DOUBLET S-PHASE SHIFT
IN THE \(n^d\)-SCATTERING BELOW THE DEUTERON BREAK-UP THRESHOLD
IN THE INDEPENDENT OF AND LINEAR IN THE NN-INTERACTION RANGE APPROXIMATION

1974
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Submitted to "Изв. АН СССР"(сер. физ.)

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Doublet S-Phase Shift in the nd-Scattering Below the Deuteron Break-Up Threshold in the Independent of and Linear in the NN-Interaction Range Approximation

The integral equations for the S-wave doublet nd-scattering in the independent of and linear in the NN-interaction range approximation are solved using the Kharchenko method of cutting the integral equation kernels. The cut-off parameter is defined through the experimental value of the doublet scattering length. Calculated S-phase shifts in the doublet nd-scattering below the deuteron break-up threshold are in good agreement with experimental data and do not depend on the cutting fashion.

Dubna. 1974
quartet S-state the Pauli principle forbids the three nucleons to be at the same point simultaneously and for in the non-zero angular momentum states the repulsive centrifugal forces appear to be "switched on" at small distances between the nucleons. The infinitely strong attraction at the negligible distances between the three nucleons in the doublet S-state results in the kernels of the integral STIM equations to decrease weakly with increasing momenta.

In the framework of the Danilov model \(^3,^6\) there have been obtained the integral equations with a sufficiently rapid decrease of the kernels at large momenta, and the doublet nd-scattering problem has been solved at zero energy of an incident neutron in the approximation independent of the nucleon–nucleon interaction range.

For treating the doublet nd-scattering problem Kharchenko \(^8\) has proposed a model which is based on solving the integral STIM equations, the kernels of which being cut off at large momenta. The cut-off parameter was defined by the following requirement. There must exist just one bound state in the three-nucleon system and the energy of the bound state must be equal to the experimental binding energy of a triton. The doublet scattering length values, calculated in \(^7\) \((a_2 = 0.48 \, \text{F})\) and in \(^8\) \((a_2 = 0.49 \, \text{F})\) are in good agreement with the experimental one \(a_2 = 0.65 \, \text{F}\) \(^9\).

2. The integral equations for three particles have been obtained and considered by Danilov in the linear in an interaction range approximation \(^6\). In this approximation the two-body t-matrix does not depend on initial and final momenta. In effect, the integral equations for three particles are possessed in the linear in the two-body interaction range approximation by the same properties as the respective STIM equations.

In the following, for treating the integral equations we use the Kharchenko method with various fashions of a high momentum cut-off.

Using the Faddeev general equations \(^4\) we are, in analogy with \(^10\), to obtain the equations for the doublet S-wave neutron-deuteron scattering in the linear in the nucleon–nucleon interaction range approximation (LNNIRA), and then to introduce the cut-off (it could be a smooth one) in the final equations. Then we have:

\[
\Psi_1(p) = \frac{8}{3\pi} (\delta_p - \delta_d) \left( 1 - \beta_1 c_1 + c_2 \gamma_p \right) \ast \left\{ \frac{\pi}{2} c_{d1} U(p, p_o, \varepsilon_o, \rho_o) + \int_0^\infty \frac{p'^2 dp'}{p'^2 - p_o^2 - i\alpha} U(p, p', \varepsilon_o, \rho_o) \ast \left[ c_{d1} \Psi_1(p') + c_{d2} \Psi_2 (p') \right] \right\} ;
\]
In the expressions written above \( \lambda = 0.2316 \text{ F}^{-1} \),
\[ a_\mu = -23.7 \text{ F} \quad r_0^t = 1.749 \text{ F} \quad r_0^s = 2.67 \text{ F} \quad \]
\( r_0^t \) and \( r_0^s \) are the effective radii of the interaction between
two nucleons in the triplet and singlet state, respectively.
In the independent of the nucleon-nucleon interaction range
approximation (INNIRA) we must fix \( r_0^t = 0 \) and \( r_0^s = 0 \).
The function \( f_d(p, p_c) \) decreases rapidly when \( p > p_c \) and
provides a cut-off for the kernels and solution of integral
equations (1) at large momenta, \( p_c \) being a phenomenological
parameter. In particular, in the sharp cut-off model, considered
in \( \theta \), this function equals
\[
f_d(p, p_c) = \Theta(p_c - p) = \begin{cases} 1 & \text{if } p < p_c \\ 0 & \text{if } p > p_c \end{cases} \tag{2}
\]
(But therein \( \theta \) there was a remark, that the cut-off function
could be a smooth one).
In our calculations this function has been taken in form (2)
as well as in the following ones:
\[
\exp(-p/p_c^2); \exp(-p/p_c^2); \frac{p_c^2}{p^2 + p_c^2},
\]
The dependence of the Saxon-Woods type also has been used for
\( f_d(p, p_c) \) with different "smearing width" values of \( \delta \).
The homogeneous equations for the three nucleon bound state
with an energy \( -\varepsilon_0 \) are as follows:
For the doublet state of the three nucleon system we have
C_{11} = C_{22} = 1/4; C_{12} = C_{21} = 3/4. The quartet nd-scattering
is described by (1) with C_{11} = -1/2; C_{22} = C_{12} = C_{21} = 0.
Equations (1) and (3) could correspond to the system of three
identical spinless particles provided one puts C_{11} = 1;
C_{22} = C_{12} = C_{21} = 0 in these equations.

Into equation (1) and (3) there enter constants originated
from the two-body interactions as well as the extra parameters,
characterized by the fashion f_d (p, p', p_c) of cutting the integral
equation kernels at large momenta. These extra parameters of the
model in question could be defined through the use of experimental
characteristics of the three particle system.

The experimental dependence of value of $k\cot\delta_{2}$
on $k^2$ for the neutron-deuteron S-wave doublet scattering

\[ k\cot\delta_{2} = -A + Bk^2 - C/(1+Dk^2) \]

which has a pole just below the elastic scattering threshold

In the model in question, with the cut-off, for instance,
taken in form (2), the values of $k\cot\delta_{2}$ at different
energies $k^2$ of the incident neutron increase with increasing
cut-off parameter $p_c$ (the range of changing the parameter
allows here the three nucleon system to have just one bound state
with an energy to be close to the triton energy). The monotone
character of curve (4) at $k^2=0$ (Fig. 1) gives us a possibility
to say that a small increase of the parameter $p$ produces a small
shift of curve (4) as a whole to the left, along the energy
axis $k^2$ (correctly, it should be said in the opposite direction to
this axis). A small increase in the parameter $p$ also produces a
can be well fitted by the curve (Fig. 1):

\[ k\cot\delta_{2} = -A + Bk^2 - C/(1+Dk^2) \]
In accordance with these reflections we choose here the cut-off parameter $p_c$ to reproduce exactly the experimental value of the doublet nd-scattering length $a_2 = 0.65 F^9$. This choice is expected to have just small divergence of the calculated triton binding energy off the experimental one, at the same time the calculated low energy S-wave phase shifts better fitting the experimental ones in the neutron-deuteron scattering.

In calculations we have used the following forms of the cut-off factor $f_d(p, p_c)$ in equations (1) and (3):

\begin{align}
(1) & \quad f_d (p, p_c) = \Theta (p_c - p), \\
(II) & \quad f_d (p, p_c) = \exp (-p / p_c^2), \\
(iii) & \quad f_d (p, p_c) = \exp (-p / p_c), \\
(iv) & \quad f_d (p, p_c) = \frac{p_c^2}{p_c^2 + p^2}, \\
(v) & \quad f_d (p, p_c) = \frac{N}{1 + \exp \left( \frac{p - p_c}{d} \right)}.
\end{align}

In item (v) the normalization constant $N$ is conditioned by

$$
\left. f_d (p, p_c) \right|_{p=0} = 1.
$$
The calculated values of \( S_\frac{1}{2} \) do not depend practically on cut-off fashion (5) and in the INNIRA the values of \( S_\frac{1}{2} \) below the deuteron break-up threshold do not depend practically on cut-off fashion (5). The values of \( S_\frac{1}{2} \) in the INNIRA and LNNIRA for different cut-off functions (5) have been determined through the experimental values of the doublet scattering length, just one bound state being demanded to exist in the three nucleon system. The binding energy \( E \) of the doublet scattering length \( a = 0.65 \) \( \, \text{g} \) in expressions (5) have been determined through the experimental value of the doublet scattering length, just one bound state being demanded to exist in the three nucleon system. The binding energy \( E \) of this state was found with the help of Eq. (5) with the parameter \( P_c \) fixed as described just before. This energy was expressed in the triton binding energy in the second and fourth columns of Table I. In the interval \( [0.05, 2.0] \, \text{g}^{-1} \), the calculated values of \( S_\frac{1}{2} \) fit the experimental ones very well, the calculated triton binding energy values exceeding the experimental ones. In the interval \( [2.0, 5.0] \, \text{g}^{-1} \), the calculated triton binding energy values exceeding the experimental ones.

### Table I

<table>
<thead>
<tr>
<th>Cut-off type (5)</th>
<th>( r_0^a = 1.940 )</th>
<th>( E_r )</th>
<th>( P_c )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>1.272</td>
<td>8.217</td>
<td>0.898</td>
<td>9.189</td>
</tr>
<tr>
<td>(ii)</td>
<td>1.980</td>
<td>8.375</td>
<td>1.287</td>
<td>9.705</td>
</tr>
<tr>
<td>(iii)</td>
<td>3.250</td>
<td>8.187</td>
<td>2.314</td>
<td>10.361</td>
</tr>
<tr>
<td>(iv)</td>
<td>3.650</td>
<td>8.402</td>
<td>1.367</td>
<td>9.896</td>
</tr>
<tr>
<td>( d = 0.05 , \text{g}^{-1} )</td>
<td>1.925</td>
<td>8.218</td>
<td>0.951</td>
<td>9.208</td>
</tr>
<tr>
<td>( d = 0.1 , \text{g}^{-1} )</td>
<td>1.985</td>
<td>8.220</td>
<td>1.010</td>
<td>9.260</td>
</tr>
<tr>
<td>( d = 1.0 , \text{g}^{-1} )</td>
<td>3.189</td>
<td>8.389</td>
<td>0.928</td>
<td>10.052</td>
</tr>
<tr>
<td>( d = 2.0 , \text{g}^{-1} )</td>
<td>3.519</td>
<td>8.468</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 2. Dependence of $k \cot \delta_2 (r^{-1})$ on $k^2(r^{-2})$

( $\delta_2$ is the doublet $nn$-scattering S-wave phase shift and $k^2$ is the incident neutron energy in the c.m.s.). The open circles are the images of the experimental data $^{11}$. The arrow marks the deuteron break-up threshold position. Curve 1 is the independent of the nucleon-nucleon interaction range approximation (INNIRA). Curve 2 is the linear in the nucleon-nucleon interaction range approximation (LNNIRA).

The considered here model is in fact the result of reducing the three-particle problem of the nucleon-deuteron scattering to the two-particle problem with an effective interaction between the neutron and deuteron. This interaction appears to be non-local and dependent on energy and spin.

We have shown earlier $^{10}$ that the LNNIRA fits well the calculated quartet $nn$-scattering S-wave phase shifts to the experiment below the deuteron break-up threshold.

Cutting factors $^{5}$ do not influence the results of calculations in the quartet case, for the functions satisfying the quartet STM equation decrease rapidly at large momenta. So the model, which is based on the use of the two-nucleon t-matrix in the LNNIRA and on cutting the kernels of the STM integral equations at large momenta would be considered as a method of constructing a quasi-optical potential for the interaction of neutron with deuteron. Such a potential may have at least one phenomenological parameter and sufficiently well fits both the bound state energy and the $nn$-scattering S-wave phase shifts (at least below the deuteron break-up threshold).

On the other hand, in the INNIRA we have good agreement with experiment for the calculated triton binding energy (that was expected from the results of $^8$) but at the same time a certain divergence of the calculated $k \cot \delta_2$ off the experimental data. We should remark that much more sophisticated models not always yield a good value for the triton binding energy, simultaneously reproducing the low energy doublet $nn$-scattering S-wave phase shifts decently.

The experimental ones (according to Table 1). On the other hand, in the INNIRA we have good agreement with experiment for the calculated triton binding energy (that was expected from the results of $^8$) but at the same time a certain divergence of the calculated $k \cot \delta_2$ off the experimental data. We should remark that much more sophisticated models not always yield a good value for the triton binding energy, simultaneously reproducing the low energy doublet $nn$-scattering S-wave phase shifts decently.

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In conclusion the authors thank V.B. Delyaev for useful discussions. One of us (E.G.T.) expresses his deep gratitude to Professor N.A.Perfilov and to Professor O.V.Loshkin for their stimulating interest in the work and their regular support. He also wants to thank Mlle S.K. for her tender attention which urged him intensely until this work was completed.

References:


Received by Publishing Department on November 29, 1974.