(\omega) E_j^{ext}(\omega), \quad M_i^{ind}(\omega) = \beta_{ij}(\omega) H_j^{ext}(\omega),$$

even if the intrinsic correspondents are zero. No discrete symmetry violation is required for that to happen. Parity and time reversal invariance may prevent systems from possessing intrinsic electric and toroid d.m.-s (this is actually the case for quantum particles, because such vector characteristics must then lie along the spin direction and the spin has opposite transformation properties), but they cannot prohibit the appearance of the corresponding induced moments (a new direction, that of the external field or current is then available). At this point we stress that γ appears on the same footing with the usual polarizabilities α, β in the description of the linear response to electromagnetic perturbations, its consideration being merely a matter of completeness when the system is interacting not only with the external field but with its source as well. There are, of course, notable differences between α and β on one side and γ in what concerns their role in physical processes. For instance, in the elastic scettering of light on the considered system, unlike α and β which, as known, establish the structure of the amplitude in the second (photon) energy order, γ enters only the fourth order. To see that in more detail, we shall look carefully to the low energy theorem for the Compton S-matrix element. With correct inclusion of the toroid dipole (like in Eq.(3)), this theorem to the desired fourth frequency order reads

$$\langle p', k' | S | p, k \rangle = \delta_{if} + i(2\pi)^4 \delta^4(p' + k' - p - k)\, e_m^*(\vec{k}')\, T_{mn}\, e_n(\vec{k})$$

with

$$T_{mn} = V^{-2} \sum_n \Big[\frac{\langle p'|F_m(-k')|n\rangle\langle n|F_n(k)|p\rangle}{n_o - p_o - k_o - i\epsilon}(2\pi)^3\delta^3(\vec{p} + \vec{k} - \vec{n}) +$$

$$+ \frac{\langle p'|F_n(k)|n\rangle\langle n|F_m(-k')|p\rangle}{n_o - p'_o + k_o - i\epsilon}(2\pi)^3\delta^3(\vec{p}' - \vec{k} - \vec{n})\Big], \qquad (5)$$

where $F_n(k) \equiv ik_0 A_n(\vec{k}) + B_n(\vec{k})$ and

$$A_n(\vec{k}) = Q_n + ik_i\,(Q_{ni} + \frac{1}{6}\,\delta_{ni}\bar{r}^2) - \frac{1}{2}\,k_i k_j\,[2Q_{nij} + \frac{1}{10}\,(\delta_{ni}\,\bar{r}_j^2 + \delta_{nj}\bar{r}_i^2)] + \dots$$

$$B_n(\vec{k}) = -ik_i\,\epsilon_{nij}\,M_j - \frac{1}{2}\,k_i k_j[-\frac{1}{3}\,(\epsilon_{ni\ell}\,M_{j\ell} + \epsilon_{nj\ell}\,M_{i\ell}) + (\delta_{ni}T_j + \delta_{nj}T^i - 2\delta_{ij}\,T_n)] + \dots \qquad (6)$$

$e_m^*(\vec{k}')$, $e_n(\vec{k})$ are the photon polarization vectors in the gauge $e_0 = 0$, $\vec{k}\cdot\vec{e}(\vec{k}) = 0$; $k = (k_0, \vec{k})$, $k' = (k_0', \vec{k}')$ and p, p' denote the photon's and target's initial and final four momenta, $n = (n_0, \vec{n})$ is that of the excited state n. All multipole operators in Eq.(6) are at zero time. In the laboratory ($\vec{p} = 0$, $k_0 \equiv \omega$, $k_0' \equiv \omega'$, $(\vec{k}\vec{k}') = \omega\omega'\cos\theta_L$), the toroid-toroid (T,T)part of T_{mn} comes out as

$$T_{mn}^{(T,T)} = (k_i'k_m' - \omega'^2\delta_{im})(k_j k_n - \omega^2\delta_{jn})\,\gamma_{ij}(0),$$

i.e., it is fixed (with proper account of the continuum normalizations adoped here) just by the static ($\omega = 0$) ana-polarizability of Eq.(4). The (T,T) part of the scattering independent of target's spin (when $\gamma_{ij}(0) \to \gamma\delta_{ij}$) is therefore $e_m^* T_{mn}^{(T,T)} e_n = \gamma\omega'^2\omega^2\,\vec{e}^*(\vec{k}')\vec{e}(\vec{k})$ and for a system of charge Ze ($e^2 \simeq 1/137$) and mass m its interference with the Thompson scattering ($T_{mn}^{Thom} = -2(Ze)^2\delta_{mn}$) will give rise to the low-energy contribution[12] $-(Ze)^2\gamma\omega^4(1+\cos^2\theta_L)/m$ in the unpolarized differential cross-section σ. This is to be contrasted with the pieces $-(Ze)^2\alpha_{E2}\omega^4\cos^3\theta_L/6m$ and $(Ze)^2\beta_{M2}\omega^4(1-3\cos^2\theta_L)/12m$ coming respectively from the interference between the Thompson scattering and the scattering due to the quadrupole electric and magnetic polarizabilities α_{E2} and β_{M2}[13]. However, in general γ will not enter alone as the coefficient of the $\omega^4(1+\cos^2\theta_L)$ structure in σ, but in the combination $\gamma + \alpha'$ with the derivative $\alpha' \equiv [d\alpha(\omega)/d\omega^2]\,\omega = 0$ (here $\alpha_{ij}(\omega) \to \alpha(\omega)\delta_{ij}$, $\alpha(\omega) = \alpha^*(-\omega)$), and therefore all one may hope to measure by Compton scattering is $\gamma + \alpha'$. Although unless the system is such that $\alpha' \ll \gamma$, its ana-polarizability cannot be directly detected in this particular process, its presence in the amplitude should not be disregarded since despite their joint appearance, γ and α' are very different in nature and exhibit different properties of the system ($\alpha(\omega)$ depends on the charge distribution, while $\gamma(\omega)$ is related to the system's current distribution, more exactly to that part of it which is not already taken into account through the usual magnetic polarizability $\beta(\omega)$). In order to

assess the size of γ in comparison with that of α', we focus now on the example of the charged pion[14]. The lowest pion's excited state to be considered in Eq.(4) is the A_1 (m_A = 1270 MeV) meson. Using the $A_1\pi\gamma$ vertex $2g_A\phi_\pi F_{\mu\nu}\mathcal{F}^{\mu\nu}$, where ϕ_π, $F_{\mu\nu}$, $\mathcal{F}^{\mu\nu}$ stand for the pion, electromagnetic and A_1 fields while g_A is related[15] to the width $\Gamma(A_1 \to \pi\gamma) \simeq 0.6$ MeV by

$$g_A = [\frac{3}{2}\,m_A^3\,\Gamma\,(A_1 \to \pi\gamma)/(m_A^2 - m_\pi^2)^3\,]^{1/2} \;,$$

the A_1 contribution to the charged pion's static dipole ana-polarizability results

$$\gamma_{(\pi)}^{(A_1)} = g_A^2(m_A + m_\pi)^4/8\,(m_A m_\pi)^3(m_A - m_\pi) \simeq 1.2 \times 10^{-5}\,\mathrm{fm}^5 \,.$$

Similarly one finds

$$\alpha'^{(A_1)}_{(\pi)} = 8\,g_A^2 m_A/m_\pi(m_A - m_\pi)^3 \simeq 0.8 \times 10^{-5}\,\mathrm{fm}^5 \,.$$

This shows that in the combination $\gamma_{(\pi)} + \alpha'_{(\pi)}$ none of the terms should be expected to overshadow the other, both being of the order of magnitude of the pion's quadrupole electric and magnetic polarizabilities[16]. This example suggests that (at least for hadrons) the effect of the induced anapole (toroid) moments in Compton scattering should be comparable in size with that of the familiar induced electric and magnetic moments of one order of multipolarity higher.

Next we shall consider the new v.d.W. forces which appear on account of γ and, skipping over technicalities, only list and shortly comment on the results[17]: (a) At very large distances (i.e., in the "retarded" case), between two bodies of (scalar, static, dipole) ana-polarizabilities $\gamma_{1,2}$, it will act a force given by the potential $V(r) = -\hbar c\,(639/4\pi)\,\gamma_1\gamma_2 r^{-11}$ (we restore for convenience the factors $\hbar$, c). This is the new (anapole or toroid) correspondent[18] of the Casimir-Polder potential[19] $V(r) = -\hbar c(23/4\pi)\alpha_1\alpha_2 r^{-7}$ (and of its magnetic analog) and expresses, this time, the effect of fluctuating toroid d.m.-s inside the interacting bodies rather than that of fluctuating electric and magnetic d.m.-s. (b) When $\gamma_{1,2}$ are three dimensional tensors rather than scalars, the previous result generalizes to[20]

$$V_{(r)}^{(\text{"ret."})} = -(\hbar c/4\pi)\,r^{-11}\gamma_{ij}^{(1)}\gamma_{k\ell}^{(2)}\,\cdot$$
$$\cdot\,[1116\,\delta_{jk}\,\delta_{\ell i} - 1683\,(\delta_{jk}\delta_{\ell 3}\delta_{i3} + \delta_{\ell i}\,\delta_{j3}\,\delta_{k3}) + 2574\,\delta_{i3}\,\delta_{j3}\,\delta_{k3}\delta_{\ell 3}\,]\,. \qquad (7)$$

(the third axis is along $\vec{r}$). In the London ("unretarded") case (i.e., at distances much smaller than the characteristic wave lengths in the spectrum but still larger than the dimensions of the bodies) one finds [21]

$$V^{(\text{"unret."})}(r) = -(\hbar/2\pi)\, r^{-6} \int_0^\infty d\omega\, (\omega/c)^4 \gamma_{ij}^{(1)}(i\omega)\, \gamma_{k\ell}^{(2)}(i\omega) \cdot$$

$$\cdot (-\delta_{jk} + 3\delta_{j3}\delta_{k3})(-\delta_{\ell i} + 3\delta_{\ell 3}\delta_{i3}).$$

Like all the other v.d.W. forces, those reported here should act irrespective of whether the bodies are macroscopic or microscopic; consequences of these results for intermolecular forces in biomolecular physics or in other applications [22] may be worth studying.

We end with the remark that if one allows for P, T-violations, the following mixed polarizabilities have to be introduced

$$f_{ij}(\omega) = i \int dt\, e^{i\omega t} \theta(t) \langle [M_i(t), T_j(0)] \rangle,$$

$$g_{ij}(\omega) = i \int dt\, e^{i\omega t} \theta(t) \langle [Q_i(t), T_j(0)] \rangle,$$

besides

$$b_{ij}(\omega) = i \int dt\, e^{i\omega t} \theta(t) \langle [Q_i(t), M_j(0)] \rangle.$$

already discussed in ref. [23]. There will be a T-violating piece in the low energy spin independent Compton amplitude $T_{mn}^{(QT+TQ)} = i\omega'\omega(\omega'-\omega) g(0)\delta_{mn}$ coming from the scalar part $g(\omega)\delta_{ij}$ of $g_{ij}(\omega)$ ($g(0)$ = real in general and vanishes if there is T-invariance). In addition to the P-violating v.d.W. forces of ref. [23], there will be also the new ones:

$$V_v^{(\text{"ret."})}(\vec{r}) = -(81/4\pi)\, \vec{n} \cdot (\vec{S}_1 \times \vec{S}_2)\, r^{-10} [\alpha_v'^{(1)}(0) f_v'^{(2)}(0) + f_v'^{(1)}(0)\, \alpha_v'^{(2)}(0)],$$

$$V_t^{(\text{"ret."})}(\vec{r}) = (8\pi)^{-1} \epsilon_{ms\ell}\, n_s (35\delta_{ik} - 63 n_i n_k) \cdot$$

$$\cdot K_{im}^{(1)} K_{\ell k}^{(2)} r^{-8} [\alpha_t^{(1)}(0) f_t^{(2)}(0) + f_t^{(1)}(0)\, \alpha_t^{(2)}(0)],$$

where $\vec{n} = \vec{r}/r$;

$$f_{ij}(\omega) = \delta_{ij} f_s(\omega) + i\epsilon_{ijk} S_k f_v(\omega) + K_{ij} f_t(\omega);$$

$$\alpha_{ij}(\omega) = \delta_{ij} \alpha_s(\omega) + i\epsilon_{ijk} S_k \alpha_v(\omega) + K_{ij} \alpha_t(\omega);$$

$$K_{ij} \equiv S_i S_j + S_j S_i - 2\delta_{ij} S(S+1)/3,$$

$\vec{S}$ = angular momentum,

$$\alpha_v'(0) = [d\alpha_v(\omega)/d\omega]_{\omega=0}, \quad f_v'(0) = [df_v(\omega)/d\omega]_{\omega=0};$$

$f_{s,v,t}$ are real by T-invariance.

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3. If in Eq. (2) one passes from $\vec{j}$ to the linear current J ($\vec{j}d^3x = Jd\vec{x}$), the classical expression of T_i for a pair of coplanar circular currents of equal radius r_T (one circulating clockwise, the other anti-clockwise) is (frame independently) $\vec{T} = \vec{n} J\pi r_T^2 R_T$ (R_T is the distance between the centers of the circles, $R_T > 2r_T$, and the unit vector $\vec{n}$ is along the symmetry axis, pointing in the (common) direction of the two currents at their closest position. The toroid d.m. of a toroidal current (with an even number of turns of winding N, small radius r_T and large radius R_T) is then easily found to be $\vec{T}$(Torus) = $\vec{n} NJV_T/4\pi$ (V_T is the volume of the torus) and so one recovers the value found in the last of refs. [2].
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9. For higher spins, the electromagnetic vertex of a Majorana fermion is richer but the anapole-type structure is always

present. In the spin 3/2 case (gravitino, for instance), supposing only CPT-invariance, we find

$$<\frac{3}{2};p+q|j^{\mu}(0)|\frac{3}{2};p> = i\bar{u}_{\rho}\{A(q^2)[q^2\gamma^{\mu}-(q\cdot\gamma)q^{\mu}]\gamma_5 g^{\rho\sigma} +$$

$$+[B(q^2)+iC(q^2)\gamma_5][q^{\mu}q^{\rho}q^{\sigma}-\frac{1}{2}q^2(g^{\mu\rho}q^{\sigma}+g^{\mu\sigma}q^{\rho})]\}u \quad ,$$

where apart from the anapole (toroid dipole) form factor A, appear also the toroid quadrupole and octupole form factors B, C. With the expression of the quadrupole toroid moment /2/

$$T_{ik} = (28)^{-1}\int[4x_i x_k x_{\ell} - 5\vec{x}^2(x_i\delta_{k\ell}+x_k\delta_{i\ell})+2\vec{x}^2 x_{\ell}\delta_{ik}]j_{\ell}(\vec{x},t)d^3x ,$$

a simple calculation shows that indeed

$$<\vec{p}=0;\ S_z=+\frac{3}{2}|T_{ik}|\vec{p}=0;\ S_z=+\frac{3}{2}>/V = B(q^2=0)\,\mathrm{diag}(\frac{1}{2},\frac{1}{2},-1).$$

10. Au C.K., Feinberg G. Phys.Rev., 1972, A6, p.2433.
11. To get an intuitive picture of the ana-polarization phenomenon, one may envisage an ensemble of randomly distributed and oriented small toroidal currents immersed in a large electrolyte. When a current density $\vec{j}^{\,ext}$ is flowing through the electrolyte, each of the small toroidal currents will tend to get aligned on the direction of $\vec{j}^{\,ext}$ and the ensemble will get so a non zero total toroid moment.
12. This result may be found also by classical electrodynamics considerations taking into account that the classical (induced) toroid dipole radiates like $\dddot{T}$.
13. An expansion of σ valid to the ω^5 order has been given in (a). Guiasu I., Radescu E.E. Phys.Rev., 1978, D18, p.651; (b) Ann.Phys.(N.Y.), 1979, 120, p.145, but no attempt was made to establish a complete contact with multipole characteristics of target's charge and current distributions. Eqs.(5), (6) furnish the necessary basis for the achievement of such a purpose and represent, in this respect, an extension (to the next relevant frequency order) of the work done by V.A.Petrukhin. Zh.Exp.Teor.Fiz., 1961, 40, p.1148; Proc.Lebedev Phys.Inst., 1968, 41, p.165.
14. This example is not too exotic after all; the pion Compton effect has by now been detected (a) Aibergenov T.A. et al. Kratkie Soobschenia po Fiz., 1982, 5, p.33 (Lebedev Inst. Short Communications, Moscow); (b) Antipov Yu.M. et al. Phys.Lett., 1983, 121B, p.445; (c) Kowalewski R.V. et al. Phys.Rev., 1984, D29, p.1000. and the pion's electric dipole polarizability has been experimentally extracted ref. (b) above and Aibergenov T.A. et al. Kratkie Soobschenia po Fiz., 1984, 6, p.31 .

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16. See numerical estimates in ref. /13/(b).
17. For comprehensive reviews on the usual v.d.W. forces see Barash Yu.S., Ginzburg V.L. Usp.Fiz.Nauk, 1975, 116, p.5; ibid, 1984, 143, p.345 (Sov.Phys.Usp.).
18. The easiest way of getting this potential is to use the dispersion method of Feinberg G., Sucher J. Phys.Rev., 1970, A2, p.2395.
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20. Eq. (7) may be found using, e.g., the method of McLachlan A.D. Proc. Royal Soc., 1963, A271, p.387.
21. Despite some formal resemblance, this Equation does not represent a higher derivative effect in the usual London case since, we emphasize again, γ_{ij} and $\dot{a}_{ij}$ describe very different properties of the system.
22. Eq. (7) leads, e.g., to the conclusion that between two perfectly conducting closed toroidal solenoids (with no macroscopic current circulating through them), there should be a long range (r^{-12}-type) force simply on account of vacuum fluctuations; specifying $\gamma_{ij}^{(1),(2)}$ for each of the two solenoids (in their respective proper frames) as $\gamma_{ij}^{(1),(2)} = \mathrm{diag}(0,0,\gamma_{||}^{(1),(2)})$, where the corresponding z-axes are directed along the axes of the tori, Eq.(7) becomes

$$V(r;\theta_1,\theta_2,\phi) = -(\hbar c/8\pi)\,r^{-11}\gamma_{||}^{(1)}\gamma_{||}^{(2)}(162\cos^2\theta_1\cos^2\theta_2 -$$

$$-567\sin\theta_1\cos\theta_1\sin\theta_2\cos\theta_2\cos\phi + 558\sin^2\theta_1\sin^2\theta_2\cos^2\phi)$$

(θ_1, θ_2 are the angles between the axes of the tori and the line $\vec{r}$ uniting their centres while ϕ is the angle between the planes formed by $\vec{r}$ with each of the two axes) and it remains to evaluate $\gamma_{||}^{(1),(2)}$; a rough estimate leads to $\gamma_{||} = -V_T R_T^2/16\pi$ ($R_T \gg r_T$; to recall notations, see footnote 3). Preliminary evaluations indicate that this effect should persist for real solenoids (with non-vanishing resistivity ρ) as well. Then $\gamma_{||}(i\omega) \approx (\omega/c)\delta_{||}$ with $\delta_{||} = NV_T R_T r_T s/16\pi c\rho$ and (for $r\to\infty$) the potential $V(r) \sim \delta_{||}^{(1)}\delta_{||}^{(2)} r^{-13}$ would depend, this time, on the numbers $N^{(1)}$, $N^{(2)}$ of turns of winding in the solenoids (s = wire's cross-section).
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Наведенные анапольные /тороидные/ моменты и физические эффекты, связанные с новым типом поляризуемости

Показано, что физические системы, не обладающие собственными анапольными /тороидными/ моментами, в присутствии внешнего тока могут приобрести соответствующие индуцированные моменты без сопутствующего нарушения Р- и Т-инвариантности. Изучается роль возникающей таким образом новой поляризуемости γ в комптоновском рассеянии и демонстрируется, что она ведет к дополнительным ван-дер-ваальсовым силам с потенциалом $V(r) = -(639/4\pi)\gamma_1\gamma_2 r^{-11}$. Оценивается дипольная γ-поляризуемость заряженного пиона. Вводятся новые тороидные поляризуемости недиагонального типа и коротко обсуждается их роль в некоторых эффектах с нарушением Р- и Т-инвариантности.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Radescu E.E. E4-85-165

Induced Anapole (Toroid) Moments, a New Type of Polarizability and Some Related Effects

Physical systems not possessing intrinsic Zeldovich's anapole moments may well acquire induced ones in the presence of an external current even if there is no P, T-violation. A new type of polarizability γ so emerges which manifests itself in the fourth frequency order Compton scattering and leads to new van der Waals forces with the potential $V(r) = -(639/4\pi)\gamma_1\gamma_2 r^{-11}$. The γ-polarizability of the charged pion is estimated. Other (mixed) toroid polarizabilities are introduced and their role in some P, T-violating effects is briefly noted.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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