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# SPECTRAL PROPERTIES OF KERNELS $h^{0}_{00,00}$ OF INTEGRAL OPERATORS OF THE SYSTEM OF INTEGRO-DIFFERENTIAL FADDEEV EQUATIONS

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## 1. INTRODUCTION

The three-body problem in nonrelativistic quantum mechanics formulated correctly for the first time by L.D.Faddeev [1] is now carefully studied within the framework of the system of integro-differential equations for functions of two independent variables [2,3]

Important results are found in studying the analytic properties of functions satisfying such systems of equations [3-7] and in solving the latter by various numerical methods [8-14]. However, both these studies are essentially complicated by the presence of nonlocal integral operators and by the sought solutions being functions of two variables. For this reason it is natural to make the next step, i.e., to separate independent variables by expanding the sought solutions over a complete set of functions of one variable. As such a basis in the problem of three-particle bound states in the case of s-wave two-body potentials it is convenient to use the systems of eigenfunctions of of kernels $\overset{0}{h}_{00,00}$ of integral operators.

In the problem of three identical particles the use of such functions [15] makes it possible to reduce the initial system of integro-differential equations to the system of second-order differential equations for functions of one variable.

This paper is devoted to the study of spectral properties of kernels $\overset{0}{h}_{00,00}$ in the general case of nonidentical particles. In section 2 the Faddeev integro-differential equations are written in polar coordinates, and the eigenvalues and eigenfunctions are found for the kernels $\overset{0}{h}_{00,00}$. Section 3 deals with the investigation of the eigenvalues and eigenfunctions obtained in Sec.2. In Sec.4 the integro-differential equations are reduced to the system of second-order differential equations for functions of one variable and some properties of solutions of that system are analysed.

## 2. EIGENVALUES AND EIGENFUNCTIONS OF KERNELS $\overset{0}{h}_{00,00}$

Consider a system of three different spinless particles interacting by means of two-body s-wave potentials $v_i$.

Following ref. [3] we introduce three sets of relative coordinates $(\vec{x}_i, \vec{y}_i)$, $i = 1,2,3$, which are expressed in terms of the radius-vectors of particles $\vec{r}_i$ by the formulae

$$\vec{x}_k = (2m_i m_j/(m_i + m_j))^{1/2} (\vec{r}_j - \vec{r}_i),$$
$$\vec{y}_k = (2m_k(m_i + m_j)/M)^{1/2} \left(\frac{m_i\vec{r}_i + m_j\vec{r}_j}{m_i + m_j} - \vec{r}_k\right). \tag{1}$$

Here indices i, j, k run over the values 1,2,3; 2,3,1; 3,1,2; $m_i$ is the mass of an i-th particle, and M is the sum of masses of three particles.

Different sets of coordinates (1) are connected by unitary transformations

$$\begin{pmatrix} \vec{x}_k \\ \vec{y}_k \end{pmatrix} = \sin\gamma_{ki} \begin{pmatrix} \mathrm{ctg}\,\gamma_{ki} & \epsilon_{ki} \\ -\epsilon_{ki} & \mathrm{ctg}\,\gamma_{ki} \end{pmatrix} \begin{pmatrix} \vec{x}_i \\ \vec{y}_i \end{pmatrix} \tag{2}$$

The angles $\gamma_{ki} \in [0, \frac{\pi}{2}]$ are determined by the ratios of particle masses

$$\mathrm{tg}^2\gamma_{ki} = m_j M/m_i m_k = m_j/m_i + m_j/m_k + (m_j/m_i)(m_j/m_k), \tag{3}$$

and numbers $\epsilon_{ki}$ are such that $\epsilon_{ki} = -\epsilon_{ik} = 1$, ik = 21,32,13.

From equalities (2) it follows that the hyper-radius $\rho = (x_k^2 + y_k^2)^{1/2}$ does not depend on the choice of the coordinate set (1), and $x_k$ and $y_k$ are functions of $x_i, y_i$ and the variable $u_i = \cos(\widehat{\vec{x}_i \vec{y}_i})$, i.e.,

$$x_k = x_k(x_i, y_i, u_i), \; y_k = y_k(x_i, y_i, u_i). \tag{4}$$

Following refs. [2,3] we write the wave function of the bound state of three particles with quantum numbers $\lambda = \ell = L = 0$ as a sum of three components $\psi = \psi_1 + \psi_2 + \psi_3$, and represent each of the components $\psi_i$, i = 1,2,3 in the coordinate set $(\vec{x}_i, \vec{y}_i)$ in the form

$$\psi_i(\vec{x}_i, \vec{y}_i) = \frac{\Phi_i(x_i, y_i)}{x_i y_i} \mathcal{Y}_{00}^{00}(\hat{y}_i, \hat{x}_i), \tag{5}$$

where $\mathcal{Y}_{00}^{00}$ is the bispherical harmonic $\mathcal{Y}_{\lambda\ell}^{LM}$ at zero indices.

Substituting functions (5) into the Faddeev equation $(H_0 - E + v_i)\psi_i = -v_i(\psi_j + \psi_k)$ we arrive at the system of equations for components $\Phi_i$ dependent only on two variables

$$\left(\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + E - v_i(x_i)\right)\Phi_i(x_i, y_i) = v_i(x_i) \sum_{k \neq i} \langle x_i, y_i |\hat{h}| \Phi_k \rangle. \tag{6}$$



Here the action of the nonlocal operator $\hat{h}$ is defined as follows:

$$\langle x_i, y_i |\hat{h}| \Phi_k \rangle = \int_{-1}^{1} du_i\, h^0_{00,00}(x_i, y_i, u_i)\, \Phi_k(x_k, y_k). \tag{7}$$

The kernel of this operator

$$h^0_{00,00}(x_i, y_i, u_i) = x_i y_i/(2x_k y_k) \tag{8}$$

is a particular case of the kernel $h^L_{\lambda\ell,\lambda'\ell'}$ at zero indices [3,14]. In equalities (7) and (8) $x_k$ and $y_k$ are functions (4) of variables $x_i, y_i$ and $u_i$. The requirement of regularity of the component $\psi_i$ (5) on the straight lines $x_i, y_i = 0$ is formulated as the boundary conditions

$$\Phi_i(0, y_i) = \Phi_i(x_i, 0) = 0, \; i = 1, 2, 3. \tag{9}$$

The square integrability of the total wave function $\psi$ is sequred by the boundary conditions

$$\Phi_i(\infty, y_i) = \Phi_i(x_i, \infty) = 0, \; i = 1, 2, 3. \tag{10}$$

Let us introduce three sets of polar coordinates

$$0 \le \rho \le \infty, \; \phi_i = \mathrm{arctg}(y_i/x_i), \; 0 \le \phi_i \le \frac{\pi}{2}, \; i = 1, 2, 3. \tag{11}$$

and from formulae (2) we obtain the connection between different angular variables:

$$\mathrm{tg}^2\phi_k(\phi_i, u_i) = (\mathrm{tg}^2\gamma_{ki} + \mathrm{tg}^2\phi_i - \eta_{ki})/(1 + \mathrm{tg}^2\gamma_{ki}\,\mathrm{tg}^2\phi_i + \eta_{ki}), \tag{12}$$

where $\eta_{ki} = 2\epsilon_{ki}\,\mathrm{tg}\,\gamma_{ki}\,\mathrm{tg}\,\phi_i u_i$.

System of equations (6) in coordinates (11) is then written in the form

$$[\Delta_i + E - v_i(\rho\cos\phi_i)]\,\Phi_i(\rho, \phi_i) = v_i(\rho\cos\phi_i) \sum_{k\neq i} \langle \rho, \phi_i |\hat{h}| \Phi_k \rangle, \tag{13}$$

where

$$\Delta_i = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial}{\partial\phi_i^2}, \; \Phi_i(\rho, \phi_i) \equiv \Phi_i(\rho\cos\phi_i, \rho\sin\phi_i).$$

Equalities (9) and (10) are reduced to the following boundary conditions

$$\Phi_i(\rho, 0) = \Phi_i(\rho, \frac{\pi}{2}) = 0, \; 0 \le \rho \le \infty, \tag{14}$$

$$\Phi_i(0, \phi_i) = \Phi_i(\infty, \phi_i) = 0, \quad 0 \le \phi_i \le \frac{\pi}{2}, \; i = 1, 2, 3. \tag{15}$$

From (7) and(8) we obtain the representation

$$<\rho, \phi_i |\hat{h}| \Phi_k> = \frac{1}{2} \int_{-1}^{1} du_i \frac{\sin 2\phi_i}{\sin 2\phi_k} \Phi_k(\rho, \phi_k). \tag{16}$$

In integrals (16) the angle $\phi_i$ is function (12) of the integration variable $u_i$ and angle $\phi_i$, whose value is fixed in the left-hand side of eq. (13). In this integral we shall make the change of variables $u_i \to \phi_k$.

Upon solving eq. (12) with tespect to the variable $u_i$ we calculate the Jacobian

$$du_i(\phi_i, \phi_k)/d\phi_k = 2\epsilon_{ki} \operatorname{coses} 2\gamma_{ki} \sin 2\phi_k / \sin 2\phi_i. \tag{17}$$

The upper $b_{k+}$ and lower $b_{k-}$ integration limits over the variable $\phi_k$ will be found from eq. (12) written in the form

$$\operatorname{tg}^2\phi_k(\phi_i, u_i = \pm 1) = \operatorname{tg}^2 b_{k\pm}(\phi_i) = \operatorname{tg}^2(\epsilon_{ki}\gamma_{ki} \mp \phi_i). \tag{18}$$

The condition $0 \le \phi_k \le \frac{\pi}{2}$ selects the only solutions to eq. (18) expressed in terms of the functions

$$c_k(\phi_i) = |\gamma_{ki} - \phi_i|, \quad d_k(\phi_i) = \min(\gamma_{ki} + \phi_i, \pi - \gamma_{ki} - \phi_i) \tag{19}$$

by the formulae

$$b_{k+} = d_k, \; b_{k-} = c_k, \; \epsilon_{ki} > 0; \quad b_{k+} = c_k, \; b_{k-} = d_k, \; \epsilon_{ki} < 0. \tag{20}$$

Functions (19) for some values $\gamma_{ki}$ are drawn in Fig. 1.

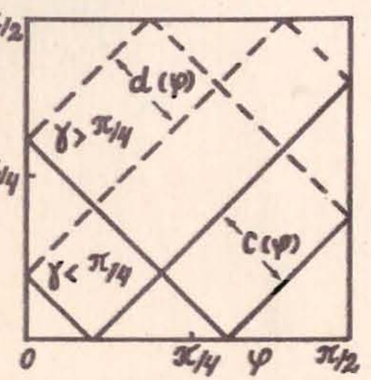


*Fig. 1*

Making use of (17), (19), (20) we rewrite (16) in the form

$$<\rho, \phi_i |\hat{h}| \Phi_k> = \operatorname{coses} 2\gamma_{ki} \int_{c_k(\phi_i)}^{d_k(\phi_i)} d\phi_k \Phi(\rho, \phi_k). \tag{21}$$

Thus, the operator $\hat{h}$ in polar coordinates (11) acts only on the angular variable and is represented by an integral with variable limits and contains as a parameter the angle $\gamma$ whose values are fixed by the ratio of particle masses (3).



Let us now proceed to study the spectral properties of the operator $h^0_{00,00}$ (all indices are omitted). We look for the solutions $y(\phi)$ to the equation

$$\lambda y = \hat{h} y, \quad \lambda y(\phi) = \operatorname{coses} 2\gamma \int_{c(\phi)}^{d(\phi)} d\xi y(\xi) \tag{22}$$

in the class of functions continuous on the interval $0 \le \phi \le \pi/2$ satisfying the Dini condition /16/ and the same boundary conditions (14) that hold for the components $\Phi$, i.e., $y(0) = y(\pi/2) = 0$. We shall denote the above class of functions by $K_{[0, \pi/2]}$.

Theorem I. In the class of functions $K_{[0, \pi/2]}$ all solutions to eq. (22) are of the form $y(\phi) = \sin 2n\phi$ and compose an orthogonal basis with the corresponding eigenvalues $\lambda_n(\gamma) = \sin 2n\gamma/(n \sin 2\gamma)$, where $n = 1, 2, \ldots$

We shall continue an arbitrary function $y(\phi) \subset K_{[0, \pi/2]}$ on the interval $[\pi/2, \pi]$ thus obtaining the function $\tilde{y}(\phi) \in K_{[0, \pi]}$ with the following properties

$$\tilde{y}(\phi) = y(\phi), \tag{23}$$

$$\tilde{y}(\frac{\pi}{2} + \phi) = -\tilde{y}(\frac{\pi}{2} - \phi), \tag{24}$$

where $0 \le \phi \le \pi/2$. On the interval $[0, \pi]$ we introduce a complete orthogonal system of functions $1$, $\cos 2n\phi$ and $\sin 2n\phi$, where $n = 1, 2, \ldots$ . Then $y(\phi)$ may be represented by the uniformly convergent series

$$\tilde{y}(\phi) = \sum_{n=1}^{\infty} b_n \sin 2n\phi, \tag{25}$$

which may be integrated term by term. According to (23) and (24) the coefficients equal

$$b_n = \frac{2}{\pi} \int_0^{\pi} d\phi \tilde{y}(\phi) \sin 2n\phi = \frac{4}{\pi} \int_0^{\pi/2} d\phi y(\phi) \sin 2\phi,$$

and coefficients of the expansion over the functions $\cos 2n\phi$, $n = 0, 1, \ldots$ are zero owing to (24). We look for the solutions to eq. (22) in the form (25) and with account of (23) we get

$$\sum_{n=1}^{\infty} b_n(\lambda - \sin 2n\gamma/(n\sin 2\gamma))\sin 2n\phi = 0.$$

with the following solutions: $b_n \neq 0$, $b_m = 0$, $m \neq n$, $n = 1,2,\ldots$ The eigenvalues

$$\lambda_n = \lambda_n(\gamma) = \sin 2n\gamma/(n\sin 2\gamma) \qquad (26)$$

depend on the parameter $\gamma$, and the corresponding eigenfunctions are determined up to arbitrary factor $b_n$ and have the form $y_n(\phi) = b_n \sin 2n\phi$. These functions form the basis in class $K[0,\pi]$, and consequently, in class $K[0,\pi/2]$.

Note that the eigenfunctions have a definite parity with respect to point $\phi = \pi/4$: $y_{2m+2}$ are odd, and $y_{2m+1}$ are even for all $m = 0,1,2,\ldots$

## 3. PROPERTIES OF EIGENVALUES OF KERNELS $h^0_{00,00}$

Let us study the properties of eigenvalues (26) as a function of the parameter $0 \leq \gamma \leq \pi/2$. Clearly, $\lambda_n(\gamma) = 0$ if there exists an integer $\ell$, $\ell = 1,\ldots, n-1$ such that $\gamma = \frac{\pi}{2}\frac{\ell}{n}$, i.e., if $q = \gamma/(\pi/2)$ is a proper rational fraction.

The latter holds valid if the particle masses obey the equality

$$\mathrm{tg}\,\gamma = \mathrm{tg}\,\gamma_{ki} = (m_j M/m_i m_k)^{1/2} = \mathrm{tg}\frac{\pi}{2}q. \qquad (27)$$

Let $n_0$ is a denominator of the fraction $q$, then zero is a multi-degenerated eigenvalue of eq. (22), ($\lambda_n(\gamma) = 0$ for all $n$ multiple to $n_0$), and the corresponding eigenfunction is an arbitrary linear combination of the functions $\sin 2n_0 m\phi$, where $m = 0,1,\ldots$

Consider two examples. Let masses of particles be equal, then $\gamma = \pi/3$, $n_0 = 3$. To the eigenvalues $\lambda_{3m}(\pi/3) = 0$ the eigenfunctions $\sin 6m\phi$ correspond. To the remaining eigenvalues

$$\lambda_{1+3m}(\frac{\pi}{3}) = (1+3m)^{-1}, \quad \lambda_{2+3m}(\frac{\pi}{3}) = -(2+3m)^{-1}$$

there correspond the functions $\sin 2(1+3m)\phi$ and $\sin 2(2+3m)\phi$ where $m = 0,1,\ldots$ . In this case eq. (22) is equivalent to the equation $\lambda^2 y''(\phi) + 4y(\phi) = 0$; $y(0) = y(\pi/2) = 0$, provided $\lambda \neq 0$, which may be obtained following ref. [17]. Now let masses of particles be related by the conditions $m_i > m_j$, $m_k = m_j(m_i+m_j)/(m_i - m_j)$ then $\gamma = \gamma_{ki} = \pi/4$, $n_0 = 2$. To the zero eigenvalues $\lambda_{2m}(\pi/4) = 0$ there correspond the functions $\sin 4m\phi$ odd with respect to the point $\phi = \pi/4$ and to others $\lambda_{2m-1}(\frac{\pi}{4}) = \frac{(-1)^{m+1}}{2m-1}$ even functions $\sin 2(2m-1)\phi$. For odd values of $n = 2k+1$, $k = 0,1,\ldots$ the eigenvalues are expressed in terms of the Dirichle kernels [16]:

$$\lambda_{2k+1}(\gamma) = \frac{2\pi}{2k+1}D_k(4\gamma) = \frac{1}{2k+1}(1 + 2\sum_{m=1}^{k}\cos 4m\gamma) \qquad (28)$$

and their sequence satisfies the limit

$$\lim(2k+1)\lambda_{2k+1}(\gamma) = 2\pi\delta(\gamma). \qquad (29)$$

It is not difficult to obtain appropriate recurrence relations

$$\lambda_{n+2}(\gamma) = \frac{n}{n+2}\lambda_n(\gamma) + \frac{2}{n+2}\cos 2(n+1)\gamma, \; n = 0,1,\ldots \qquad (30)$$

and representations for eigenvalues for even $n$:

$$\lambda_{2k}(\gamma) = \frac{1}{k}\sum_{m=1}^{k}\cos(4m-2)\gamma, \quad k = 1,2,\ldots \qquad (31)$$

From definition (3) written in the form

$$\mathrm{tg}^2\gamma = \alpha + \beta + \alpha\beta, \qquad \alpha = m_j/m_i, \quad \beta = m_j/m_k$$

it follows that $\gamma \to 0$ if $\alpha, \beta \to 0$; $\gamma \to \pi/2$ if $\alpha \to \infty$ or $\beta \to \infty$. Consequently, at finite and nonzero masses of particles the parameter $\gamma$ satisfies the inequality $0 < \gamma < \pi/2$. Note that $\lambda_n(0) = 1$, $\lambda_n(\pi/2) = (-1)^{n+1}$ for all integer $n$ and prove by induction the following

Theorem II. Provided that $0 < \gamma < \frac{\pi}{2}$ the inequalities

$$|\lambda(\gamma)| < 1, \quad n = 2,3,\ldots \qquad (32)$$

are valid.

Clearly, $|\lambda_2(\gamma)| = |\cos 2\gamma| < 1$, let $|\lambda_n(\gamma)| < 1$ for an even $n > 2$, i.e., $\epsilon - 1 < \lambda_n(\gamma) < 1 - \epsilon$, where $0 \leq \epsilon \leq 1$. From (30) we obtain the upper bound $|\lambda_{n+2}(\gamma)| < 1 - n\epsilon/(n+2) < 1$ that proves the theorem. For odd $n$ the proof is similar. Let $K_1 \subset K[0,\pi/2]$ be the class of functions orthogonal to the first eigenfunction $y_1(\phi) = \sin 2\phi$ corresponding to the eigenvalue $\lambda_1(\gamma) = 1$.

According to theorem II, for any physical values of masses of particles $0 < m_i < \infty$, $i = 1,2,3$ the operator $\hat{h}$ in the class $K_1$ is a contracting mapping [16]. The iteration sequence of functions $z_{k+1} = \hat{h}z_k + f$, where $z_0$ and $f$ are arbitrary functions from the class $K_1$, converges in metrices $L_2$ to the exact solution $z(\phi)$

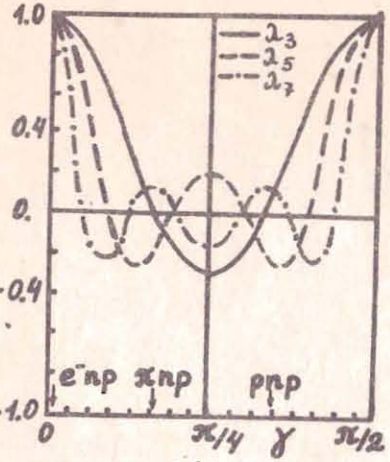


Fig.2

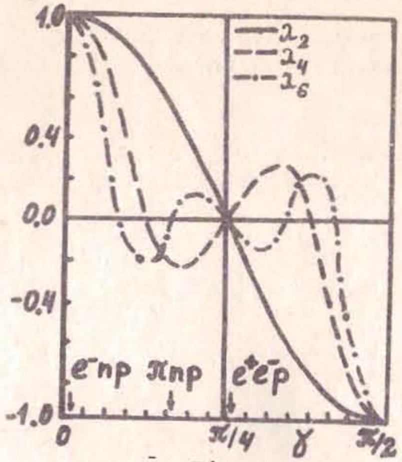


Fig.3

of the inhomogeneous equation $z(\phi) = \cos es\, 2\gamma \int_{c(\phi)}^{d(\phi)} d\xi\, z(\xi) + f(\phi)$. Let us mention one more property:

$$\lim_{n\to\infty} \lambda_n(\gamma) = 0, \quad 0 < \gamma < \frac{\pi}{2}. \tag{33}$$

Some eigenvalues as functions of the parameter $\gamma$ are drawn in Fig.2 and 3. Arrows mark the values of $\gamma = \gamma_{ki}$ for some three-particle systems when $m_j < m_i, m_k$.

## 4. CONCLUSION

We shall search the solution to system (13) in the form

$$\Phi_i(\rho, \phi_i) = \sum_{n=1}^{\infty} \rho^{-1/2} f_{in}(\rho) \sin 2n\phi_i, \quad i = 1, 2, 3. \tag{34}$$

For the functions $f_{in}(\rho)$ we may readily find the system of second-order differential equations

$$f''_{in}(\rho) + (E + (1-16n^2)/4\rho^2) f_{in}(\rho) - \sum_{m=1}^{\infty} v_{inm}(\rho) f_{in}(\rho) =$$

$$= \sum_{m=1}^{\infty} v_{inm}(\rho)\,(\lambda_m(\gamma_{ji}) f_{jm}(\rho) + \lambda_m(\gamma_{ki}) f_{km}(\rho)), \quad n = 1, 2, \ldots \tag{35}$$

and the boundary conditions $f_{in}(0) = f_{in}(\infty) = 0$, $n = 1,2,\ldots$ generated by the conditions (15).



Instead of the exact and, generally, infinite systems of eqs. (35) one may consider a finite and approximate system of equations ($n, m = 1, \ldots N$) for the following three reasons. Matrix elements of the two-body nonsingular potential

$$v_{inm}(\rho) = \frac{\pi}{4} \int_0^{\pi/2} d\phi \sin 2n\phi\, v_i(\rho\cos\phi) \sin 2m\phi$$

are decreasing [15] with increasing $|n-m|$ not slower than $|n-m|^{-1}$. The eigenvalues $\lambda_m \to 0$ for $m\to\infty$ (33), and the functions $\rho^{-1/2} f_{im}(\rho)$ should decrease not slower than $m^{-2}$ with increasing m as to provide the continuity of the second partial derivative with respect to the angular variable of the component (34). The terms in the r.h.s. of system (35) describing the influence of the third particle on the couple of particles decrease with $m\to\infty$ faster than the sum of terms in the left-hand side. We point out some properties of a three-body system following from the representation (34). Substituting (35) into (5) we arrive at the following representation of the Faddeev component:

$$\psi_i(\rho, \phi_i) = (8\pi)^{-1} \rho^{-5/2} \sum_{n=1}^{\infty} n f_{in}(\rho)\, \omega_n(\phi_i), \tag{36}$$

which results in the known [4,5] asymptotic behaviour

$$\psi_i(\rho, \phi) \underset{\rho\to\infty}{\longrightarrow} O(\rho^{-5/2}).$$

The functions $\omega_n(\phi) = \sin 2n\phi/(n \sin 2\phi)$ have the same dependence on the angle $\phi$ as the eigenvalues on the parameter $\gamma$ and, consequently, possess the same properties (28-33). For the total wave function, all owing for (5), (16), (22) and (36) we obtain the expression

$$\psi(\rho, \phi_i) = (8\pi)^{-1} \rho^{-5/2} \sum_{n=1}^{\infty} n\omega_n(\phi_i) \sum_{k=1}^{3} \sigma_{nik} f_{kn}(\rho), \tag{37}$$

where $\sigma_{nii} = 1$, $\sigma_{nik} = \lambda_n(\gamma_{ki})$, $k \neq i$. The wave function (37) does not contain terms $\sim f_{km}(\rho)$, where m are integers obeying equalities (27), i.e., such that $\lambda_m(\gamma_{ki}) = 0$. This kinematic property is specific for all three-body systems masses of which obey conditions (27). The system of equations (35) may have nontrivial solutions noncontributing to the total wave function (37). These solutions satisfy the condition $\sum_{k=1}^{3} \sigma_{2ik} f_{k2}(\rho)$ and will be studied elsewhere.

In conclusion we note once again that the eigenfunctions of kernels $h^0_{00,00}$ do not depend on the particle masses unlike the eigenvalues. In the class of functions in which we look for the solution of the Faddeev equation in the case of the bound state with $\lambda = \ell = L = 0$ the eigenfunctions form the angular basis. Thus, we believe that the use of the results obtained will greatly simplify the analysis of solutions of the configuration formulation of the three-body problem.

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Пупышев В.В. E4-84-808

Спектральные свойства ядер $h^{0}_{00,00}$ интегральных операторов системы интегро-дифференциальных уравнений Фаддеева

Для системы трех различных частиц найдены и исследованы собственные значения и функции ядер $h^{0}_{00,00}$ интегральных операторов, содержащихся в интегро-дифференциальных уравнениях Фаддеева. Показано, что в задаче на связанные трехчастичные состояния, в случае s-волновых парных потенциалов, такие уравнения сводятся к системам дифференциальных уравнений второго порядка для функций одной переменной.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Pupyshev V.V. E4-84-808

Spectral Properties of Kernels $h^{0}_{00,00}$ of Integral Operators of the System of Integro-Differential Faddeev Equations

For a system of three different particles we have found and studied eigenvalues and eigenfunctions of kernels $h^{0}_{00,00}$ of integral operators in the integro-differential Faddeev equations. It is shown that in the problem on bound three-particle states with s-wave two-body potentials these equations reduce to systems of second-order differential equations for the function of one variable.

The investigation gas been performed at the Laboratory of Theoretical Physics, JINR.

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