ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E4 - 8414

V.N.Efimov, E.G.Tkachenko

DOUBLET S-PHASE SHIFT IN THE nd-SCATTERING BELOW THE DEUTERON BREAK-UP THRESHOLD IN THE INDEPENDENT OF AND LINEAR IN THE NN-INTERACTION RANGE APPROXIMATION

1974

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E4 - 8414

V.N.Efimov,[1] E.G.Tkachenko[2]

DOUBLET S-PHASE SHIFT IN THE nd-SCATTERING BELOW THE DEUTERON BREAK-UP THRESHOLD IN THE INDEPENDENT OF AND LINEAR IN THE NN-INTERACTION RANGE APPROXIMATION

Submitted to "Изв. АН СССР"(сер. физ.)

[1] LNP, JINR
[2] V.G.Khlopin Radium Institute, Leningrad, USSR.

Ефимов В.Н., Ткаченко Э.Г. E4 - 8414

Дублетная S-фаза nd-рассеяния ниже порога развала дейтрона в нулевом и первом порядках по радиусу взаимодействия

Дублетная S-фаза nd-рассеяния ниже порога развала дейтрона, рассчитанная в рамках модели интегральных уравнений с обрезанием ядер при больших импульсах, воспроизводит экспериментальные данные качественно в нулевом порядке по радиусу нуклон-нуклонного взаимодействия и очень хорошо количественно в первом порядке.

Препринт Объединенного института ядерных исследований.
Дубна, 1974

Efimov V.N., Tkachenko E.G. E4 - 8414

Doublet S-Phase Shift in the nd-Scattering Below the Deuteron Break-Up Threshold in the Independent of and Linear in the NN-Interaction Range Approximation

The integral equations for the S-wave doublet nd-scattering in the independent of and linear in the NN-interaction range approximation are solved using the Kharchenko method of cutting the integral equation kernels. The cut-off paremeter is defined through the experimental value of the doublet scattering length. Calculated S-phase shifts in the doublet nd-scattering below the deuteron break-up threshold are in good agreement with experiment and do not depend on the cutting fashion.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1974

Nucleon -nucleon resonant interaction provides, in the Skornyakov-Ter-Martirosyan (STM) model [1] (which corresponds to the zero range nucleon-nucleon interaction), a collapse ("the fall into the centre") of the three nucleon system in the S-state with $S = 1/2$, $T = 1/2$. The collapse exhibits itself both in the non-existence of the lowest energy level for the three-nucleon system [2] and in the existence of the homogeneous STM equation solution at any fixed energy. The latter leads to the non-uniqueness of the STM equation solution for the neutron-deuteron S-scattering in the $S = 1/2$, $T = 1/2$ state (i.e., for doublet scattering). Danilov [3] had shown that the extra requirement, the wave functions be orthogonal when corresponding to different energies, eliminates the ambiquity in the solution. Into the consideration an extra parameter then should be involved which could be defined through experimental characteristics of the three nucleon system, for example, through the triton binding energy. In that case there arises the discrete energy spectrum in the three nucleon system with levels $E_n < 0$, and $E_n \xrightarrow[n \to \infty]{} -\infty$ if the interaction range r_0 is diminishing. However, as was noted in [6], in the approximation which corresponds to the small value $|E|^{1/2} r_0$ (E is the three nucleon energy and r_0 is a range of the nucleon-nucleon interaction) we should concern only with the highest level of this spectrum, for the other states of the spectrum violate the condition $\left|\frac{E_n M}{\hbar^2}\right|^{1/2} r_0 < 1$. In the framework of the consideration $-E_1$ is the triton binding energy. In the STM model " the fall into the centre" is a result of the infinitely strong attraction at negligible distances and in the three nucleon case it is characteristic for the doublet S-state only, for in the quartet S-state the Pauli principle forbids the three nucleons to be at the same point simultaneously and for in the non-zero angular momentum states the repulsive centrifugal forces appear to be "switched on" at small distances between the nucleons. The infinitely strong attraction at the negligible distances between the three nucleons in the doublet S-state results in the kernels of the integral STM equations to decrease weakly with increasing momenta.

In the framework of the Danilov model [3,6] there have been obtained the integral equations with a sufficiently rapid decrease of the kernels at large momenta, and the doublet nd-scattering problem has been solved at zero energy of an incident neutron in the approximation independent of the nucleon-nucleon interaction range.

For treating the doublet nd-scattering problem Kharchenko [8] has proposed a model which is based on solving the integral STM equations, the kernels of which being cut off at large momenta. The cut-off parameter was defined by the following requirement. There must exist just one bound state in the three-nucleon system and the energy of the bound state must be equal to the experimental binding energy of triton. The doublet scattering length values, calculated in [7] (a_2 = 0.48 F) and in [8] (a_2 = 0.49 F) are in good agreement with the experimental one a_2 = 0.65 F [9].

2. The integral equations for three particles have been obtained and considered by Danilov in the linear in an interaction range approximation [6]. In this approximation the two-body t-matrix does not depend on initial and final momenta. In effect, the integral equations for three particles are possessed in the linear in the two-body interaction range approximation by the same properties as the respeotive STM equations.

In the following, for treating the integral equations we use the Kharchenko method with various fashions of a high momentum cut -off.

Using the Faddeev general equations [4] we are, in analogy with [10], to obtain the equations for the doublet S-wave neutron--deuteron scattering in the linear in the nucleon-nucleon interaction range approximation (LNNIRA), and then to introduce the cut--off (it could be a smooth one) in the final equations. Then we have:

$$\Psi_1(p)=\frac{8}{3\pi}(\gamma_p-\beta_1)(1-\beta_1 c_1+c_1\gamma_p)*\tag{1}$$
$$*\Big\{\frac{\pi}{2}c_{11}*\mathcal{U}(p,p_0,\varepsilon_0,p_c)+$$
$$+\int_0^\infty\frac{p'^2dp'}{p'^2-p_0^2-i\delta}\mathcal{U}(p,p',\varepsilon_0,p_c)*$$
$$*\left[c_{11}\Psi_1(p')+c_{12}\Psi_2(p')\right]\Big\};$$

$$\Psi_2(p) = \frac{2}{\pi}\frac{p^2-p_0^2}{\beta_2+\gamma_p}\left(1-\beta_2 c_2 + c_2\gamma_p\right) *$$

$$* \left\{ \frac{\pi}{2} c_{21} * \mathcal{U}(p, p_0, \varepsilon_0, p_c) + \right.$$

$$+ \int_0^\infty \frac{p'^2\,dp'}{p'^2 - p_0^2 - i\delta}\, \mathcal{U}(p, p', \varepsilon_0, p_c) *$$

$$\left. * \left[c_{21}\Psi_1(p') + c_{22}\Psi_2(p') \right] \right\};$$

$$\gamma_p = \sqrt{\varepsilon_0 + \tfrac{3}{4}p^2}\ ; \quad \varepsilon_0 = \alpha^2 - \tfrac{3}{4}p_0^2;$$

$$\beta_1 = -\alpha; \quad \beta_2 = -\frac{1}{a_s} - r_0^s \frac{1}{2a_s^2};$$

$$c_1 = \frac{1}{2} r_0^t\ ; \quad c_2 = \frac{1}{2} r_0^s\ ;$$

$$\mathcal{U}(p, p', \varepsilon_0, p_c) = f_d(p, p_c) * f_d(p', p_c) *$$

$$* \frac{1}{pp'} \ln \frac{p^2 + p'^2 + pp' + \varepsilon_0}{p^2 + p'^2 - pp' + \varepsilon_0}\ ;$$

In the expressions written above $\alpha = 0.2316\ F^{-1}$, $a_s = -23.7$ F , $r_0^t = 1.749$ F, $r_0^s = 2.67$ F,

r_0^t and r_0^s are the effective radii of the interaction between two nucleons in the triplet and singlet state, respectively.

In the independent of the nucleon-nucleon interaction range approximation (INNIRA) we must fix $r_0^t = 0$ and $r_0^s = 0$.

The function $f_d(p, p_c)$ decreases rapidly when $p > p_c$ and provides a cut-off for the kernels and solution of integral equations (1) at large momenta, p_c being a phenomenological parameter. In particular, in the sharp cut-off model, considered in [8], this function equals

$$f_d(p, p_c) = \Theta(p_c - p) = \begin{cases} 1 & \text{if } p < p_c \\ 0 & \text{if } p > p_c \end{cases} \tag{2}$$

(But therein [8] there was a remark, that the cut-off function could be a smooth one).

In our calculations this function has been taken in form (2) as well as in the following ones:

$$\exp(-p^2/p_c^2); \quad \exp(-p/p_c); \quad \frac{p_c^2}{p_c^2 + p^2}.$$

The dependence of the Saxon-Woods type also has been used for $f_d(p, p_c)$ with different "smearing width" values of d.

The homogeneous equations for the three nucleon bound state with an energy $-\varepsilon_0$ are as follows:

$$\psi_1^0(p) = \frac{2}{\pi} \cdot \frac{1-\beta_1 c_1 + c_1 \gamma_p}{\beta_1 + \gamma_p} *$$

$$* \int_0^\infty u(p,p',\varepsilon_0,p_c) *$$

$$* \left[c_{11} \psi_1^0(p') + c_{12} \psi_2^0(p') \right] p'^2 dp' \; ; \qquad (3)$$

$$\psi_2^0(p) = \frac{2}{\pi} \cdot \frac{1-\beta_2 c_2 + c_2 \gamma_p}{\beta_2 + \gamma_p} *$$

$$* \int_0^\infty u(p,p',\varepsilon_0,p_c) *$$

$$* \left[c_{21} \psi_1^0(p') + c_{22} \psi_2^0(p') \right] p'^2 dp' .$$

For the doublet state of the three nucleon system we have $C_{11} = C_{22} = 1/4$; $C_{12} = C_{21} = 3/4$. The quartet nd-scattering is described by (1) with $C_{11} = -1/2$; $C_{22} = C_{12} = C_{21} = 0$. Equations (1) and (3) could correspond to the system of three identical spinless particles provided one puts $C_{11} = 1$; $C_{22} = C_{12} = C_{21} = 0$ in these equations.

Into equation (1) and (3) there enter constants originated from the two-body interactions as well as the extra parameters, characterized by the fashion f_d (p,p_c) of cutting the integral equation kernels at large momenta. These extra parameters of the model in question could be defined through the use of experimental characteristics of the three particle system.

3. The experimental dependence of value of $k \, ctg\, \delta_2$ on k^2 [11] for the neutron-deuteron S-wave doublet scattering can be well fitted by the curve (Fig.1):

$$k \, ctg\, \delta_2 = -A + Bk^2 - C/\ (1+Dk^2) \qquad (4)$$

which has a pole just below the elastic scattering threshold [12]. (In (4) δ_2 is the doublet S-phase shift in the neutron scattering off deuteron, and the coefficients A,B,C,D are determined by the best fit of (4) to the experimental points [11]).

In the model in question, with the cut-off, for instance, taken in form (2), the values of $k \, ctg\, \delta_2$ at different energies k^2 of the incident neutron increase with increasing cut-off parameter p_c (the range of changing the parameter allows here the three nucleon system to have just one bound state with an energy to be close to the triton energy). The monotone character of curve (4) at $k^2 > 0$ (Fig.1) gives us a possibility to say that a small increase of the parameter p produces a small shift of curve (4) as a whole to the left, along the energy axis k^2 (correctly, it should be said in the opposite direction to this axis). A small increase in the parameter p_c also produces a small increase in the binding energy of three-nucleon system bound state, as well as a small replacement in the position of the pole of functional dependence (4). This pole in functional dependence (4) of $k \, ctg\, \delta_2$ on k^2 is situated very closely to the elastic nd-scattering threshold at $k^2=0$ and small changes in the position of this pole yield considerable changes in the value of the doublet nd-scattering length. So the doublet nd-scattering length, being defined by the nearest to the pole point on the energy axis k^2 , is the most sensitive value with respect to the change of the cut-off parameter p_c.

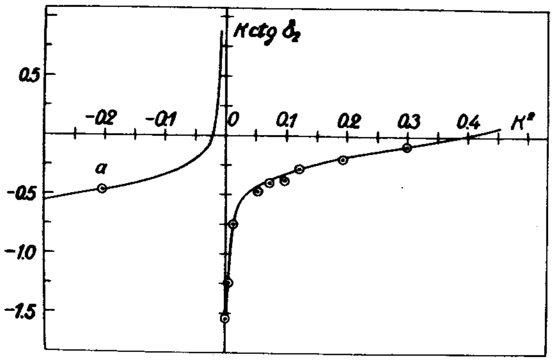

Fig.1. Dependence of $k\,\mathrm{ctg}\,\delta_2$ (F^{-1}) on $k^2(F^{-2})$ (δ_2 is the doublet nd-scattering S-wave phase shift and k^2 is the incident neutron energy in the c.m.s.). Experimental data [11] are pictured as open circles. The point a is concerned with the triton binding energy. The curve represents the best fit of the formula (4) parameters to the experimental data.

In accordance with these reflections we choose here the cut-off parameter p_c to reproduce exactly the experimental value of the doublet nd-scattering length $a_2 = 0.65$ F [9]. This choice is expected to have just small divergence of the calculated triton binding energy off the experimental one, at the same time the calculated low energy S-wave phase shifts better fitting the experimental ones in the neutron-deuteron scattering.

In calculations we have used the following forms of the cut-off factor $f_d(p,p_c)$ in equations (1) and (3):

$$
\begin{aligned}
&(i) \quad f_d(p,p_c) = \theta(p_c - p)\,, \\
&(ii) \quad f_d(p,p_c) = \exp(-p^2/p_c^2)\,, \\
&(iii) \quad f_d(p,p_c) = \exp(-p/p_c)\,, \\
&(iv) \quad f_d(p,p_c) = \frac{p_c^2}{p_c^2 + p^2}\,, \\
&(v) \quad f_d(p,p_c) = \frac{N}{1 + \exp\left(\frac{p-p_c}{d}\right)}\,.
\end{aligned}
\tag{5}
$$

In item (v) the normalization constant N is conditioned by

$$
f_d(p,p_c)\Big|_{p=0} = 1.
$$

The "smearing width" of cutting d is given several fixed values in the interval $[0.05, 2.00]\ F^{-1}$.

As was explained above, the values of the parameter p_c in expressions (5) have been determined through the experimental value [9] of the doublet nd-scattering length, just one bound state being demanded to exist in the three nucleon system. The binding energy of this state was found with the help of Eqs. (3) with the parameter p_c fixed as described just before. This energy was regarded as the triton binding energy. The parameter p_c values for different cut-off fashions (5) are obtained by the use of the experimental value of the doublet scattering length $a_2=0.65$ F [9] and exhibited in Table I for the independent of and linear in the interaction range approximation in the first and third column, respectively. In the second and fourth column of Table I there are written the calculated values for the triton binding energy in the INNIRA and LNNIRA.

Fig.2 shows two dependences of $k\,ctg\,\delta_2$ on k^2 (δ_2 is the doublet nd-scattering S-wave phase shift and k^2 is the incident neutron energy in the c.m.s.) below the deuteron break-up threshold. These two dependences are calculated on the basis of Eq. (1) in the INNIRA and LNNIRA for different cut-off functions (5). It has appeared that both in the INNIRA and in the LNNIRA the values of $k\,ctg\,\delta_2$ below the deuteron break-up threshold do not depend practically on cut-off fashion (5) and ould not be distinguished in the Fig.2 scale. In the LNNIRA the calculated values of $k\,ctg\,\delta_2$ fit the experimental ones very well, the calculated triton binding energy values exceeding

Table I

The parameter $P_c\,(F^{-1})$ values and the triton binding energy E_T(MeV) for different cut-off functions (5) in the independent of and linear in the interaction range approximation. The experimental triton binding energy is regarded to be $E_T = 8.48$ MeV.

Cut-off type (5)	P_c $r^s_o=r^t_o=0$	E_T $r^s_o=r^t_o=0$	P_c $r^s_o=2.67F$ $r^t_o=1.749F$	E_T $r^s_o=2.67F$ $r^t_o=1.749F$
(i)	1.872	8.217	0.898	9.189
(ii)	3.980	8.375	1.587	9.705
(iii)			2.314	10.361
(iv)	3.650	8.402	1.367	9.896
(v) $d=0.05F^{-1}$	1.925	8.218	0.951	9.208
$d=0.1F^{-1}$	1.985	8.220	1.010	9.260
$d=1.0F^{-1}$	3.189	8.389	0.928	10.052
$d=2.0F^{-1}$	3.519	8.468		

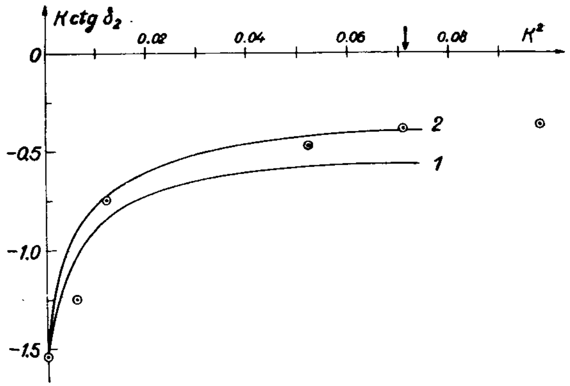

Fig.2. Dependence of $k \, ctg \, \delta_2$ (F^{-1}) on $k^2(F^{-2})$ (δ_2 is the doublet nd-scattering S-wave phase shift and k^2 is the incident neutron energy in the c.m.s.). The open circles are the images of the experimental data [11]. The arrow marks the deuteron break-up threshold position. Curve 1 is the independent of the nucleon-nucleon interaction range approximation (INNIRA). Curve 2 is the linear in the nucleon-nucleon interaction range approximation (LNNIRA).

the experimental ones (according to Table I). On the other hand, in the INNIRA we have good agreement with experiment for the calculated triton binding energy (that was expected from the results of [8]) but at the same time a certain divergence of the calculated $k \, ctg \, \delta_2$ off the experimental data. We should remark that much more sophisticated models not always yield a good value for the triton binding energy, simultaineously reproducing the low energy doublet nd-scattering S-wave phase shifts decently.

The considered here model is in fact the result of reducing the three-particle problem of the neutron-deuteron scattering to the two-particle problem with an effective interaction between the neutron and deuteron. This interaction appears to be non-local and dependent on energy and spin.

We have shown earlier [10] that the LNNIRA fits well the calculated quartet nd-scattering S-wave phase shifts to the experiment below the deuteron break-up threshold.

Cutting factors (5) do not influence the results of calculations in the quartet case, for the functions satisfying the quartet STM equation decrease rapidly at large momenta. So the model, which is based on the use of the two-nucleon t-matrix in the LNNIRA and on cutting the kernels of the STM integral equations at large momenta would be considered as a method of constructing a quasi-optical potential for the interaction of neutron with deuteron. Such a potential may have at least one phenomenological parameter and sufficiently well fits both the bound state energy and the nd-scattering S-wave phase shifts (at least below the deuteron break-up threshold).

In conclusion the authors thank V.B.Belyaev for useful discussions. One of us (E.G.T.) expresses his deep gratitude to Professor N.A.Perfilov and to Professor O.V.Lozhkin for their stimulating interest in the work and their regular support. He also wants to thank Mlle S.K. for her tender attention which urged him intensely until this work was completed.

References:

1. G.V.Skornyakov, K.A.Ter-Martirosyan. DAN USSR, 106, 425, 1956 ; JETP, 31, 775, 1956.
2. L.H.Thomas. Phys.Rev. 47, 903, 1935.
3. G.S.Danilov. JETP, 40, 498, 1961.
4. L.D.Faddeev. JETP, 39, 1459, 1960.
5. R.A.Minlos, L.D.Faddeev. DAN USSR, 141, 1335, 1961; JETP, 41, 1850, 1961.
6. G.S.Danilov. JETP, 43, 1424, 1962.
7. G.S.Danilov, V.I.Lebedev. JETP, 44, 1509, 1963.
8. V.F.Kharchenko. Yad.Fiz. 16, 318, 1972.
9. W.Dilg, L.Koester, W.Nistler. Phys.Lett. 36B, 208, 1971.
10. V.N.Efimov, E.G.Tkachenko. Yad.Fiz. 18, 62, 1973.
11. W.T.H.van Oers and J.D.Seagrave. Phys.Lett. 24B, 562, 1967.
12. R.S.Christian and J.L.Gammel. Phys.Rev. 91, 100, 1953; A.C.Phillips. Phys.Rev. 142, 984, 1966; L.M.Delves. Phys.Rev. 118, 1318, 1961.

Received by Publishing Department on November 29, 1974.