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FOUR-BODY WAVE FUNCTION OF π^3He-SYSTEM AT THE THRESHOLD ENERGY

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INTRODUCTION

In spite of a significant success achieved in the quantum few-body problem, the description of πA-interactions on the basis of exact many-body equations is restricted at present by the consideration only of the simplest case, namely, πd-system[1].

The existing microscopic approximate methods such as the coupling-constant-evolution treatment[2], the Δ-hole model[3], the variational approach[4], the nuclear-Hamiltonian finite-rank-approximation method[5,6], the multiple-scattering theory[7] and others used for the investigation of pion interactions with three-nucleon and more heavier nuclei in many cases sufficiently correctly describe the πA-dynamics. However, these methods demand a considerable computational work. That is why in the calculations where the πA-dynamics is only a part of any large problem one uses, as a rule, some highly simplified methods. As the most commonly used of them there is the distorted-wave method based on a phenomenological optical πA-potential[8]. One of the substantial assumptions of that method consists in the representation of the whole many-body πA-wave-function as a product of the two-body relative-motion πA-function and the nuclear function. Such a factorization means that the nucleon motion inside the nucleus is assumed negligibly perturbed by the presence of the pion. At high pion energies when the interaction time is significantly less than the nucleon mean-free-path time inside the nucleus those perturbations are likely small. But at low energies, for example, in the processes of threshold pion photoproduction and absorption of stopped pions by nuclei there are no sufficient reasons to assume the smallness of the perturbations. A theoretical analysis of these processes involving lightest nuclei is especially interesting in view of the meson-current and other degree-of-freedom investigations. However final conclusions may be done only just after a sufficiently exact description of the pure potential part of the problem.

In the present work, using the method of the nuclear-Hamiltonian finite-rank-approximation, we calculate (without inclusion of the Coulomb forces) the whole 4-body wave function of $\pi(3N)$-system at zero pion-kinetic-energy and explore how significantly pion distorts the wave-function of $(3N)$-subsystem. Moreover, we present the calculated 4-body function in a semianalytical form which enables one to use it in different calculations.

1. FORMALISM

We write the total π(3N)-Hamiltonian as the sum $H = H_0 + V + H_A$. Here H_0 describes the free relative motion of pion and the c.m. of the (3N)-subsystem governed by the Hamiltonian H_A, $V = \sum_{i=1}^{3} V_i$ is the sum of πN-potentials. Further we will need the following Green functions:

$$G(z) = (z - H)^{-1}, \quad G_A(z) = (z - H_0 - H_A)^{-1}, \quad G_0(z) = (z - H_0)^{-1}.$$

The π(3N)-scattering amplitude is the asymptotic state average of the operator $T = V + VGV$, which obeys the equation[5]

$$T = \tau + \tau G_0 H_A G_A T, \tag{1}$$

where τ is a solution of the auxiliary equation

$$\tau = V + V G_0 \tau. \tag{2}$$

As is well-known[7], eq. (2) describes scattering in the fixed-scatterer-approximation (FSA).

We stress that the couple of equations (1,2) is an exact formulation of the problem. The introduction of the FSA-amplitude is only a convenient formal trick. Obtaining τ and then correctly solving eq.(1) one may, in principle, find an exact solution even for such a problen where FSA is certainly inapplicable. However, the presence of the complicated many-body operators H_A and G_A makes it impossible to solve eq.(1) without any approximation. Following the proposal of ref.[6] we approximate the Hamiltonian H_A by the first term of its spectral expansion

$$H_A \simeq \epsilon_0 |\phi\rangle\langle\phi|, \tag{3}$$

where ϵ_0 is the energy of the bound state $|\phi\rangle$ of the 3-nucleon system. A sufficient condition of applicability of such an approximation is the absence (at a given collision energy) of open nucleus-breakup channels[6].

Therefore, investigating π(3N)-scattering at the threshold energy (i.e., at zero relative π(3N)-motion kinetic-energy) we may perfectly use the approximation (3). Then from eq.(1) we obtain for the elastic amplitude $\langle \vec{k}, \phi | T | \vec{k}, \phi\rangle$ a rather simple integral (over variable $\vec{k}$) equation[5]

$$\langle T\rangle = \langle\tau\rangle + \langle\tau\rangle G_0(z)\epsilon_0 G_0(z - \epsilon_0)\langle T\rangle, \tag{4}$$

where the brackets $\langle\ \rangle$ denote the average over $|\phi\rangle$.

Solving the elastic scattering problem (2,4) we can obtain the corresponding wave function by means of the equation

$$|\psi_{\vec{p}}\rangle = \lim_{\epsilon \to 0+} i\epsilon\, G(\epsilon_0 + \frac{p^2}{2\mu} + i\epsilon)\,|\vec{p}, \phi\rangle.$$

Representing the full propagator in the form $G = G_A + G_A T G_A$ and taking into account that the asymptotic state $|\vec{p}, \phi\rangle$ (where $\vec{p}$ is a pion momentum in the 4-body c.m.-frame) is an eigenstate of $(H_0 + H_A)$ we arrive at the following exact expression for the π(3N)-wave-function

$$|\psi_{\vec{p}}\rangle = |\vec{p}, \phi\rangle + G_A T |\vec{p}, \phi\rangle. \tag{5}$$

Now we use approximation (3) to rewrite the propagator $G_A = G_0 + G_0 H_A G_A$ as follows

$$G_A(z) = G_0(z) + \epsilon_0 G_0(z)|\phi\rangle\langle\phi| G_0(z - \epsilon_0)$$

and to obtain from (5) the following approximate expression

$$|\psi_{\vec{p}}\rangle = |\vec{p}, \phi\rangle + G_0(z)T(z)|\vec{p}, \phi\rangle + |\phi\rangle\epsilon_0 G_0(z) G_0(z - \epsilon_0) \times \\ \times \langle\phi|T(z)|\vec{p}, \phi\rangle, \tag{6}$$

where $z = E_p + \epsilon_0 + i0$, $E_p = p^2/2\mu$.

Let $\vec{\xi} = (\vec{r}, \vec{\rho})$ be a set of the Jacobi coordinates of 3N-subsystem $\vec{r} = \vec{\zeta}_1 - \vec{\zeta}_2$, $\vec{\rho} = 3/2\, \vec{\zeta}_3$, where $\vec{\zeta}_i$ is the radius-vector of an i-th nucleon with respect to the 3N-c.m. The four-body wave function (6) in the mixed representation is

$$\langle\vec{k}, \vec{\xi}|\psi_{\vec{p}}\rangle = \langle\vec{k}, \vec{\xi}|\vec{p}, \phi\rangle + \langle\vec{k}, \vec{\xi}|T|\vec{p}, \phi\rangle(z - E_k)^{-1} + \\ + \epsilon_0 \phi(\vec{\xi})\langle\vec{k}, \phi|T|\vec{p}, \phi\rangle(z - E_k)^{-1}(z - \epsilon_0 - E_k)^{-1}. \tag{7}$$

Besides the elastic amplitude eq.(7) involves a matrix $\langle\vec{k}, \vec{\xi}|T|\vec{p}, \phi\rangle$ for which, analogously to eq.(4), we obtain

$$\langle\vec{k}, \vec{\xi}|T|\vec{p}, \phi\rangle = \phi(\vec{\xi})\langle\vec{k}|\tau(\vec{\xi})|\vec{p}\rangle + \\ + \epsilon_0\phi(\vec{\xi})\int\frac{d\vec{q}}{(2\pi)^3}\frac{\langle\vec{k}|\tau(\vec{\xi})|\vec{q}\rangle}{z - E_q}\frac{\langle\vec{q}, \phi|T|\vec{p}, \phi\rangle}{z - \epsilon_0 - E_q}. \tag{8}$$

Here we have used the diagonality of the matrix τ with respect to states $|\vec{\xi}\rangle$, i.e., $\langle\vec{\xi}'|\tau|\vec{\xi}\rangle = \delta(\vec{\xi}' - \vec{\xi})\tau(\vec{\xi})$.

Adding and subtracting the term

$$\phi(\vec{\xi})\, f_{\vec{p}}(\vec{k}) \equiv \phi(\vec{\xi}) \langle \vec{k}, \phi | T | \vec{p}, \phi \rangle (z - \epsilon_0 - E_k)^{-1},$$

in the right-hand side of eq.(7) we arrange that expression as follows

$$\langle \vec{k}, \vec{\xi} | \psi_{\vec{p}} \rangle = A_{\vec{p}}(\vec{k}, \vec{\xi}) + B_{\vec{p}}(\vec{k}, \vec{\xi}), \qquad (9)$$

$$A_{\vec{p}}(\vec{k}, \vec{\xi}) = [(2\pi)^3 \delta(\vec{k} - \vec{p}) + f_{\vec{p}}(\vec{k})]\, \phi(\vec{\xi}), \qquad (10)$$

$$B_{\vec{p}}(\vec{k}, \vec{\xi}) = [\langle \vec{k}, \vec{\xi} | T | \vec{p}, \phi \rangle - \phi(\vec{\xi}) \langle \vec{k}, \phi | T | \vec{p}, \phi \rangle](z - E_k)^{-1}. \qquad (11)$$

The first, factorized in $\vec{k}$ and $\vec{\xi}$, term of eq.(9) corresponds to the usual representation of the full wave-function in the distorted-wave method. That term describes the scattering process without changing the initial (3N)-configuration. The presence of the pion inside the nuclear subsystem apparently distorts the nucleon-motion. The term $B_{\vec{p}}$ describes that distortion, i.e., the scattering going through changed (3N)-configurations. Really, multiplying eq.(11) by $\phi^*(\vec{\xi})$ and integrating it over $\vec{\xi}$, one easily verifies that the function $B_{\vec{p}}(\vec{k}, \vec{\xi})$ is orthogonal to $\phi(\vec{\xi})$. Consequently, function (9) takes account of the continuous spectrum of (3N)-subsystem. At the first sight this seems to contradict the used approximation (3). But we search the amplitude in the form $T = \tau + \theta$, where θ is the integral term of eq.(1), and get the approximation (3) only to obtain θ. The part of the problem (i.e., eq.(2)) will be solved exactly with inclusion of all the (3N)-configurations.

2. SCHEME OF THE WAVE FUNCTION CALCULATION AND RESULTS

Having in mind that we restrict ourselves to the threshold π(3N)-state consideration, let us search only the S-wave component of wave function (9). For the same reason we assume that the pion interacts with a nucleon only in two channels, namely S_{11} and S_{31}, and the state $|\phi\rangle$ has only the S-component.

To solve eq.(2), we define the following 4-particle basis states

$$|k, \xi, (t_\pi((t_1 t_2) t_{12} t_3)\, t) I I_z \rangle \equiv |k, \xi, \alpha, I, I_z\rangle, \qquad (12)$$

where $\xi = (r, \rho)$, and the isospins are coupled according to the following scheme $\vec{I} = \vec{t}_\pi + ((\vec{t}_1 + \vec{t}_2) + \vec{t}_3)$. In (12) we don't write the angular variables because all the orbital ones are equal to

zero and, as a result, operators τ and V don't depend on nucleon spins.

Taking into account the diagonality of eq.(2) with respect to ξ and I we write it in the form

$$\langle k', \alpha' | \tau^I(\xi) | k, \alpha \rangle = \langle k', \alpha' | V^I(\xi) | k, \alpha \rangle + \frac{1}{2\pi^2} \sum_{\alpha''} \int_0^\infty dq\, q^2 \langle k', \alpha' | V^I(\xi) | q, \alpha'' \rangle \frac{\langle q, \alpha'' | \tau^I(\xi) | k, \alpha \rangle}{z - E_q}. \qquad (13)$$

Let us express the many-body operater V in terms of the channel two-body potentials v_μ, operators P_μ^i of projecting of a πN_i-state onto a μ-th channel and the space-translation operators $Q_i = \exp(i \vec{\zeta}_i \hat{\vec{K}})$ shifting the 3N-c.m. by the vector $\vec{\zeta}_i$ (here $\hat{\vec{K}}$ is the operator corresponding to the momentum $\vec{k}$)

$$V = \sum_{i=1}^{3} \sum_{\mu=1}^{2} Q_i v_\mu P_\mu^i Q_i^+ .$$

Expressing $\vec{\zeta}_i$ in terms of $\vec{r}$ and $\vec{\rho}$ we obtain for the S-wave projection of the operators Q_i the following

$$Q_i^S(k, \xi) = j_0\left(\frac{kr}{2}\right) j_0\left(\frac{k\rho}{3}\right), \quad i = 1, 2, \quad Q_3^S(k, \xi) = j_0\left(\frac{2}{3} k\rho\right).$$

Therefore, we have

$$\langle k', \alpha' | V^I(\xi) | k, \alpha \rangle = \sum_{i=1}^{3} \sum_{\mu=1}^{3} Q_i^S(k', \xi)\, Q_i^S(k, \xi)\, v_\mu(k', k) \langle \alpha' I I_z | P_\mu^i | \alpha I I_z \rangle . \qquad (14)$$

The calculation of matrix elements of projection operators consists in the recoupling of the isospins

$$\langle \alpha' I I_z | P_\mu^1 | \alpha I I_z \rangle = (-1)^{3(t'+t)+1}\, \hat{t}_\mu^2\, \hat{t}'_{12}\, \hat{t}_{12}\, \hat{t}'\, \hat{t} \; \times$$

$$\times \sum_{t_{23}} (-1)^{t'_{23} + t_{23}}\, \hat{t}_{23}^2 \begin{Bmatrix} t_\pi & I & t' \\ t_{23} & t_1 & t_\mu \end{Bmatrix} \begin{Bmatrix} t_\pi & I & t \\ t_{23} & t_1 & t_\mu \end{Bmatrix} \begin{Bmatrix} t_{23} & t_1 & t' \\ t'_{12} & t_3 & t_2 \end{Bmatrix} \begin{Bmatrix} t_{23} & t_1 & t \\ t_{12} & t_3 & t_2 \end{Bmatrix},$$

$$\langle \alpha' I I_z | P_\mu^2 | \alpha I I_z \rangle = (-1)^{3(t'+t)-(t'_{12}+t_{12})+1}\, \hat{t}_\mu^2\, \hat{t}'_{12}\, \hat{t}_{12}\, \hat{t}'\, \hat{t} \; \times$$

$$\times \sum_{t_{31}} \hat{t}_{31}^{2} \begin{Bmatrix} t_\pi & I & t' \\ t_{31} & t_2 & t_\mu \end{Bmatrix} \begin{Bmatrix} t_\pi & I & t \\ t_{31} & t_2 & t_\mu \end{Bmatrix} \begin{Bmatrix} t_{31} & t_2 & t' \\ t'_{12} & t_3 & t_1 \end{Bmatrix} \begin{Bmatrix} t_{31} & t_2 & t \\ t_{12} & t_3 & t_1 \end{Bmatrix},$$

$$\langle \alpha' I I_z | P^3_\mu | \alpha I I_z \rangle = (-1)^{2(t_\pi + I) + t' + t} \, \hat{t}_\mu^2 \hat{t}' \hat{t} \, \times$$

$$\times \begin{Bmatrix} t_\pi & t_3 & t_\mu \\ t_{12} & I & t' \end{Bmatrix} \begin{Bmatrix} t_\pi & t_3 & t_\mu \\ t_{12} & I & t \end{Bmatrix},$$

where $\hat{\sigma} = \sqrt{2\sigma + 1}$, t_μ is the πN-isospin in the channel μ, t_{23} and t_{31} are the isospins of the corresponding nucleon pairs.

As $v_\mu(k', k)$ we use the separable potential

$$v_\mu(k', k) = \lambda_\mu h_\mu(k') h_\mu(k), \quad h_\mu(k) = (k^2 + \beta_\mu^2)^{-1}.$$

Defining the functions

$$g_{n\mu}(k, \xi) = h_\mu(k) Q^{s}_{n+1}(k, \xi), \quad n = 1, 2$$

and the matrix

$$L^{n\mu}_{\alpha'\alpha} = \lambda_\mu \langle \alpha', I, I_z | (P^1_\mu + P^2_\mu) \delta_{n1} + P^3_\mu \delta_{n2} | \alpha, I, I_z \rangle, \quad n = 1, 2$$

we write eq.(14) in the form

$$\langle k', \alpha' | V^I(\xi) | k, \alpha \rangle = \sum_{n\mu} g_{n\mu}(k', \xi) L^{n\mu}_{\alpha'\alpha} g_{n\mu}(k, \xi).$$

A solution of eq. (13) will be searched in the form

$$\langle k', \alpha' | \tau^I(\xi) | k, \alpha \rangle = \sum_{n'\mu' n\mu} g_{n'\mu'}(k', \xi) W^{n'\mu' n\mu}_{\alpha'\alpha}(\xi) g_{n\mu}(k, \xi). \tag{15}$$

Then for the coefficients W we obtain the system of linear algebraic equations

$$W^{n'\mu' n\mu}_{\alpha'\alpha} = L^{n\mu}_{\alpha'\alpha} + \sum_{n''\mu''\alpha''} L^{n'\mu'}_{\alpha'\alpha''} J_{n'\mu' n''\mu''} W^{n''\mu'' n\mu}_{\alpha''\alpha}, \tag{16}$$

where

$$J_{n'\mu' n\mu}(\xi, z) = \frac{1}{2\pi^2} \int_0^\infty dk \frac{k^2}{z - E_k} g_{n'\mu'}(k, \xi) g_{n\mu}(k, \xi).$$

The dimension of the matrix W is $\bar{n} \cdot \bar{\mu} \cdot \bar{\alpha} = 2 \cdot 2 \cdot 4 = 16$. The S-component of a three-nucleon nucleus wave function has the form $\phi(\xi)(\chi''\eta' - \chi'\eta'')/\sqrt{2}$, where η', and η'' are the isospin functions with $t_{12} = 0$ and $t_{12} = 1$, and χ', χ'' are the analogous spin functions. Using the independence of τ on the spin states and the orthogonality of χ' and χ'' we find that the amplitude τ^I averaged over the nuclear function has the form

$$\tau^I(k', k) = \int_0^\infty dr \, d\rho \, |r\rho\phi(r, \rho)|^2 \, \tau^I(k', k, \xi),$$

where

$$\tau^I(k', k, \xi) = \frac{1}{2} \sum_{t_{12}} \langle k', \alpha | \tau^I(\xi) | k, \alpha \rangle. \tag{17}$$

The corresponding full amplitude obeys the equation

$$T^I(k, p) = \tau^I(k, p) + \epsilon_0 \int_0^\infty \frac{dq}{2\pi^2} q^2 \frac{\tau^I(k, q)}{z - E_q} \cdot \frac{T^I(q, p)}{z - \epsilon_0 - E_q} \tag{18}$$

which is a specific form of eq.(4).

For the S-component of the full wave function (9) we obtain from (8-11) the expressions

$$\psi^I_p(k, \xi) = A^I_p(k, \xi) + B^I_p(k, \xi), \tag{19}$$

$$A^I_p(k, \xi) = [2\pi^2 \delta(k - p)/k^2 + f^I_p(k)] \, \phi(\xi), \tag{20}$$

$$B^I_p(k, \xi) = [T^I(k, p, \xi) - \phi(\xi) T^I(k, p)]/(z - E_k), \tag{21}$$

$$T^I(k, p, \xi) = \phi(\xi) \left[\tau^I(k, p, \xi) + \frac{\epsilon_0}{2\pi^2} \int_0^\infty dq \, q^2 \frac{\tau^I(k, q, \xi) T^I(q, p)}{(z - E_q)(z - \epsilon_0 - E_q)} \right], \tag{22}$$

which are just the formulae we have used for the calculations. To obtain numerical results, we use πN-potential range-parameters $\beta_{1/2} = \beta_{3/2} = 2.629$ fm^{-1} [9] which permit a satisfactory description both for πN [9] and for πd [10] and π^3He [11] elastic scattering. The depth-parameters λ were chosen to reproduce the experimental πN-scattering-lengths [12] $a^{1/2}_{\pi N} = -0.257$ fm, $a^{3/2}_{\pi N} = 0.154$ fm. As $\phi(\xi)$ we use the Irving function [13]

$$\phi(\xi) = N \exp(-\kappa\eta), \quad \kappa = 0.5745 \text{ fm}^{-1}, \quad \eta^2 = \frac{3}{2} r^2 + \frac{1}{2} \rho^2.$$

For the energy ϵ_0 we take its experimental value -7.718 MeV.

Equation (18) will be solved for the threshold energy $p = 0$, $E = \epsilon_0$ by a standard method. We replace the upper limit of the

Table

i	$-T_i^{1/2}\cdot 10^6$	$T_i^{3/2}\cdot 10^6$	i	$T_i^{1/2}\cdot 10^6$	$-T_i^{3/2}\cdot 10^6$
0	275306	910263	13	-175	-204
1	275091	909449	14	48	87
2	269494	888129	15	64	62
3	243302	789330	16	62	55
4	186520	581018	17	58	52
5	118287	332879	18	48	46
6	56992	146841	19	45	41
7	23492	51573	20	39	35
8	8629	15489	21	38	31
9	3119	4782	22	37	29
10	854	350	23	34	27
11	488	233	24	29	26
12	466	209	25	26	24

integral by a sufficiently great value $b = 12\ fm^{-1}$. This choice is caused by the fact that the kernel of eq.(18) decreases by six orders with increasing k, q from 0 to 6 fm^{-1}.

On the segment $[0, b]$ we construct the net

$$\Delta_k :\ 0 = k_0 < k_1 < \ldots < k_{N-1} < k_N = b, \qquad (23)$$

where $k_1, \ldots, k_{N-1}$ are the Gauss knots $(N-1 = 24)$. Eq. (18) written for inner knots of net (23) transforms (after replacing the integral by the Gauss sum) into a system of linear equations solved by the usual method of matrix inversion.

Values $T_i^I = T^I(k_i, 0)$ thus computed are represented in the Table. The numbers T_0^I are proportional to $\pi\,^3$He-scattering-lengths in the corresponding isotopic states

$$a^{1/2}_{\pi\,^3He} = \frac{\mu}{2\pi}\, T_0^{1/2} = -0.0876\ fm, \quad a^{1/2}_{\pi\,^3He} = \frac{\mu}{2\pi}\, T_0^{3/2} = 0.2895\ fm.$$

Further, the function $T(k, 0)$ is interpolated on the segment $[0, b]$ by the cubic spline which is a function $S(k) \in C^2_{[0,b]}$ obeying the equations $S(k_i) = T_i$, $i = 0, \ldots, N$ and being a cubic polynomial on each segment $[k_i, k_{i+1}]$ /14/

$$S(k) = \psi_i(t)\,(T_i,\ T_{i+1},\ m_i,\ m_{i+1})^T,$$

where $t = (k - k_i)/h_i$, $h_i = k_{i+1} - k_i$ and elements of the row are as follows

$$\psi_{i1}(t) = 2t^3 - 3t^2 + 1, \quad \psi_{i2}(t) = -2t^3 + 3t^2,$$
$$\psi_{i3}(t) = h_i\, t\,(1-t)^2, \quad \psi_{i4}(t) = h_i\,(t^3 - t^2). \qquad (24)$$

Such a representation guarantees the continuity of the first derivatives of $S(k)$ everywhere on $[0, b]$. The continuity condition of the second derivatives at the inner points of the net $S''(k_i - 0) = S''(k_i + 0)$ is expressed by $(N-1)$ equations for the unknown coefficients $m_i = S'(k_i)$

$$\lambda_i m_{i-1} + 2m_i + \mu_i m_{i+1} = 3[\mu_i h_i^{-1}(T_{i+1} - T_i) + \lambda_i h_{i-1}^{-1}(T_i - T_{i-1})], \qquad (25)$$

where $\mu_i^{-1} = 1 + h_i h_{i-1}^{-1}$, $\lambda_i = 1 - \mu_i$, $i = 1, \ldots, N-1$. The partial derivatives $\partial/\partial k$, $\partial/\partial k'$, $\partial^2/\partial k \partial k'$ of functions (15), (17) are equal to zero on the lines $k = 0, \infty$, $k' = 0$, and $k' = \infty$, consequently, from eq. (18) we have $T_k(k, 0) = 0$, $k = 0, \infty$ which gives us the boundary conditions $m_0 = m_N = 0$ for the spline building.

Solving eq. (25) we obtain

$$m_i = 3 \sum_{j=0}^{N} B_{ij} T_j, \quad i = 1, \ldots, N-1, \quad m_0 = m_N = 0, \qquad (26)$$

where the matrix B is completely defined only by the choice of net (23) and is as follows

$$B_{ij} = h_{j-1}\bar{\delta}_{j0}[\mu_{j-1}\Gamma_{ij-1}\bar{\delta}_{j1} + (2-\lambda_j^{-1})\Gamma_{ij}\bar{\delta}_{jN}] + \lambda_{j+1} h_j^{-1}\Gamma_{ij+1}\bar{\delta}_{jN-1}\bar{\delta}_{jN},$$
$$i = 1, \ldots, N-1, \quad j = 0, \ldots N, \quad \bar{\delta}_{k\ell} = 1 - \delta_{k\ell}. \qquad (27)$$

Here Γ is the inverse matrix of system (25). For each value of ξ the function $\tau^I(k', k, \xi)$ (17) is computed at the knots of the two-dimensional net $\Delta_\omega = \Delta_{k'} \times \Delta_k$, where $\Delta_{k'}$ and Δ_k are nets of type (23). Taking the knot values $\tau_{ij} = \tau(k'_i, k_j, \xi)$, $i, j = 0, \ldots, N$ we interpolate that function on the square $\omega = \{k', k : 0 \le k' \le b,\ 0 \le k \le b\}$ by the bicubic spline $S(k', k) \in C^2_\omega$. On each cell $\omega_{ij} = [k'_i, k'_{i+1}] \times [k_j, k_{j+1}]$ such a spline is a bicubic polynomial $S(k', k) = \psi_i(t')\, F\, \psi_j^T(t)$, where the elements of the rows $\psi_i(t')$ and $\psi_j(t)$ are functions (24) of the variables $t' = (k' - k'_i)/h_i$ and $t = (k - k_j)/h_j$. The matrix F (4x4) contains coefficients which are the knot spline-values and the corresponding derivatives

$$F_{k\ell} = \tau_{nm} = S(k'_n, k_m), \quad F_{k\ell+2} = N_{nm} = S_{k'}(k'_n, k_m),$$

$$F_{k+2\ell} = M_{nm} = S_k(k'_n, k_m), \quad F_{k+2\ell+2} = K_{nm} = S_{k'k}(k'_n, k_m),$$

where $k = 1(2)$ for $n = i(i+1)$, $\ell = 1(2)$ for $m = j(j+1)$. Due to the equations

$$\tau_{k'}(k', k, \xi) = \tau_k(k', k, \xi) = \tau_{k'k}(k', k, \xi) = 0, \quad k', k = 0, \infty$$

all the coefficients K , N , M at the boundary knots equal zero, and the remaining ones are to be computed step by step by the formula

$$M_{ij} = 3\sum_{\ell=0}^{N} B_{j\ell}\tau_{i\ell}, \quad N_{ij} = 3\sum_{\ell=0}^{N} B_{i\ell}\tau_{\ell j}, \quad K_{ij} = 3\sum_{\ell=0}^{N} B_{i\ell}M_{\ell j}.$$

We calculate function (19) of the π^3He-system by formulae (19-22) where instead of $T^I(k, 0)$ and $\tau^I(k', k, \xi)$ we take the corresponding splines approximating that with the accuracy $O((\max_{i=0,\dots N-1} h_i)^4)$.

Further, carrying out the Fourier transformation of function (19) with respect to variable k, we obtain it in the (x, ξ)-representation, where x is a distance between the pion and the (3N)-c.m.

In the Figure the ratio is displayed for two parts of the π^3He-wave function $|B_0/A_0| \cdot 100\%$ as a function of x. This function is shown for different values of the nucleon coordinates (r, ρ).

As is seen, at $x \leq 2$ fm, that is, when the pion is placed inside the nucleus, the orthogonal addenda A_0 and B_0 are comparable with each other. Consequently, the factorization of the total function in that region is not correct.

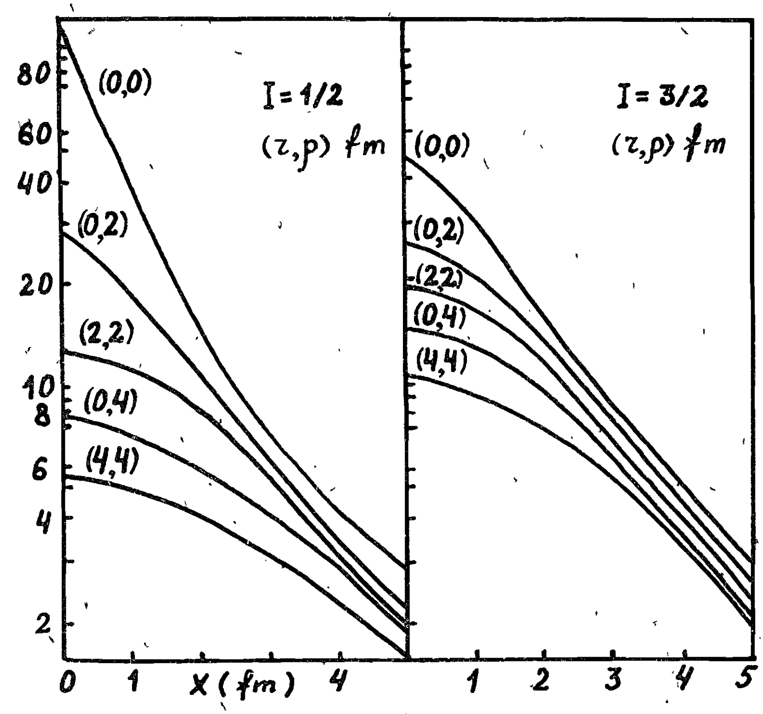
Formulae (15-22) give an opportunity of computing a correct (in the framework of approximation (3)) π^3He-wave function with modest computer-time consuming.

The computation process may be divided into five steps:

1) the solution of the set of linear equations (16) for the matrix W with dimension (16x16);

2) the construction of the FSA-amplitude $\tau^I(k', k, \xi)$ by formulae (15,17);

3) the calculation of $T^I(k, 0)$ by interpolating its knot values displayed in the Table (if one intends further to diffe-



rentiate the π^3He-function, then it is convenient to use a spline-interpolation);

4) the calculation of $T^I(k, 0, \xi)$ by formula (22);

5) the final calculation of $\psi^I(k, \xi)$ by formulae (19-21).

CONCLUSION

Therefore, the configuration of a 3-nucleon subsystem is significantly distorted due to the nucleon interaction with the pion when all the distances among the four particles are comparable with the size of ^{3}He-nucleus. As a result, calculations of the matrix elements of any operators which mainly act in the nucleus inner-region may denamd to take into account the distortions, i.e., to know the correct wave function.

The scheme presented is sufficiently simple for a correct description of the four-body-system dynamics in the framework of the used approximation (3).

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Четырехчастичная волновая функция π^3He-системы для пороговой энергии

На основе приближенных четырехчастичных уравнений вычислена волновая функция π^3He-системы при нулевой кинетической энергии движения пиона относительно ядра. Обнаружены сильные искажения волновой функции ядерной подсистемы, вносимые присутствием пиона, когда расстояния между всеми четырьмя частицами сравнимы с размером ^{3}He. Вычисленная 4-частичная функция представлена в полуаналитическом виде, что позволяет использовать ее в различных расчетах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Pupyshev V.V., Rakityansky S.A. E4-85-122

Four-Body Wave Function of π^3He -System at the Threshold Energy

On the basis of approximate 4-body equations the wave function of π^3He-system is calculated at zero kinetic energy of the pion. In the case when distances between all four particles are comparable with the nucleus size a strong distortion of the wave function of (3N)-subsystem caused by the presence of the pion is found. The calculated 4-body function is represented in a semianalytical form, which makes it possible to apply it in different calculations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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