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QUARTET S-WAVE PHASE SHIFTS IN THE nd-SCATTERING ABOVE THE DEUTERON BREAK-UP THRESHOLD IN THE APPROXIMATION LINEAR IN THE NN-INTERACTION RANGE

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1. The wave functions for the nd-scattering can be found by solving the Faddeev integral equations [1] in the momentum space. The kernels of these equations are expressed via the two-body t-matrices off the energy shell. These two-body t-matrices depend both on the initial and final momentum and on the energy. For solving the Faddeev equations are widely applied the methods which use an approximate factorization of the two-body t-matrices. These methods allow one to obtain from the Faddeev equations a set of one-dimensional integral equations for a three-particle system.

In the effective range approximation to the two-body interaction the t-matrix does not depend both on an initial and on a final momentum, so one can extremely easily obtain the one-dimensional equations for the momentum representation of the wave functions.

Skornyakov and Ter-Martirosyan (STM) considered the nd--scattering in the zero approximation on the NN-interaction range (ZANNIR). They obtained the one-dimensional integral equations for the nd-scattering amplitudes [2]. The kernels of the STM equations (STME) have a simple analytical structure which is originated just from the kinematics of the system of three free particles.

For the S-wave nd-scattering in the doublet state (the total spin equals $\mathcal{S} = 1/2$ and the isotopic spin of the system equals $T = 1/2$) the STME fail to have a unique solution [3,4] so do the equations in the approximation linear in the NN-interaction range (ZANNIR) [5].

For the quartet state ($S=3/2$, $T=3/2$) the solution both of the STME and of the equations in the ALNNIR is unique and thus it is possible to solve these equations numerically to find the nd-scattering phase shifts.

2. The integral equations in the approximation linear in the interaction range were obtained by Danilov [5]. In ref. [6] such equations for the quartet nd-scattering were obtained straightforward from the Faddeev equations, just the S-component of the nucleon-nucleon t-matrix in the triplet state having been taken into account.

(It would be remarked here, that for reason of the quartet state symmetry the set of the Faddeev equations degenerates into the single integral equation for this state).

For the S-component of the nucleon-nucleon t-matrix we might **apprehend** the following expression

$$t(k,p;z) \equiv t(z) = \frac{1}{-(\alpha+i\sqrt{z})+\frac{1}{2}\tau_0(\alpha^2+z)} . \tag{1}$$

This expression is well-known in the effective range theory of nucleon-nucleon interaction and is valid if

$$\alpha\tau_0 \ll 1 \quad \text{and} \quad \tau_0\sqrt{|z|} \ll 1 , \tag{2}$$

where α^2 (F^{-2}) is the deuteron binding energy and $\tau_0(F)$ is the effective range for the NN-interaction in the triplet state.

The substitution of expression (1) in the Faddeev integral equation for the quartet state should in fact allow one to reduce this equation to the one-dimensional integral equation in the momentum space for the function $\Psi(p,p_0)$, where p_0 is the incident neutron momentum. However expression (1) has an extra irrelevant pole at the energy $z=-\left(\frac{2}{\tau_0}-\alpha\right)^2$ which does not relate to any physical state in the two-nucleon system. The nucleon-nucleon t-matrix enters into the Faddeev equation as the function $t(z_p)$,where $z_p = E-(3/4)p^2$ and E is the energy of the three-nucleon system. So the straightforward use of eq. (1) is seen to be impossible in the relevant Faddeev equation. To obtain the correct equation linear in τ_0 one must make use of the following expression (instead of (1)):

$$t(z) = -\frac{1}{\alpha+i\sqrt{z}}\left[1+\frac{1}{2}\tau_0(\alpha-i\sqrt{z})\right] .$$

This expression is equivalent to (1) if conditions (2) are fulfilled and, on the other hand, does not contain the superfluous non-physical pole. Another way to tackle the problem in question is to split the function $\Psi(p,p_0)$ into two parts [5]

$$\Psi(p,p_0) = \psi_0(p,p_0)+\psi_1(p,p_0) , \tag{4}$$

where $\psi_0(p,p_0)$ is the solution of the problem at $\tau_0=0$ (i.e. ,the solution of the respective STM equation) and $\psi_1(p,p_0)$ is the correction due to the finite range of the NN-interaction.

We have followed here the second way.

3. For the quartet S-scattering of neutron with the initial momentum p_0 on deuteron the equations in the ALNNIR for the functions $\psi_0(p,p_0)$ and $\psi_1(p,p_0)$ are as folllows [6] :

$$\frac{3}{4}\cdot\frac{1}{\alpha-i\sqrt{z_p}}\,\psi_0(p,p_0)=-\frac{G_0(p,p_0)}{2pp_0}-$$

$$-\frac{1}{\pi}\int_0^\infty p'^2dp'\,\frac{G_0(p,p')}{pp'}\cdot\frac{1}{p'^2-p_0^2-i0}\,\psi_0(p',p_0);\tag{5}$$

$$\frac{3}{4}\cdot\frac{1}{\alpha-i\sqrt{z_p}}\,\psi_1(p,p_0)=\frac{3}{8}\,r_0\,\psi_0(p,p_0)-$$

$$-\frac{1}{\pi}\int_0^\infty p'^2dp'\,\frac{G_0(p,p')}{pp'}\cdot\frac{1}{p'^2-p_0^2-i0}\,\psi_1(p',p_0),$$

where

$$\frac{G_0(p,p')}{pp'}=\int_{-1}^{+1}\frac{dx}{p^2+p'^2+pp'x-z}\tag{6}$$

and

$$z=E+i0;\qquad z_p=z-(3/4)p^2.\tag{7}$$

$E=-\alpha^2+(3/4)p_0^2$ is the total energy of the system in units F^{-2}.

The function $\psi(p,p_0)$ in (4)is normalized by the condition

$$\psi(p_0,p_0)=\frac{1}{2ip_0}\cdot\left(e^{2i\delta}-1\right),\tag{8}$$

where δ is the nd-scattering S-phase shift in the quartet state.

The energy of the n+d system is positive $(E>0)$ above the deuteron break-up threshold. Therefore the value of $G_0(p,p')$ is complex and one can split $G_0(p,p')$ into the real and imaginary part

$$G_0(p,p')=G^{(1)}(p,p')+i\pi G^{(2)}(p,p'),\tag{9}$$

where

$$G^{(1)}(p,p')=\ln\left|\frac{p^2+p'^2+pp'-E}{p^2+p'^2-pp'-E}\right|\tag{10}$$

and

$$G^{(2)}(p,p')=\theta\left(\frac{p^2+p'^2-E}{pp'}\right)$$

with the wide-spread designation

$$\theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases} .$$

So the kernels of equations (5) are possessed at $E > 0$ by the logarithmic singularities of which the positions on the (p, p') plane are **determined** by the equations

$$p^2 + p'^2 \pm pp' - E = 0$$

in the domain, where $0 \leqslant p, p' \leqslant \frac{2\sqrt{E}}{\sqrt{3}}$.

To treat correctly these "moving" singularities we have applied, for solving equations (5) at $E > 0$, the interpolation method that was considered in ref. [7] .

We solved eqs. (5) at different energies of the incident neutron above the deuteron break-up threshold to get the values of the quartet nd-scattering S-phase shifts.

4. Figure 1 shows the dependence of $\kappa\, ctg(Re\,\delta)(F^{-1})$ (δ is the quartet S- phase shift as defined in (8)) on the incident neutron c.m.s. energy $\kappa^2 (F^{-2})$. Curves 1 and 2 relate to the zero and linear in the triplet effective range approximation, respectively.

The following experimental values were used in our calculations:

$$\alpha = 0.2316\ F^{-1}\ ;\quad r_0 = 1.75F\ ,$$

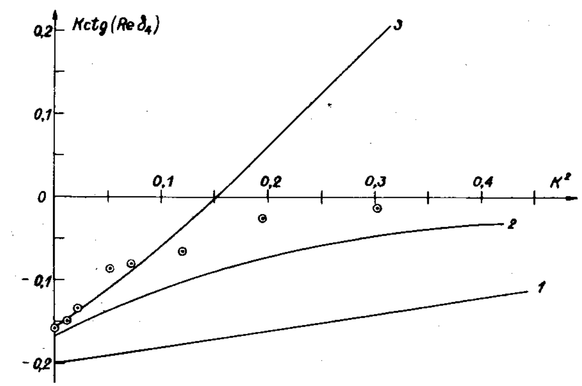

Fig.1. Dependence of $\kappa\, ctg(Re\,\delta)(F^{-1})$ on $\kappa^2(F^{-2})$ (δ is the quartet nd-scattering S-phase shift and κ^2 is the incident neutron c.m.s. energy). Curve 1 is for the **zero approximation on the NN-interaction** range (ZANNIR). Curve 2 is for the approximation linear in the NN interaction range (ALNNIR). Curve 3 is from ref. [11] . The experimental data [8] are pictured here as the circled points.

The calculated data below the deuteron break-up threshold were taken from ref. [6] (the threshold is situated at $\kappa^2 = \frac{4}{3}\alpha^2 = 0.0717 F^{-2}$ on the energy axis). The part of curve 1 at energies above the threshold is taken from ref. [7].

As one can see from fig.1 the theoretical results for $\kappa\, ctg\,(Re\,\delta)$ in the ALNNIR (curve 2) fit well the experimental data [8] (the latter are represented by the circled points on fig.1). In the ZANNIR (curve I) we have only qualitative agreement between the theoretical results and the experimental dependence of $\kappa\, ctg\,(Re\,\delta)$ on κ^2. In the ZANNIR the quartet scattering length is equal to $a_4 = 5.09\, F$ [2,9]. This value diverts considerably off the experimental one $a_4 = 6.35 \pm 0.02$ F [10]. In the ALNNIR we have $a_4 = 6.06$ F [6].

For comparison with our results in fig.1 there is drawn the curve 3 exhibiting the dependence of $\kappa\, ctg\,(Re\,\delta)$ on κ^2 which has been obtained by solving the Faddeev equation for the case of the two-body square well potentials [11]. The calculations with the potentials of Yamaguchi type produce analogous results [12], so do the ones with the soft-core potentials [12].

Eqs. (5) in the ALNNIR are correct if, strictly speaking, conditions (2) are fulfilled, i.e. if in the wave function the important momenta are of the value $p \leq \bar{p}$, where $\bar{p}\tau_0 \ll 1$ and $\bar{p}$ is the greatest of the characteristic momenta of the system p+d at a fixed energy. These conditions are fulfilled for the quartet nd-scattering below the deuteron break-up threshold [6].

The correction $\psi_1(p,p_0)$ is small enough in that case as compared with $\psi_0(p,p_0)$ [6].

Above the deuteron break-up threshold the important momentum range for the wave function $\psi_0(p,p_0)$ is $p \leq \bar{p} \sim \sqrt{E}$ as results from ref. [7]. Not always the value of $\bar{p}\tau_0$ is small then, and, for example, at the incident neutron lab.energy $E_n =$ 14.1 MeV the value of $\bar{p}\tau_0$ is of the order of unity. So the condition (2) is violated and the correction $\psi_1(p,p_0)$ to the $\psi_0(p,p_0)$ (see (4)) appears, in our calculations, to be of the order of the function $\psi_0(p,p_0)$ itself.

However, the ALNNIR in the form of (5) provides good agreement for the calculated quartet nd-scattering S-phase shifts with experiment both below and above the deuteron break-up threshold.

We may consider the scheme used here for calculations as a species of an optical model adjusted to the nd-scattering problem. From this point of view it seems to be more reasonable to insert into the relevant Faddeev equation just the expression (3) for the nucleon-nucleon t-matrix and not to make the splitting (4) of the wave function into two parts.

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