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APPLICATION OF THE SINGULARITY SUBTRACTION METHOD FOR (d,p) REACTIONS ON LIGHT NUCLEI

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Применение метода вычитания ближайшей сингулярности для анализа реакций (d,p) на легких ядрах

Показано, что метод вычитания ближайшей сингулярности может быть успешно применен для обработки данных о (d,p) реакциях. Полученная при этом информация о структуре ядер в дальнейшем уже может быть использована для более эффективного исследования механизма реакций традиционными методами.

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Application of the Singularity Subtraction Method for (d,p) Reactions on Light Nuclei

The applicability of the singularity subtraction method to the analysis of (d,p) reactions is demonstrated. Using the structure information given by the subtraction method one can make "traditional" mechanism studies more effective.

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For various transfer processes it has been attempted to extract the strength of the nearest singularity to the physical region in the $\cos\theta$ plane using the analyticity of the differential cross-section at fixed energy [1-7]. In this paper I apply the singularity subtraction method to (d,p) reactions on light nuclei and using the resulting structure information I perform some mechanism studies. On the whole a new scheme for analysing nuclear reaction data emerges.

To save space I give here only a simplified description of the singularity subtraction method, the reader is referred to ref. [6] as concerning the general background and the details of the method.

For (d,p) and (d,t) types of reaction the transfer pole (at $z = z_p$) is usually the nearest singularity to the physical region in the $z = \cos\theta$ plane. Therefore the pole determines the asymptotics of the expansion coefficients of the differential cross section according to a set of regular functions. Using this fact one can determine the strength of the pole singularity as follows. First of all, one should remove the pole from the differential cross section by a suitably chosen factor and using the least squares procedure should fit the obtained result according to some set of polynomials. These polynomials might be the well known orthogonal polynomials $B_n(z)$ which are orthonormal with respect to the weights of the least squares procedure. Then one has:

$$(z_p - z)^2 \frac{d\sigma}{d\Omega} = \sum_{n=1}^{N+1} a_n B_n(z), \qquad (1)$$

where only the significant terms are included into the sum. If one removes only the interference term of the pole with the background amplitude then one has

$$(z_p - z)\frac{d\sigma}{d\Omega} = \frac{P}{z_p - z} + \sum_{n=1}^{N} b_n B_n(z) = \sum_n A_n B_n(z). \qquad (2)$$

Here the $B_n(z)$ polynomials differ from that in (1) but this is of no importance. From (2) it follows that if one analyses $(z_p - z)d\sigma/d\Omega - P/(z_p - z)$ with a guess strength P, then at the correct strength the A_n coefficients with $n > N$ become insignificant. To increase the effectiveness of the method all these are carried out not in the $z = \cos\theta$ plane, but in a new variable received by the so-called optimal conformal mapping [8,9].

I do not maintain my previous statement that the results of the subtraction method are necessarily free of any systematical error [6]. The determination of the value of N in the expansion series of (2) involves an assumption: in some cases within their errors the b_n background coefficients might obey the same (recurrence) relation which the pole contribution coefficients obey and which assures that the pole contribution is not present in the highest order terms of the expansion series of (1). As a consequence, one determines a wrong value of N. One should especially be aware of it if the background singularities lie near to the pole and if one is able to extract only the lowest order coefficients from the experimental data due to their large errors. Mostly it is of academic interest, but in some unfavourable cases one virtually can get wrong results.

One can also determine the strength of the pole by inserting $z = z_p$ into the right-hand side of formula (1). This is the continuation method generally used by other authors [1-5]. This procedure has some practical disadvantage, which I discuss later.

Once the strength of the pole is found, one has to calculate the spectroscopic information from it. For this purpose the formulae of the peripheral model can be used [10,12], with no cut-off one gets back the pure pole.

The result is:

$$(z_p - z)^2 \frac{d\sigma}{d\Omega} \to \frac{5}{8\pi^2} \frac{m_a^2 c^4}{E_i \cdot E_f} \frac{k_f}{k_i} \frac{2J_B + 1}{(2J_A + 1)(2J_a + 1)} G_x^2 G_B^2, \qquad (3)$$

where I consider the $A + x \to B + y$ $(x = a + y, B = A + a)$ transfer process with zero orbital momentum in the x vertex. $E_i (E_f)$ and $k_i (k_f)$ are the CM relative kinetic energy and momentum in the initial (final) state; m and J denote the mass and spin of the corresponding particle; G_x^2 and G_B^2 are the vertex constants containing information on the structure of particles. The differential cross section is measured in mb/sr, while the vertex constants in fermi. The $G_{\ell j}(q)$ decay amplitudes (here q is the relative momentum in the vertex, ℓ is the orbital momentum, while $\vec{j} = \vec{\ell} + \vec{J}_a$) have the following connection with the vertex constants:

$$G_x^2 = [G_{0 J_a}^{(x)}(i\kappa_x)]^2,$$

$$G_B^2 = \sum_{\ell_B} (-1)^{\ell_B} G_{\ell_B}^2; \quad G_{\ell_B}^2 = \sum_{j_B} [G_{\ell_B j_B}(i\kappa_B)]^2, \qquad (4)$$

where $i\kappa$ denotes the wave number corresponding to the binding energy. If the wave function of the corresponding particle in the decay channel characterized by the quantum numbers of ℓ and j has the asymptotic behaviour of $A_{\ell j} \exp(-\kappa r)/r$, then

$$G_{\ell j}(i\kappa) = A_{\ell j} \, h\sqrt{\pi}/\mu c, \qquad (5)$$

where μ is the reduced mass. Later on I use the notation $G = G_x^2 G_B^2$.

As it follows from (4), the sign of the residue is determined by the parity of the orbital momenta in the vertices. The residue itself is the sum of contributions with different possible orbital momenta in the vertex, so determining its value no information could be received on the number and values of these orbital momenta.

It cannot be regarded as a disadvantage of the subtraction method because, as a consequence, the method becomes very effective in extracting structure information and extremely insensitive to other factors. Note that in favourable cases the resulting structure information is given with such a small error which was unfeasible untill now (see[6,7] and the results discussed later). If one wants to learn something about the orbital momenta in the vertices and about the mechanism of the reaction, then one should make some additional analysis with the well-known methods (like the peripheral model, DWBA and so on) fixing the structure information given by the subtraction method.

In this paper I study the $^{16}O(d,p)^{17}O(0.87)E_d$=13.3MeV[10] and the $^{9}Be(p,d)^{9}Be(0.0,2.9)E_p$ = 17, 21, 25, 29 MeV[11] reactions in the above described way. At the end I briefly present the results for the $^{31}P(d,p)^{32}P$ E_d = 10.0 MeV reaction[12].

Table 1

Results for the $^{16}O(d,p)^{17}O(0.87)$ reaction: a) expansion coefficients in formula (1) (note that they are uncorrelated and have an rms error of ± 1) and χ^2 values when fitting terms only up to the given index are included. b) the guess strength for the pole (G in fermi^2) when the corresponding coefficient disappears in (2).

	$(z_p - z)^2 \frac{d\sigma}{d\Omega}$		$(z_p - z)\frac{d\sigma}{d\Omega}$	
n	a_n	χ^2/N_f	G	Δ G
9	62.4	36.6		
10	28.4	15.2		
11	19.5	4.7	0.425	0.010
12	-2.5	4.6	0.315	0.019
13	5.8	3.7	0.591	0.036
14	-3.5	3.5		

I give more details for the ^{16}O case to illustrate some problems in connection with the application of the method. The coefficients in the expansion series of (1) and the guess strength of the pole when the corresponding expansion coefficients in (2) disappear, are presented in Table 1. In this case the typical error for the experimental points (there are 46 of them) is ± 3%, but this is an estimated value mainly coming from various corrections. This implies, firstly, that the value of the errors is fairly uncertain, secondly, that there is a strong correlation present in the data, which was not taken into account during the analysis.

Therefore, χ^2 as a function of the number of fitting terms in formula (1) does not reach its expectation value N_f (the number of degrees of freedom in the fit) at $N+1=11$ after a steep decrease, but continues decreasing with a considerably slighter slope. The sudden change in the steepness makes it clear that after it one fits only the statistical noise and some systematical error present in the data. It follows that one can unambigously choose $N=10$ for the expansion series (2).

This behaviour of χ^2 in itself shows that the errors present in the values of G are larger than those calculated from the given errors of the experimental data. In addition, there is the strong correlation between the data. In principle, one can always easily include correlations into the analysis but, in practice, this is always a problem due to the lack of reliable information. The best thing one can do is to take the weighted average of the values with their given errors, and calculate the error of this average from the actual deviation of the G values from it. The result got in this way is $G = 0.41 - 0.04\ f^2$. Its sign is in accordance with the even parity of the final state (l_n = is the only possible orbital momentum in the vertex).

If the given errors of the experimental points really corresponded to the actual errors and the points were uncorrelated, then the resulting structure information would have a remarkably small error: about 2% (!). It points out the extreme power of the subtraction method

and also its only disadvantage, its strong dependence on the statistical quality of the experimental data.

In the continuation method, as one has no statistically independent values for G, one cannot average different results and one possible way of statistical control is lacking. I think that this is the only practical difference between the two methods, otherwise the results are extremely similar, because in the pole only the highest order terms give the dominant contribution to the sum of formula (1).

As I mentioned above, if one wants to learn about the given reaction more than the single figure for the vertex constant, then one should make some additional analysis. For this purpose I have chosen the peripheral model /13/ due to its extreme simplicity. In this model the contribution of the pole singularity is taken into account in the higher partial wave amplitudes, while the effect of other mechanisms and the coupling to other channels is taken into account phenomenologically by cutting off the low partial wave amplitudes. For more detail see ref. /14/ and the literature cited there. In its simplest form the peripheral model (PM) has only one parameter: the L value at which the cut-off is made in that channel where the $L \sim kR$ prediction is larger. In our case the prediction in the deuteron channel is $L = k_d (R + R_d) = 4.5$ with $R = 1.1 \sqrt[3]{A} + 0.75$ /15/ and $R_d = 1.0$f.

In the PM analysis I fixed the vertex constant as it was given by the subtraction method and changing the cut-off parameter I tried to describe the forward-angle region indicated by arrows on Fig. 1. The fitted value for the cut-off parameter $L = 4.65$ is extremely near to the prediction indicating its physical meaning, and the description at larger angles is very good too. It bears out the correctness of the structure information given by the subtraction method and also shows that very simple physical assumptions are enough to describe the angular distribution. Keeping in mind this fact it is remarkable that DWBA is generally regarded as unable to describe reliably the (d,p) reactions in this atomic mass region.

It is interesting to determine the spectroscopic factor

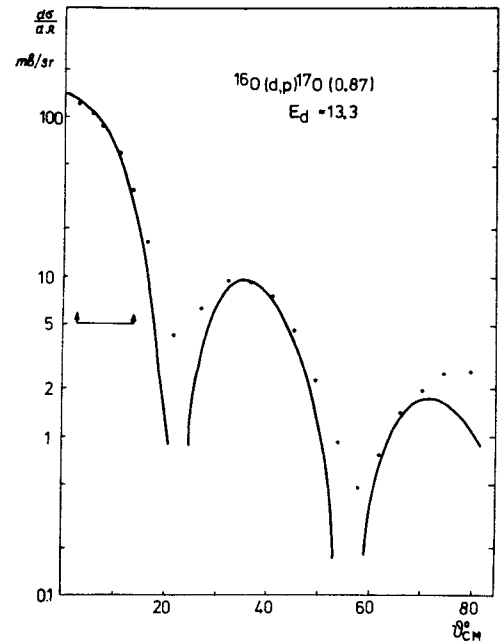

Fig. 1. Differential cross section for the $^{16}O(d,p)^{17}O(0.87)$ $E_d = 13.3$ MeV reaction. The points are the experimental data, while the solid curve is the PM fit with $G = 0.41 f^2$ fixed. The fitted region indicated by arrows, the fitted cut-off parameter is $L = 4.65$.

from the vertex constant. The spectroscopic factor, appart from the vertex constant, is not a model independent quantity. It measures to which extent a model wave-function, the so-called single-particle wave function used in the DWBA or a similar analysis, is present in the actual wave function of the final nucleus. As the contribution of the nuclear interior is usually strongly suppressed /14/, the information on the bound state wave function got by a DWBA analysis concerns its peripheral part. Therefore it is natural to assume that

$$G_B^2 = S \cdot G_0^2 , \qquad (6)$$

where S is the spectroscopic factor and G_0^2 is the single particle vertex constant calculated from the asymptotics of the single particle wave function by formula (5). I have made the calculation by means of the well-known well depth fitting procedure and used two different geometries for the Saxon-Woods potential: $R = 1.25\sqrt[3]{A}$ and $R = 1.1\sqrt[3]{A} + 0.75$ [15], while in both cases $a = 0.60$. Using the deuteron vertex constant $G_d^2 = 0.43 \pm 0.01$ f [14], the spectroscopic factor I got was $S = 0.66$ and $S = 0.80$, respectively. It is in accordance with the generally assumed single

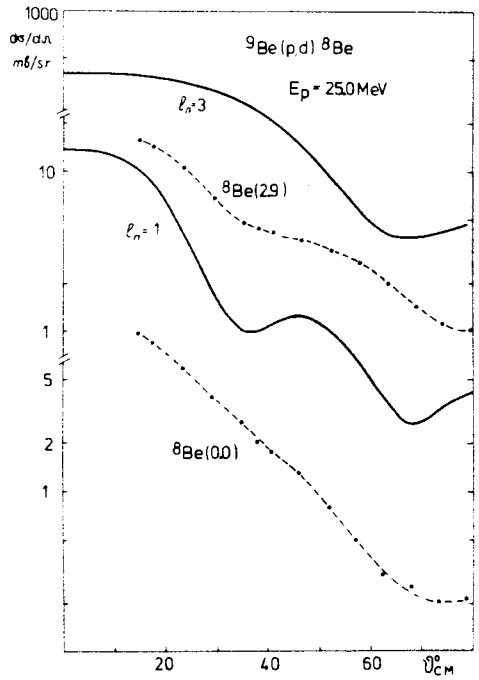

Fig. 2. Differential cross section for the $^9Be(p,d)^8Be$ reaction at $E_p = 25.0$ MeV. The results of the PM calculations for the reactions leading to the $^8Be(2.9)$ state with $G = 0.071 f^2$ and with the predicted cut-off parameter of $L = 5.0$ are shown by solid line for the two kinematically possible vertex orbital momenta.

particle nature of the $^{17}O(0.87)$ state. In such a way one can get information on the average field in the nucleus: if somebody is sufficiently convinced that for the given state the spectroscopic factor is known, then from (6) the single particle vertex constant could be determined and could be compared with the constant which follows from the assumed average field.

For the $^9Be(p,d)^8Be(0.0)$ reaction the subtraction method did not work, i.e., in the expansion series of (1) there were more significant terms than in (2). At $E_p = 25.0$ and 29.1 MeV I got a positive residue with a 25% error, but it must be attributed to some systematical error and/or to the effect of correlations in the experimental data.

For the $^9Be(p,d)^8Be(2.9)$ reaction the results are presented in Table 2. The different G values cannot be regarded as contradicting one another if one multiplies their presented errors by a factor of $(\chi^2/N_f)^{1/2}$. The $E_p = 25.0$ MeV results are the most reliable, their weighted average is $G = -0.071 \pm 0.007$ f^2. The sign is in accordance with the odd parity of ^9Be: the possible vertex orbital momenta are $\ell_n = 1$ and $\ell_n = 3$. The G value itself contains contribution from these two transitions. PM calculations, the results of which are presented on Fig. 2, show that only a very small $\ell_n = 3$ admixture can be present in the ^9Be nucleus, but it could have an important effect on the differential cross section. The correlation between the second maximum of the $\ell_n = 1$ PM angular distribution and the bump in the experimental data at $\theta \sim 50°$ explains why the subtraction method was unsuccessful in the $^9Be(0.0)$ case: the absence of any similar bump shows that in this case the transfer pole has no visible effect (note that only $\ell_n = 1$ possible). It follows that the attempt made in ref. [11] to describe these reactions by DWBA is quite hopeless. If the transfer pole is not dominant then one should explicitly take into account other mechanisms, too, and it is not sufficient to include them phenomenologically into the analysis with the aid of optical potentials. Besides, in the $^8Be(2.9)$ case one should take into account $\ell_n = 3$, too.

Table 2

Results for G (in $10^{-2} f^2$) for the $^9Be(p,d)^8Be(2.9)$ reaction. The χ^2 values received using the expansion series (1) show the goodness of the experimental data

E_p	n=9	n=10	n=11	n=12	χ^2/N_f
17.0	-7.1±0.4	-8.24±0.18	-17.9±1.5		5.04
21.0		-9.40±0.7	-14.8±1.4		4.8
25.0		-6.3±0.7	-9.2±1.4	-10.5±2.8	1.8
29.1			-8.1±1.1	-17.5±2.1	5.0

I have also succeeded in analysing the $^{31}P(d,p)^{32}P$ E_d = 10.0 MeV reaction leading to different excited states of ^{32}P /12/. The statistical quality (i.e., the reliability of the errors) of these data was the highest one, but as the above given examples illustrate quite well the application of the subtraction method, I give a short summary of the results. I applied the method to each of the ten cases when the differential cross section exceeded the 1 mb/sr limit. In one case I got a clearly wrong result, in six cases it was possible to extract the structure information from two or three coefficients, which assures that the results are correct. In four cases when the differential cross section was higher than 4 mb/sr a peripheral model analysis, as I described above in the oxygen case, was successful. Therefore, a very important by-product of these investigations is the direct evidence that the peripheral model fit gives correct structure information providing the angular distribution is well described.

One can conclude that the singularity subtraction method gives correct structure information for (d,p) reactions on light nuclei. This method can also be applied to havier nuclei and to reactions of the $(d,{}^3He)$ type, because in these cases a similar physics is involved.

The subtraction method with its extreme effectiveness for extracting structure information does not replace the other methods for the analysis of nuclear reactions but does make them more powerful when they are combined. I hope that the method will soon become widespread in analysing nuclear reactions.

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